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Research paper

Life settlements pricing based on fuzzy interest rates arisen from life insurance premiums

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Abstract:

In this article, fuzzy random variables are used to model interest rate uncertainty used in the calculation of whole life insurance premiums, and calculate the effect of this uncertainty on the price of life settlements. The fuzzy results obtained from deterministic and probabilistic pricing approaches have been compared with the results of the stochastic approach. Also, the results have been analyzed for Iran life table, which has been issued to insurance companies since 1400, and for France life table, which was previously used by insurance companies. In addition, since 5-year survival probability for each cancers in Iran was lower than in the United States, the probability adjustment coefficient for Iran was higher than that of the United States. In addition, the interval obtained for the fuzzy probability price and the stochastic price for both Iran and France life tables are close to each other. But in most cases, the fuzzy price obtained based on the deterministic approach has a significant distance from the stochastic and fuzzy probability approaches. Also, the findings of the research indicate that the price calculated using the fuzzy deterministic approach for Iran life table is higher than France life table. While the results for fuzzy probabilistic approach and stochastic approach are completely opposite. In the other words, the price calculated for the Iran life table is lower than the France life table. This difference between the prices obtained by Iran and France life tables comes from the fact that in this article, the adjustment coefficients for these life tables are not assumed to be constant and are calculated for each person separately from related life tables. *Keywords*: Life

expectancy, Life settlements, Secondary market, Fuzzy random variable, Interest rate.

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1 Introduction

A life settlement (LS) is a financial transaction that enables the sale of life insurance policies by policyholders to investors in the secondary life insurance market. The secondary life insurance market is an alternative to surrender a life insurance policy, which usually allows policyholders to sell their policies to investors at a higher price than the surrender value. In this way, after the sale of the life insurance policy, the investor undertakes to pay, if any, the future premiums and obtain the death benefits at the time of the insured's death [13].

The secondary life insurance market creates conditions that increase policy liquidity, demand for life insurance and the efficiency of the primary market [8]. It also allows the policyholders to sell the policies at a value higher than the surrender value if the insured's health status is below average, and on the other hand, it ends the monopoly of insurers as the only potential buyers of policies [9].

As the secondary life insurance markets are rapidly growing and developing, the pricing LSs become one of the most challenging problems for policyholders and investors. In the pricing LSs, components such as mortality probability, interest rate or technical interest rate, insured life expectancy and death benefits are used [6], which are the sources of uncertainty. Among the mentioned components, the interest rate is a factor whose changes affect the premiums of life insurances. For this purpose, regulators usually set an upper limit for the technical interest rate and update it periodically, but interest rate changes are still one of the most important concerns of insurance company managers and regulators.

Although the interest rate is determined at the time of signing the life insurance contract for the duration of the contract and is a fixed value, in reality it can change and cause uncertainty in pricing life insurances [1, 10, 14, 17]. Furthermore, Fuzzy numbers have been used in a few articles to model uncertainty in LSs pricing. Aalaei in [3] has implemented the theory of fuzzy sets to model the internal rate of return and has calculated the price of life settlement using three deterministic, probabilistic and stochastic approaches. In [15], in addition to the internal rate of return, the mortality rate is also considered to be fuzzified.

The purpose of this article is to calculate the premium based on the fuzzy technical interest rate at the beginning of the insurance policy and its effect on the price of life settlement. Therefore, the fuzzy variable used in this research is different from the aforementioned articles. This issue differentiates this research from other researches and is also considered as the innovative aspect of this article.

Therefore, in this article, we are going to model the uncertainty resulting from the technical interest rate in calculating the initial premium of life insurance using fuzzy methods. So, instead of calculating a specific price for life settlement, a range of all possible values is provided to the actuary and an appropriate price is determined based on the available economic information. In order to achieve this goal, this article is classified into 5 sections. In section 2, definitions and basic concepts such as fuzzy random variables, the single premium at the start of the insurance policy, the adjustment multiplier for the mortality probability, life expectancy and deterministic, probabilistic and stochastic pricing approaches for pricing LSs are provided. Section 3 includes research methodology and fuzzy pricing approaches. Numerical test results for fuzzy pricing LSs are given in Section 4. Finally, our conclusions are given in Section 5.

