Journal of Mathematics and Modeling in Finance (JMMF)
Vol. 1, No. 1, Winter & Spring 2021
Research paper

Impacts of no short selling and noise reduction on portfolio allocation

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Abstract:
Noise presence in financial series, often as a result of existence of fraudulent transactions, arbitrage and other factors, causes noise in financial data, which will lead to a false estimation of the parameter and hence distorts the portfolio allocation strategy. In this paper, wavelet transform is used for noise reduction in mean-variance portfolio theory. I apply conditional estimation of the mean and variance of returns along with the simple one obtaining "optimal weights" is applied which later combines with smooth and non-smooth series, result in four optimal portfolio weights and therefore four portfolio returns. After this, the non-negativity constraint (for weights) deduced from the Kuhn-Tucker approach is imposed to Exchange. Weights and portfolio returns changed dramatically in this step but the main result (which asset to hold) did not. Comparing Sharp ratios, it is observed that Regardless of the psychological characteristics of the investor, holding the risk-free asset is almost the optimal choice in this case.

Keywords:: wavelet transform, weigh matrix, smooth, covariance matrix.
JEL Classification: C53, G10, G11, G17

1 Introduction

It has been said that the goal of any government is to reach a higher rate of economic growth (Tobin, 1964) which is the key to higher levels of welfare. Investment is one of the most important roads for increasing GDP and economic growth because of the necessities of infrastructures and facilities which only investment can bring. It is important to any investor to be able to choose the correct allocation of wealth and have the opportunity to profit the most from his investment while minimizing the risk associated with it. Stock markets are one of the most important institutions for both individuals and corporations to take part in, in order to reach their goals (maximizing profit and financing projects). To benefit from the diversification, investors normally invest in a portfolio consists of risky and risk-free securities. And for more discretion, they usually choose securities that are less dependent to each other. This idea seems very logical and simple now but when Harry Markowitz

[1]Corresponding author
Received: 2020-09-05 Approved: 2020-11-20
http://dx.doi.org/10.22054/jmmf.2020.55023.1015

(1952) published his paper of Portfolio Selection it was an interesting idea which changed the financial world from then on. It was in Markowitz's work (1959) and (1952) that the distinction between asset risk and portfolio risk was clarified for the first time; He is known as the founder of modern portfolio theory, as Rubinstein (2002) argues, because the idea of diversification was explained in a mathematical approach for the first time in his paper of Portfolio Selection. His theory is also known as the Mean Variance Portfolio Theory and it leads to the optimal portfolio which is in fact based on the expected return of each security, variance of securities, and correlation among securities. In fact, the classical framework of modern portfolio theory is based on the assumption that the investor only cares about the first two moments of the return distribution: mean and variance (Rigamonti, 2020). Markowitz believed that an investor should:

"Consider the expected return a desirable thing and the variance of return an undesirable thing."

In other words, at a given risk one should maximizes the profit and for a given profit one should minimizes the risk. The most important point in his work was that an asset cannot be chosen only because of its own characteristics, in fact co-movements and correlations among securities must also be taken into account and a portfolio must be built based on mean and variance of the portfolio not on the one of the very single asset individually. When combining the portfolio of interest, we need to estimate the efficient weights, choosing a combination of assets and their fraction, in a way that as possible maximizes portfolio return and minimizes its risk. Using mean and variance of the portfolio is just due to its simplicity and if we use other moments of return, like skewness, it would help the description of the return distribution to be more realistic but we are not sure about the efficiency of the final portfolio and that's why the mean variance portfolio theory is still the basis for new theories.

In order to estimate the expected return and variance of securities, one can apply many approaches. One approach is based on the historical mean and variance of securities. Since calculating the correlation coefficients in large portfolios is very inefficient, and leads to a very huge correlation matrix, estimation of correlation coefficients among securities is difficult (Behradmehr,2010). To palliate this problem, Elton and Gruber (1973) propose several efficient models for computing correlation coefficients between securities and full historical model, using past data to estimate future correlation coefficients, is one of them.

These models made estimating the correlation coefficients among a set of securities, more efficient however Laloux, Cizeau, Bouchaud, and Potters (1999) showed that high amounts of noise are present in covariance and correlation matrix estimations and this renewed the quest for better modeling. They argue that they can treat an empirical correlation as a random matrix since the amount of noise is high. Therefore, as they note, one should take into consideration the effect of noise present in the empirical correlation and covariance of the financial models (like mean variance portfolio theory). So, for obtaining more accurate results, one could minimize the noise present in the financial series.

Every market has its own characteristics and Tehran Stock Exchange (TSE) from which my data sets are obtained, is no exception. For example, exerting a price limit is one of these characteristics which can affect statistical characteristics of the financial series and hence the portfolio allocation decisions. Some kind of regulatory limitations related to short-selling is another difference between TSE and other stock markets. Short-selling, selling the asset we do not have in order to benefit from further price reduction in the future, is one of the commonly used financial instruments which causes the weights to be negative in estimations. In most of the papers the weights are automatically set to be some specific amount (mostly a positive amount) but we are going to investigate in how imposing the no short selling constraint affect efficient weight matrices and hence the combination of the efficient portfolio.