2 Basic concepts

Considering that in this article, the pricing LSs will be discussed using the fuzzy interest rate in the secondary market; In the following, the definitions and concepts used are introduced. Suppose a x^* -year-old person has purchased a whole life insurance policy with C monetary unit benefits. After several years, due to economic pressure or health problems, he/she tends to sell his insurance policy in the secondary market at a suitable value.

Insured life expectancy (LE) and interest rate are the most key parameters for calculating the value of the insurance policy, which should be calculated in the first step: The life expectancy of an insured applying to sell his policy on the secondary market is reflected in substandardized mortality rates. In other words, the insured's LE will be a reflection of his health status. Therefore, to calculate LE, adjusted mortality probabilities should be obtained from the standard life table, that is compatible with the personal health status and lifestyle of the insured person and the related life expectancy. In order to calculate the mortality probability according to the health status of insured, an adjustment multiplier is used. Because the insured with health problems has a different LE than people with standard health, and using the standard life table shows the LE more than it is. Therefore, using the adjustment multiplier makes the mortality probability and life expectancy (LE) closer to reality for these insureds [2].

Then, from the adjusted mortality probabilities q_{x+t}^* , $t = 1, 2, \dots, \omega - x$, we can find the adjusted probability that the insured of age x survives k years as follows:

$$_{k}P_{x}^{*} = \prod_{t=0}^{k-1} (1 - q_{x+t}^{*})$$
(1)

As a result, the insured's LE can be calculated as follows:

$$e_x^* = \sum_{k=1}^{\omega-x} {}_k P_x^* = \sum_{k=1}^{\omega-x} \prod_{t=0}^{k-1} (1 - q_{x+t}^*)$$
(2)

Next, the method of calculating the adjustment multiplier should be determined. As mentioned by [13], the adjustment multiplier is calculated as follows:

$$\beta = 1 + \sum_{j=1}^{m} \rho_j \tag{3}$$

The coefficient ρ_j represents the positive or negative factors in mortality in the assessment made in the case of j which is measured based on the insured's lifestyle. Considering β as the adjustment multiplier, the modified mortality probability q_x^* is considered as a linear function of the standard probability q_x and is calculated as follows:

$$q_x^* = \beta q_x = (1 + \sum_{j=1}^m \rho_j) q_x$$
(4)

Since $0 \le q_x^* \le 1$, we should have $-1 \le \sum_{j=1}^m \rho_j \le \frac{1}{q_x} - 1$. However, since this inequality may not hold at all ages in the standard mortality table that serves as the basis, [16] proposed the following relationship instead of (4) as in the source [13] has also been used:

$$q_{x+t}^* = \min\{1, (1 + \sum_{j=1}^m \rho_j)q_x\}, t = 1, 2, \cdots, \omega - x.$$
 (5)

It should be kept in mind that the purpose of this article is to investigate the uncertainty resulting from the technical interest rate using fuzzy methods on the pricing of life settlements. Therefore, following [1], a fuzzy set denoted by \tilde{A} can be defined as follows:

$$\tilde{A} = \left\{ (x, \mu_{\tilde{A}(x)}) | x \in X \right\}$$

 \tilde{A} is a subset defined on the reference set X, where $\mu_{\tilde{A}}$ is a membership function defined as follows:

$$\mu_{\tilde{A}}: X \to [0,1].$$

Also, a fuzzy set can be represented by its α -cuts which is a set \tilde{A} as follows:

$$A_{\alpha} = \left\{ x \in X | \mu_{\tilde{A}(x)} \ge \alpha \right\}, \forall \alpha \in (0, 1]$$

The α -cuts of the fuzzy set \tilde{A} are in the form of bounded closed intervals which are defined as follows:

$$A_{\alpha} = \left[\underline{A}_{\alpha}, \overline{A}_{\alpha}\right] = \left[\inf_{x \in X} \left\{\mu_{\tilde{A}}(x) \geq \alpha\right\}, \sup_{x \in X} \left\{\mu_{\tilde{A}}(x) \geq \alpha\right\}\right],$$