In this paper the wavelet transform is applied to smooth the financial series and minimizing the noise. Then the effect of noise reduction on the mean, correlation and variance matrices of the financial series and therefore, the effect of noise reduction on the allocation of the portfolios based on the mean-variance portfolio theory is studied. Furthermore, to investigate the main source of changes in allocation of portfolios, different combinations of raw and smooth series and simple and conditional estimations are applied to acquire the mean and variance used in estimating the optimal portfolio. To investigate the impact of no short-selling, weight constraints are imposed and the study is done with and without weight constraints using Kuhn-Tucker approach. Results reveal that regardless of the psychological characteristics of the investor, holding the risk-free asset is almost the optimal choice in this case. The remainder of this paper is organized as follows. Section 2 discusses wavelet transform, and reviews proper financial literature relating to wavelet transform. Section 3 elaborates the methodology used in employing the wavelet transform to increase the portfolio return and how weight constraints were imposed using Kuhn-Tucker approach. Section 4 describes the data sets as well as the evaluation results following by a conclusion in Section 5.

2 Wavelet Transform

It has been well appointed that by representing time series in other domains (i.e. frequency, wavelet, Z transform, etc.), certain characteristics which are invisible in the time domain are highlighted. Such characteristics can be used to better understand the underlying time series. For example, it is difficult to describe a complex time series created by the superposition of a few sine series. However, by illustrating this complex time series in the frequency domain, one could observe simply the frequency of the sinusoidal components, which form such a series. With wavelet analysis and by observing frequency characteristics in different time resolutions, we obtain an additional level of insight into the characteristics of the signal. This is contrary to simple frequency analysis in which one observe the frequency characteristics over the entire time series (Behradmehr, 2010). This very wavelet transform of course does not change the information of the time series (signal) and for this reason, wavelets are considered a powerful tool for time series analysis as observed by Ramsey (1999) and Ramsey (2002). Fourier transform is considered to be the basis for the rest of the transforms. Using Fourier transform our information (data) would be a function of frequency and therefore there would be no time information so this transform is better to be used for nonstationary time series. However, with wavelet transform we have the analysis of different frequencies at different scale (Resolution) using multi-resolution techniques so the analysis of the time series which are nonstationary in different frequencies will be done with high efficiency. In the following section I point out the basic concept of wavelet transform and after that some of the related work in the economic literature which have employed wavelets and weight analysis will be discussed.

2.1 Wavelet Theory

Unlike the Fourier transform, where sine is the only basis function, there are many wavelet basis functions with different shapes all of which are compactly supported with finite energy. Wavelet transform converts a time series to the frequency domain, using these very basis functions, and represents the series at different time and scale resolutions. As noted above, these characterizes make dealing with nonstationary and transient series possible for wavelet transform in addition to the ability to decompose time series to different components at different scales. The basis function is called the mother wavelet and other bases are obtained from the dilation (size) and the translation (location) of the mother wavelet. For a continues time series, one would employ the continuous wavelet transform (CWT). According to Kvasnicka (2015), the most used wavelet transforms are Continuous Wavelet Transform (CWT), Discrete Wavelet Transform (DWT) and Maximum Overlap Discrete Wavelet Transform (MODTW). Genay et.al. (2001) represent the continuous wavelet transform, W(u, s) as:

$$W(u,s) = \int_{-\infty}^{+\infty} x(t)\psi_{u,s}(t)dt \tag{1}$$

and the continuous mother wavelet, $\psi_{u,s}$, as:

$$\psi_{u,s} = \frac{1}{\sqrt{s}}\psi(\frac{t-u}{s}) \tag{2}$$

As shown in equation 1, continuous wavelet transform $W(u,s)$, a projection of time series $x(t)$ on to the basis wavelet (i.e.$\psi_{u,s}$), is a function of two continuous variables s and u. The size of wavelet (dilation) is described by the parameter s, and u is a parameter for its translation (location). The wavelet transform decomposes the time series into a different scale and time resolution components by obtaining the wavelet with different dilation and translation values. There are two kinds of basis functions for wavelets; scaling function also known as father wavelet (ϕ) and mother wavelets (ψ). Equations 3 and 4 show father and mother wavelets for continuous wavelet, respectively.

$$\Phi_{J,K} = s^{-\frac{J}{2}}\Phi(\frac{t-s^J k}{s^J}) \tag{3}$$

$$\Psi_{j,k} = s^{-\frac{j}{2}}\Psi(\frac{t-s^j k}{s^j}), j=1,..J \tag{4}$$

Discrete wavelet transform is more useful because most financial series are in discrete form. Putting S=2 in equations 3 and 4 , father and mother wavelets (equations 5 and 6) for discrete wavelet are obtained

$$\varphi_{j,k}(t) = 2^{\frac{-j}{2}}\varphi(2^{-j}t-k) \quad j,k \in Z \tag{5}$$

$$\psi_{j,k}(t) = 2^{-\frac{j}{2}}\psi(2^{-j}t-k) \quad j,k \in Z \tag{6}$$

where j is the index for dilation (size) and scale parameter in the Jth level of decomposition and k is the index for translation (location) of the wavelet. For example, as j increases, the wavelet becomes more compact (i.e. smaller in length), hence the time resolution will increase since smaller time durations are analyzed. Apparently, s parameter is the difference between wavelet and fourier analysis; changing j in $2^{\frac{-j}{2}}$, suitable scale will be obtained (Abassi Nejad & Mohammadi, 1385). The set of two dimensional discrete wavelet transform coefficients, $d_{j,k}$, can be obtained by the inner product of series x(t) and mother wavelet $\psi_{j,k(t)}$, as in equation 7:

$$d_{j,k} =< X(t), \psi_{j,k} >= \int X(t)\psi_{j,k}(t)dt \tag{7}$$

Furthermore, Mallat (1989) proposes the multiresolution analysis, applying which a time series can be decomposed to an approximation and detailed components at various resolutions. In order to demonstrate multiresolution analysis of a time series, besides the mother (basis) wavelet function, which capture the detailed component, one need another function to capture the approximation component and scaling function, $\varphi_{j,k(t)}$ in equation 5, represented by Burrus and Gopinath (1997), is that another function. Scaling functions always satisfy the following condition:

$$\varphi_{j,k}(t) = 1 \tag{8}$$

Therefore, any square integrable function, $g(t) \in L2(R)$, can be expressed as a combination of the scaling functions and mother wavelets.

$$g(t) = \sum c_{j.}(k)\Phi_{c_{j.}k}(t) + \sum_k \sum_{j=j.}^{\infty} d_{j.}(k)\Psi_{c_{j.}k}(t) \tag{9}$$

where $dj(k)$ represents the detailed coefficients (noisy) and $cj(k)$ donates the approximation coefficients (smooth). So far, I have presented a brief review of wavelet transforms and different functions. In what follows in this section, a review of the corresponding economic and financial literatures that have employed wavelet transform and weight analysis as part of their work, will be conducted.

2.2 Related works

Wavelets and the idea of noise reduction using different functions, have been employed in a number of research literature. In this research wavelet transform is used as a tool for data preparation. On the contrary, the number of researches investigating on the impact of weight on the results, is really limited. In what follows, I first review related literatures in which the wavelet transform is used and afterwards, I cover works in which weight analysis have been employed.

Wavelet Analysis

Understanding the relation between variables in economic and financial models has always been important. Wavelet transform can provide researchers with a tool to understand the relation between variables in the short and the long periods using the ability to decompose the time series into different components.

Ramsey (1999) notes that we can use wavelets for noise smoothing and denoising but denoising is done by thresholding the wavelet coefficients before reconstructing the time series which is in contrast to smoothing. In smoothing only the smooth coefficients are used to reconstruct the time series and the detailed coefficients are removed. Ramsey notes that when the underlying time series includes regime shifts and discontinuities, denoising would be the better approach rather than smoothing.

Dajcman (2013) investigate in the Interdependence Between Some Major European Stock markets using a Wavelet Lead/Lag Analysis.

Ramsey and Lampart (1998) employ wavelet decomposition to study the relation between consumption and income. They found that the relation between money and income at different level of time series decomposition is not the same hence arguing the wavelet transform is beneficial in capturing these variations.

Ramsey (2002) defines the difference between denoising and smoothing and explaines useful applications of wavelet in current financial and economic literature to predict possible inventions in future.

Behradmehr (2007) uses wavelet for minimizing the noise present in financial data (smoothing) along with different methods for estimating moments of return. The result of this research is that the most efficient model is obtained applying AR(1)-GARCH(1,1) and smoothing can improve the results taking into account it's sensitivity to the level of smoothing.

Abassi nejad & Mohammadi (1386), applied wavelet analysis along with Artificial Neural Network (ANN) to predict exchange rate. They found out that using this combination of methods (for 1-5 step predictions) would result in better predictions rather applying just the ANN or ARIMA.

Weight Analysis

Roncalli (2010) analyzes the impact of weight constraints on the difference between the optimal weight deduced from Markowitz optimization and the one obtained from constrained optimization. By comparing mean and variance-covariance matrixes and using quadratic programming, he finds out that implying weight constraints would dramatically affect the optimal variance-covariance matrix and in turn optimal portfolio.

Jagannathan & Ma (2002) try to explain why implying weight constraints can help the efficiency of the results. They believe that although this could cause specification error, as Green et.al. (1999) say, the benefits of the reduction in sampling error may cover that. Using simulation technique, they find that implying non negativity constraints is equal to the applying shrinkage estimation.

3 Methodology

In the previous section I discussed the wavelet transform and a brief review of the in hand literature. In this section I will discuss the methodology applied to appraise the effect of noise reduction using wavelet smoothing on the combination of portfolios. First, I review the framework of the modern portfolio theory, and the process of obtaining the optimal portfolio weights; The very process is conducted through a number of models I consider for estimating the parameters used to calculate the portfolio weight. Besides, there is a need to consider parameters required for the smoothing operation and the level of smoothing employed. In addition to the models used for estimating the parameters of interest, section 3.1 reviews mean-variance portfolio theory and section 3.2 elaborates the methodology used for smoothing financial series.

3.1 Mean-Variance Portfolio Theory

The Mean-variance portfolio theory also known as modern portfolio theory was introduced by Markowitz (1952). He believed that investors must maximize expected return while minimizing variance of the portfolio. Researchers (e.g, see Levy and Markowitz (1979)), have shown that the optimal portfolio in this framework could be obtained by solving an optimization problem, assuming that the distribution of asset returns is normal or using a quadratic utility function. The optimization problem using Okhrin and Schmid (2007) notations is

$$\max_{w} EU(R_p), \qquad s.t. w^{'}1 = 1, \tag{10}$$

where $R_p$is the portfolio return which is formed by combination of risky assets and a risk-free asset:

$$R_p = w^{'}(X - r_f 1) + r_f \tag{11}$$

and X denotes the normally distributed ($X \sim N(\mu, \Sigma)$) of K-dimensional vector of asset returns and r_f is a risk-free asset; Furthermore w denotes the vector of the portfolio weights. Then the maximization problem of 10, using quadratic utility function, is transformed to

$$\max_w E(R_p) - \frac{\gamma}{2} Var(R_p), s.t.w^{'}1 = 1 \tag{12}$$

where $\gamma > 0$ is the risk aversion coefficient which measures the attitude of the investor towards risk and by substituting R_p from equation 11 to equation 13, the maximization problem can be rewritten as:

$$\max_w w^{'}(\mu - r_f 1) - \frac{\gamma}{2} w^{'}\Sigma w \tag{13}$$

Therefore, the optimal weight of a risky asset is

$$w_{op} = \frac{\mu - r_f 1}{\gamma \Sigma} \tag{14}$$

Hence the weight of a risk-free asset is $w_{r_f} = 1 - 1^{'}w_{op}$.

As shown in equation 14, the optimal portfolio weight depends on the inverse of the mean and variance covariance matrix of assets. Below two different models used to estimate the mean and covariance matrix are reviewed.

Benchmark Model

Using the historical mean (sample mean) and sample covariance matrix of each series, as well as the sample mean of the risk-free asset, a simple benchmark model is built to first estimate portfolio weights and then the its returns. This simple model is used for comparative purposes basically.

Conditional Mean & Variance

Secondly, applying the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model, introduced by Bollerslev (1986), conditional mean and conditional variance matrices are obtained. For efficiency's sake, the AR(1)-GARCH(1,1) model is used in this step.

3.2 Smoothing Financial Series

As mentioned before, the wavelet transform can be used as a tool for either smoothing or denoising time series and in this paper, wavelet smoothing is employed instead of wavelet de-noising to avoid any further complications; when applying wavelet denoising, extra parameters need to be estimated (i.e. denoising threshold). I have experimented with different basis wavelets such as Haar, Daubechies, and Symlet, and have observed better results with the Daubechies wavelet which is one of the most important and applicable basis wavelets. The time series are first decomposed to detail and approximate coefficients, using the Daubechies wavelet, and then to reconstruct the series I use the wavelet synthesis function. In this step, the reconstruction is done from only the approximate coefficients i.e. I remove the second part of the equation 9 (9), which represents the detail (noise) coefficients.

4 Data Set and Empirical Analysis

The data sets are obtained from the Tehran Stock Exchange website (tse.ir) amongst companies from different industries, which their corresponding assets and shares are among the ones with the most trade volumes in the desirable period and also with the most compatible trading halts. The stock prices of these six companies which cover the period of last days of March2006 through 2013 are chosen, which consist a total of 1400 daily observation for each series turning into weekly observations later (weekly returns). A five-year deposit in government (state) banks is chosen as a risk-free asset and its rate is gotten (for the same period) from the Central Bank of Iran website (cbi.ir). Annually rates converted to weekly rates using:

$$(1 + Annualrate)^{\frac{1}{52}} - 1 = weeklyreturn$$

I use the simple mean of this series as a risk-free rate. Name and index of the six companies can be seen in table (1).

Table 1: Selected companies and their symbols

Symbol	**Company**
STEH1	Tehran Cement
SNMA1	Ind. & Mine Inv
PARK1	Shazand Petr.
NOVN1	EN Bank
IKCO1	Iran Khodro
FAJR1	Amirkabir Steel

So, our final portfolio is a combination of six companies' stock, noted in table (1) and a risk-free asset.

Table 2 reports the summary statistics for the raw weekly returns obtained using Matlab software and I will decompose and smooth the data using wavelet transform in what follows.

The mean of all weekly returns are positive and small amounts, except for the returns of the 1,2 ,&5^{th}comapies which are negative. The maximum and minimum

Table 2: Statistical characteristics of companies' raw return (Source: Research findings)

Data	Mean	Standard Error	kewness	Kurtosis	Maximum Amount	Minimum Amount
STEH1	-0.0027	0.0463	-0.0760	10.8169	0.2307	-0.2076
SNMA1	-0.0018	0.0527	-0.2760	19.2502	0.3307	-0.3561
PARK1	0.0006	0.0646	-2.7486	28.5239	0.3506	-0.5724
NOVN1	6.5e5	0.0681	2.0718	57.0451	0.7391	-0.5493
IKCO1	-0.0029	0.0596	-1.8919	17.1331	0.2111	-0.4562
FAJR1	0.0045	0.4077	0.1004	165.4577	5.2989	-5.2628

amount of return of the 6^{th} company are respectively the highest (5.29) and the lowest (-5.26) and it also has the highest amount of mean and standard deviation. The kurtosis statics of all returns are more than three meaning that their distributions are taller than normal; The assumption that can be relaxed according to Beyhaghi & Hawley (2012) and as Bradly & Taqqu (2003) argue it is commonly accepted that financial asset returns are, in fact, heavy-tailed. In what follows, the effect of wavelet smoothing on the statistics of the data sets used will be discussed and then the impact of wavelet smoothing on the combination of portfolios will be detected.