Now consider the interest rate as a fuzzy number with α -cuts as $i_{\alpha} = [\underline{i}_{\alpha}, \overline{i}_{\alpha}]$. As a result, the fuzzy discount rate \tilde{d}_t for the payment of a monetary unit in t years, with α -cuts for each $\alpha \in [0, 1]$, will be a decreasing function of the interest rate:

$$\tilde{d}_{t_{\alpha}} = [\underline{d}_{\underline{t}_{\alpha}}, \overline{d}_{t_{\alpha}}] = [(1 + \overline{i}_{\alpha})^{-t}, (1 + \underline{i}_{\alpha})^{-t}].$$
(6)

If the triangular fuzzy number of the interest rate is (i_l, i_c, i_u) , the discount function in relation (6) will be as follows:

$$\tilde{d}_{t_{\alpha}} = [\underline{d}_{t_{\alpha}}, \overline{d}_{t_{\alpha}}] = [(1 + i_u - (i_u - i_c)\alpha)^{-t}, (1 + i_l + (i_c - i_l)\alpha)^{-t}].$$
(7)

Since the technical interest rate is a factor affecting the premium, the premium should be calculated using the fuzzy discount function. The single premium, Π_x , is calculated as follows based on the principle of equality in the book [7] and C is the money that the insurance company pays to the beneficiaries at the end of the year of his death:

$$\Pi_x = C \sum_{k=0}^{\omega - x^*} (1+r)^{-(k+1)} \times_{k|} q_{x^*}, \qquad (8)$$

On the other hand, the annual premium, payable during the life of the insured, P, is calculated using (8) as follows:

$$P\sum_{k=0}^{\omega-x^*} (1+r)^{-k} \times_k p_{x^*} = \Pi_x = C\sum_{k=0}^{\omega-x^*} (1+r)^{-(k+1)} \times_{k|} q_{x^*},$$

$$P = \frac{C\sum_{k=0}^{\omega-x^*} (1+r)^{-(k+1)} \times_{k|} q_{x^*}}{\sum_{k=0}^{\omega-x^*} (1+r)^{-k} \times_k p_{x^*}},$$
(9)

Now consider a term life insurance. If the x-year-old insured dies in the nth year of the contract, at the end of the year of his death, 1 unit of money will be paid by the insurer to the beneficiaries. In general, for whole life insurance $n = \omega - x + 1$ is considered, where ω is the maximum age in the life table.

2.1 Pricing approaches

The deterministic economic value of LS for a x-year-old person is represented by VED_x and by calculating the life expectancy e_x^* and for a given IRR, according to [13], we will have:

$$VED_x = \frac{C}{(1 + IRR)^{e_x^*}} - \sum_{k=1}^{e_x^*} \frac{P}{(1 + IRR)^k}$$
(10)

The probabilistic approach is the most usual valuation formula in the secondary life insurance market. In this case, all cash flows that may be collected and paid are considered according to the weight of probability of occurrence. Therefore, the probable economic value of LSs for the insured at age x, according to [13], VEP_x , is:

$$VEP_x = \sum_{k=1}^{\omega - x^*} \frac{C}{(1 + IRR)^k} \times_{k|} q_x^* - \sum_{k=1}^{\omega - x^*} \frac{P}{(1 + IRR)^k} \times_k p_x^*$$
(11)

The third valuation method is the stochastic approach. In this case, the future life expectancy random variable for the insured at current age x is T_x^* , which can be obtained from the adjusted mortality probabilities for the insured person. The possible results of T_x^* are:

$$0, 1, 2, \cdots, \omega - x - 1$$

with respective probabilities as follows:

$$q_x^{*},_{1|} q_x^{*},_{2|} q_x^{*}, \cdots,_{\omega-x-1|} q_x^{*}$$

Therefore, the stochastic economic value of LSs for the insured at age x, according to [13], VEE_x , is:

$$VEE_x = \frac{C}{(1 + IRR)^{T_x^* + 1}} - \sum_{k=1}^{T_x^*} \frac{P}{(1 + IRR)^k}$$

According to the concepts of actuarial mathematics, it can be shown that the mathematical expectation of the random variable VEE_x is consistent with the probabilistic economic value, VEP_x . The results obtained using the random method are different each time the program is run. Therefore, the program should be repeated an acceptable number of times and using the mean and standard deviation to calculate the confidence interval for the price of LSs.