Wavelet Decomposition of Financial Series

As discussed before in Section 3.2, in order to decompose the raw data into detail and approximate coefficients, the Daubechies1 wavelet is used as the basis function (mother wavelet) and afterwards, the smooth series are reconstructed by using only the approximate coefficients. The smooth series are reconstructed from the approximate coefficients obtained at different levels of decomposition to scavenge the effect of different levels of smoothing. At first, raw series are decomposed by one level, and the smooth series are reconstructed from only the approximate coefficients. Then, for approximate coefficients obtained from the first level decomposition resulting in the second level detail and approximate coefficients, wavelet decomposition is used and after that smooth data are reconstructed; The reconstruction is based on only the second level approximation coefficients (Behradmehr, 2010). I do not go any further for next levels of smoothing since in third level, I observed that the mean of the time series is changing meaning data are over-decomposed because noise generally has a zero mean and therefore its reduction should not impact the mean of the series. Table 3, reports summary statistics of the two-level smooth series.

Comparing sample means in Table 2 and Table 3, it is observed that the sample mean for all six returns remain unchanged, as expected, with respect to the sample mean of the raw series. Now comparing estimated standard deviations in Table 2

Table 3: Statistical characteristics of companies' smooth return (Source: Research findings))

Data	Mean	Standard Error	kewness	Kurtosis	Maximum Amount	Minimum Amount
STEH1	-0.00274	0.0229	0.276332	4.59742	0.07673	-0.06571
SNMA1	-0.00188	0.0262	-0.01154	4.22161	0.07307	-0.08931
PARK1	0.00069	0.0353	-1.13801	6.18588	0.08248	-0.13565
NOVN1	6.6e-5	0.0347	1.214301	13.3163	0.18510	-0.13789
IKCO1	-0.00299	0.0302	-0.07748	6.04831	0.09352	-0.11274
FAJR1	0.00455	0.0293	0.940046	4.95490	0.10141	-0.06608

and Table 3, it is observable that as the data is smoothed the estimated standard deviations decreases in a way that the average standard deviation for raw series is 0.116 while for the second level smooth series the average standard deviation decreases to 0.029. This clearly points to a reduction of noise in the data and in fact is the reason we apply wavelets. Furthermore, it is observed that sample kurtosis clearly decreases through smoothing; It may seem that return distributions are getting closer to a normal one. Figure 1 plots the multiresolution analysis of the first company's weekly returns using Daubechies1 wavelet. Top panel plots the raw serie and the second panel plots the second level smooth serie (the general trend). The third panel plots the residual (noise) from the first level decomposition and the fourth panel plots the noise extracted from the second level of decomposition.

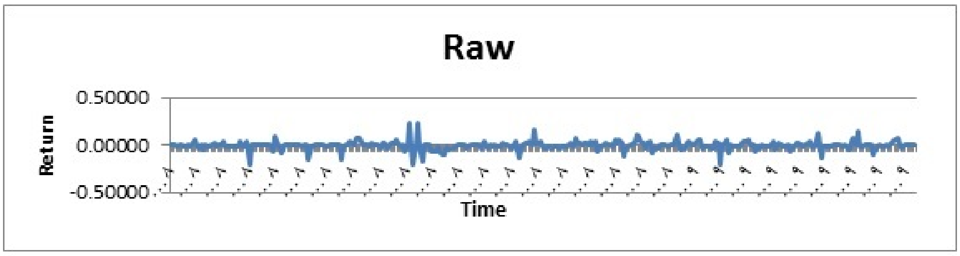

Figure 1: Raw serie _ STEH1's return

Since not all the economic variables and parameters are stationary therefore it is suggested that we run the Augmented Dicky Fuller (ADF) test and examine the stationarity unless, the results may not be valid enough. A time series is stationary if it's mean, variance and auto-covariance are time independent and because this might not be the case for many financial time series, I applied the Augmented Dicky Fuller (ADF) test. Table 4 reports the results.

As it is observable from Table 4, all the time series are stationary and differenti-

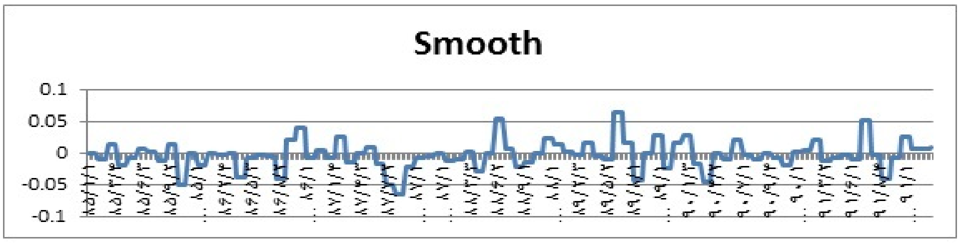

Figure 2: Second level smooth serie (the general trend _ STEH1's return

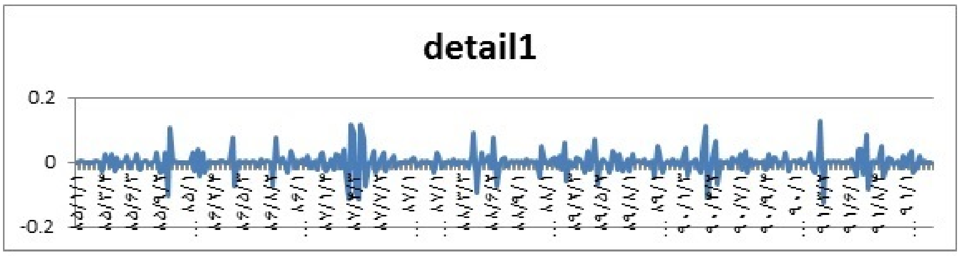

Figure 3: Residuals from the first level decomposition _ STEH1's return

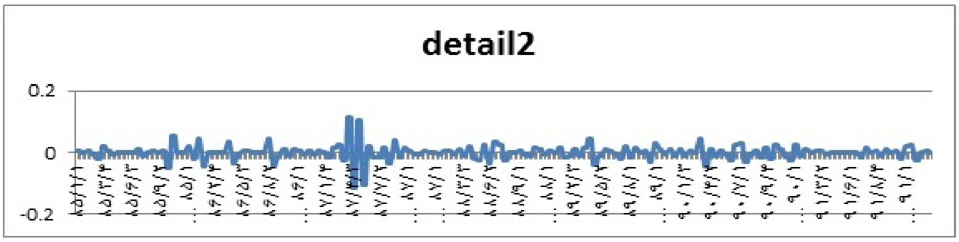

Figure 4: Noise extracted from the second level of decomposition_ STEH1's return

Figure 5: Multiresolution analysis using Daubechies1 wavelet_ STEH1's return (Source: Research findings

ation is not necessary. Obviously, noise series are stationary too since they are all residuals.

Table 4: Augmented Dickey-Fuller test for raw returns (Source: Research findings)

	STEH1	SNMA1	PARK1	NOVN1	IKCO1	FAJR1
t	-14.64	-17.71	-16.65	-17.82	+16.12	-13.58
Significance	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

4.1 Weight Estimation

As noted above, the estimation of the efficient weight matrices (equation 14) is necessary in order to obtain the proper portfolio allocation and in this research this estimation is done applying not just one approach. For comparison purposes, I decided to form a portfolio of the underlying assets with equal weights. In other words, each of the risky and risk-free assets will get a fraction of 1/7 of the final portfolio. Table 5 reports the summary statistics of this portfolio.

Table 5: Equal weights- portfolio's parameters (Source: Research findings)

Return	Standard error	Sharp ratio
0.050	0.061	0.762

As we can see in Table 5, sample mean, standard deviation and sharp ratio of the corresponding portfolio is positive. Based on the benchmark and conditional estimation models, discussed in Section 3.1, and using wavelet transform, I am going to estimate the efficient weight in four scenarios (cases). In other words, the simple and estimated conditional mean and variance values are obtained independently from both the raw and smooth series, resulting in four cases of interest:

- Case one: both the simple mean and the variance are estimated from raw series.
- Case two: both simple mean and simple variance are estimated from the smooth series.
- Case three: both the conditional mean and the conditional variance are estimated from raw series.
- Case four: smooth series are used to estimate the conditional mean and variance.

Note that before applying GARCH estimation, existence of ARCH effect was tested and the results are reported in table 6.

As it is shown in table 6, ARCH effect is observed but not for all the companies which is due to some conditions with tse data, for example applying maximum and minimum amount for price volatilities, price limit, in a way that real volatilities cannot be seen causing results to be biased. But since there are two kinds of assets

Table 6: ARCH test (Source: Research findings)

Symbol	STEH1	SNMA1	PARK1	NOVN1	IKCO1	FAJR1
Significance Level	0.91	0.002	0.92	0.82	0.69	0.0000

in the portfolio, applying GARCH for risk index estimation is appropriate. Best results can be obtained by employing GARCH(1,1).

Estimations Without Weight Constraint

In order to estimate portfolio parameters, weight matrixes must be estimated first (equation 14). With the first 341 observations weight matrix will be estimated and then using the last observation the mean of the return will be estimated. In order to estimate weight matrix, risk aversion coefficient (γ) is needed. In references as we looked there is no specified limit for γ (Paolo Brandimarte, (2002)) but it is common to define γ as an amount between one to five. We applied different amounts of γ ($\gamma = 1, 3, 5$). Since the main results of this research did not change with different γ , estimations with $\gamma = 1$ are reported only. Table 7 reports the efficient weight matrices for the six companies and Table 8 reports portfolio parameters considering risk-free asset.

Table 7: Weight estimations without constraint with $\gamma = 1$ (Source: Research findings)

	STEH1 Weight	SNMA1 Weight	PARK1 Weight	NOVN1 Weight	IKCO1 Weight	FAJR1 Weight
Case 1	-2.714	-1.0556	-0.1839	-0.7007	-1.5436	0.0051
Case 2	-23.2295	-6.2408	3.2102	-0.3831	-10.8445	8.8884
Case 3	-45063.9	-6566.41	294742.4	-272627	-28711.8	-2925.35
Case 4	-1.2e13	1.76e13	3.67e13	-4.5e12	-2.2e13	-2.7e13

Table 8: Portfolio parameters - no constraint , $\gamma = 1$ (Source: Research findings)

	Return	Standard error	Sharp ratio
Case 1	-5.16E-01	0.1857	-2.7918
Case 2	-3.49E+00	3.8874	-0.8996
Case 3	-6.03E+03	2.65E+04	-2.28E-01
Case 4	-3.42E+12	1.12E+13	-3.06E-01

As it is shown in tables 7 and 8, all the returns and sharp ratios are negative. Negative sharp ratio means that the risk-free rate is greater than portfolio return and therefore holding risk-free asset is the optimal choice. Standard divisions are also large amounts and almost all of the weights are negative meaning short- selling. In addition to the tse's inherent problem, as mentioned in Okhrin & Schmid (2007) negative weights or weights greater than one occurs because of using historical data in estimating moments of return, in fact, it is completely predictable. So Okhrin & Schmid (2007) propose a solution which we discuss in the next section.

Estimations with weight constraint

One of the solutions suggested for solving the weight estimation issue (discussed in previous section) is imposing some weigh constraints. One of the most common constraints is the non-negativity one along with the equalization of the sum of the weight to one. The first one is no short selling constraint. As mentioned earlier, the impact of this rule on weight and portfolio estimations in Tehran Stock Exchange is investigated in this paper. For this purpose, I use Matlab software. At first, constraints should be defined along with the efficient frontier which is built using a few numbers of portfolios. The allocation decision consisting of risky assets is chosen from these portfolios which are on the efficient frontier in a way that it has the minimum variance for a given mean. Then, risk-free rate (rf) is entered in the calculations. By drawing Capital Market Line (CML), the line drawn from rf to the efficient frontier, and then the tangency point of these two (CML & efficient frontier), our portfolio allocation strategy will be determined consisting of risky portfolio and a risk-free asset (the efficient allocation between risky portfolio and rf) moving along CML when γ changes, getting closer to or farther from rf. In other words, it is γ (risk aversion coefficient) deciding how much rf should be involved, higherγ means that the investor is more risk averse and the allocation decision would be closer to rf. Estimations are done like pervious section; Table 9 reports the efficient weights for all of the γs.

Table 9: Weight estimations with weight constraint and $\gamma = 1, 3, 5$ (Source: Research findings)

	STEH1 Weight	SNMA1 Weight	PARK1 Weight	NOVN1 Weight	IKCO1 Weight	FAJR1 Weight
Case 1	$\cong 0$	0	0	0	0	1
Case 2	$\cong 0$	0	0	0	0	1
Case 3	–	–	–	–	–	–
Case 4	–	–	–	–	–	–

As it is shown, estimated weights for case one and case two are exactly the same. We can say smoothing in constrained situations does not affect weight matrices estimated with simple estimators. In these cases, almost all of the wealth will be invested in sixth company (Fajr) and therefore there will be no diversification. Before justifying cases three and four, another result must be presented: Risky

Fraction is the fraction of Optimal Risky Portfolio in Optimal Overall Portfolio. Optimal Overall Portfolio is the investor's final choice consisting of risk-free asset and Optimal Risky Portfolio (explained before). Table 10 reports Risky Fraction for all the cases and different levels of risk aversion coefficient.

Table 10: Risky fraction for all the cases and different levels of risk aversion coefficient (Source: Research findings)

	$\gamma = 1$	$\gamma = 3$	$\gamma = 5$
Case 1	0.0082	0.0027	0.0016
Case 2	2.97	0.99	0.59
Case 3	$\cong 0$	$\cong 0$	$\cong 0$
Case 4	$\cong 0$	$\cong 0$	$\cong 0$

As it is obvious, this fraction is very small (almost zero) for cases three and four. This amount is greater than one for the second case ($\gamma = 1$) which means borrowing. Considering 3^{th} and 4^{th} rows of the Table 10, we can justify the results of Table 9. When applying conditional moments of returns for either of the raw or smooth data, weights of the risky asset would be equal to zero and therefore the portion of the risky portfolio is indefinable in cases three and four (table 9) and the investor will be better off holding just the risk-free asset in his optimal overall portfolio. As we can see, the results of the first and second case in Table 10 is changing when γ changes and we will explain this in what follows.

Table 11: Portfolio parameters with constraint, $\gamma = 1$ (Source: Research findings)

	Return	Standard error	Sharp ratio
Case 1	0.0031	0.0034	0.0034
Case 2	0.0071	0.0638	0.0638
Case 3	0.0031	$\cong 0$	0
Case 4	0.0031	$\cong 0$	0

In Table 11 portfolio parameters are reported and as we can see portfolio return in three cases is the same and equal to 0.0031 which is rf and this is completely as we expected due to the results of Tables 9 and 10. In other words, because no fractions were assigned to the risky portfolio, optimal overall portfolio return is the same as the risk-free rate. Likewise, because case two has greater fraction of risky portfolio it has a different return, 0.0071 which is higher than rf. Note that standard deviation of this case is also higher than the rest of the cases which does not cause its sharp ratio to be any lesser than other sharp ratios of other cases. Clearly this parameter (sharp ratio) for the cases three and four is zero knowing that portfolio return in these cases is equal to rf and we hold just the risk-free asset in our final portfolio. Generally speaking, implying weight constraints causes both portfolio return and sharp ratio to be higher and get improved in comparison to unconstrained situation. Figure 2 plots CML and efficient frontier and the position of the optimal risky and overall portfolios for case one ($\gamma = 1$) which helps understanding latter results better. Efficient frontier is the lower curve and the tangency point of CML and this curve shows the optimal risky portfolio. We can see that Optimal overall portfolio is almost on the vertical axe, near to rf.

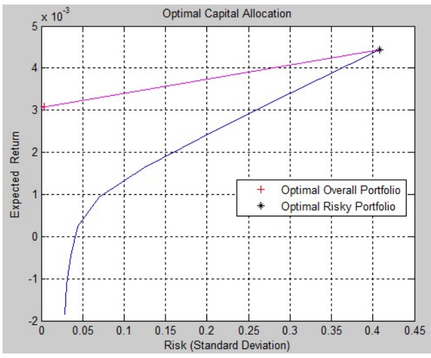

Figure 6: CML, efficient frontier and the position of the optimal risky and overall portfolios with constraint_ Case 1 ($\gamma = 1$)

From Table 10, as γ increases and becoming more risk averse (moving horizontally) risky fraction is decreasing noticeably which means the investor is holding lesser and lesser of risky portfolio and therefore more and more of the risk-free asset each time. For more clarification, figure 3 and figure 4 plot the second case in constrained estimation for $\gamma = 3$ and $\gamma = 5$ respectively. As it is shown in figure 3, $\gamma = 3$, optimal overall and risky portfolios are almost in the same position. Next, putting $\gamma = 5$ and getting more risk averse, the position of the optimal overall portfolio got closer to rf on CML while optimal risky portfolio did not move. This shows the impact of risk aversion coefficient on choosing among risky portfolio and risk-free asset which in this very situation the investor invests mostly in the risk-free asset.

5 Conclusion

In order to form a portfolio, one can employ the mean-variance portfolio theory which would further need an investor to estimate a set of statistical characteristics in the portfolio of interest. But it not as simple as this because the noise present in the underlying securities may affect the estimated parameters (statistical characteristics), as it has been shown, and therefore affect the resulting portfolio allocation strategy (Behradmehr, 2010). Also, the impact of any weight constraints on the portfolio allocation strategy has not been clearly demonstrated yet.

In this research, as noted above, the wavelet transform was employed to inves-

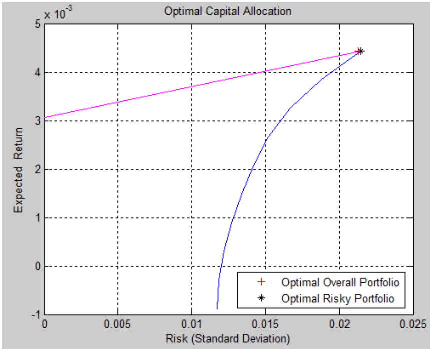

Figure 7: CML, efficient frontier and the position of the optimal risky and overall portfolios with constraint _ Case 2 ($\gamma = 3$)

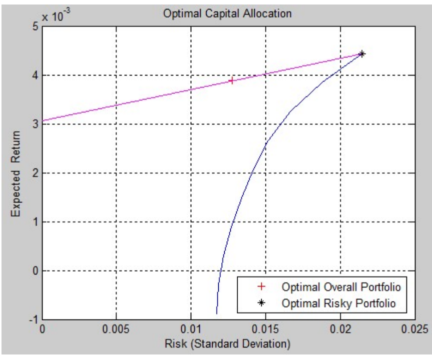

Figure 8: CML, efficient frontier and the position of the optimal risky and overall portfolios with constraint _ Case 2 $\gamma = 5$

tigate the effect of noise through a set of empirical experiments. This purpose was accomplished by minimizing the effect of noise in the financial series before estimating their statistical characteristics. Afterwards, the smoothed statistics are used to estimate the optimal portfolio weights along with the noisy ones. In other words, as part of the investigation, the effect of noise reduction when the mean and variance matrices are obtained from smooth or raw datasets was evaluated independently. More specifically I investigated four cases of raw mean and variance/simple estimation, smooth mean and variance/simple estimation, raw mean and variance/-conditional estimation, and smooth mean and variance/conditional estimation. All the noted process is done once with and again without weight constrains in order to understand the effect of no *short selling condition;* Imposing weight constraints is also one of the solutions suggested for solving weight estimation issue which is negative weights or weights greater than one predictably occurring because of using historical data in estimating moments of return.

Results reveal that the standard deviation estimates are changed after reducing the noise present in the financial series. In fact, as observed in the Section 4.2 these adjusted statistics affect the allocation of the portfolio. In the best case, it is observed that using the AR(1)-GARCH(1,1) model is the best decision. Clearly, this indicates that the accuracy of our variance estimates has improved by the smoothing operation.

The impact of smoothing on weight estimations and portfolio return are as follows: If we use simple estimation and imposing weight constraints, then we can say smoothing will result in choosing risky portfolio, although this may seem to be sensitive to data, risk aversion coefficient and the desirable period. So, it can be deduced that noise effects in this market is deniable and the main reason for that is the limits for daily price changes present in tse.

The impact of constraints on efficient weights and portfolio return are also as follows: improvement in efficient weights (0 or one), increase in portfolio returns (almost the same as rf for most cases) and positive sharp ratios. Because of the nearly zero risky fractions in the constrained approach like the unconstrained one, holding risk-free asset is suggested.

One interesting result in this paper is that if we assign all the assets the same and equal weight, what most of the amateur investors do, we even get better results (portfolio returns and sharp ratios increase). In other words, with this data and this period of investigation, if we want to allocate the wealth among *risky assets*, it would be better to do so without any estimation and just simply put every asset the same weight!

The estimation method of mean and variance in weight matrices affect (not strongly) the allocation of the wealth and how much risk-free asset we hold. Another important result in this paper is that implying weight constraints, would change the weights in the way that it will be positive or zero (risk-free asset has the biggest portion) and sharp ratio will increase and become positive and this has nothing to do with the risk aversion coefficient (results do not change in respect to γ).

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How to Cite: Soudeh Sheybanifar[1], *Impacts of no short selling and noise reduction on portfolio allocation*, Journal of Mathematics and Modeling in Finance (JMMF), Vol. 1, No. 1, Pages:63–82, (2021).