3 Methodology

In this research, the interest rate and pricing approaches of life settlement are stated in a fuzzy state. The interest rate is one of the most important factors affecting the insurance premium, which is assumed to be constant in numerical calculations, but in reality it is a variable and ambiguous parameter that cannot be measured specifically. In this regard, to model this ambiguity, instead of using a crisp interest rate, a fuzzy interest rate can be used to create an allowed range for the interest rate. So that insurance companies can determine the amount of insurance premium in the desired period according to their risk aversion. In this section, the implemented fuzzy set theory has been used to model the interest rate and premium to calculate the price of life settlement on deterministic, probabilistic and the results will be compared to the prices arise using stochastic approach in the secondary market. Now, in order to obtain the Fuzzy deterministic economic value, it is enough to apply the fuzzy premium equation according to equations (7) and (9) in equation (10). In this case, the fuzzy deterministic economic value of life settlement ($V \tilde{E} D$) is calculated as follows:

$$V\tilde{E}D_x = [\underline{VED}, \overline{VED}] = \frac{C}{(1+IRR)^{e_x^*}} - \sum_{k=1}^{e_x^*} \frac{\tilde{P}}{(1+IRR)^k}$$
$$= [\frac{C}{(1+IRR)^{e_x^*}} - \sum_{k=1}^{e_x^*} \frac{\overline{P}}{(1+IRR)^k}, \frac{C}{(1+IRR)^{e_x^*}} - \sum_{k=1}^{e_x^*} \frac{\underline{P}}{(1+IRR)^k}].$$

To obtain the fuzzy probabilistic economic value, it is sufficient to apply the fuzzy premium equation according to equations (7) and (9) in equation (11). In this case,

the fuzzy probabilistic economic value of life settlement $(V\tilde{E}P)$ is calculated as follows:

$$V\tilde{E}P_{x} = [\underline{VEP}, \overline{VEP}] = \sum_{k=1}^{\omega - x^{*}} \frac{C}{(1 + IRR)^{k}} \times_{k|} q_{x}^{*} - \sum_{k=1}^{\omega - x^{*}} \frac{\tilde{P}}{(1 + IRR)^{k}} \times_{k} p_{x}^{*}$$
$$= [\sum_{k=1}^{\omega - x^{*}} \frac{C}{(1 + IRR)^{k}} \times_{k|} q_{x}^{*} - \sum_{k=1}^{\omega - x^{*}} \frac{\overline{P}}{(1 + IRR)^{k}} \times_{k} p_{x}^{*}]$$
$$, \sum_{k=1}^{\omega - x^{*}} \frac{C}{(1 + IRR)^{k}} \times_{k|} q_{x}^{*} - \sum_{k=1}^{\omega - x^{*}} \frac{P}{(1 + IRR)^{k}} \times_{k} p_{x}^{*}].$$

4 Numerical test results

The purpose of this research is to price life settlement based on the fuzzy technical interest rate used in life insurance premium calculation. Therefore, in this section, the regulations related to the determination of interest rates in life insurance will be mentioned first, and then the results of the research according Iran life tables (ILT1400) which has been notified for use by the Iran Central Insurance since the beginning of 1400 and the French life table (TD8890) which was used by insurance companies before the Iran life table. According to the Supplement to Insurance Regulation No. 68, the maximum rate of technical interest rate for the first two years of the insurance policy is 16%, for the next two years, 13%, and for the years after the first four years, 10%. Now suppose that for each $\alpha \in [0, 1]$ the fuzzy interest rate is considered as follows:

$$\tilde{i} = \begin{cases} [0.15 + 0.01\alpha, 0.17 - 0.01\alpha], & \text{if } t \le 2; \\ [0.115 + 0.015\alpha, 0.145 - 0.015\alpha], & \text{if } 2 < t \le 4; \\ [0.085 + 0.015\alpha, 0.115 - 0.015\alpha], & \text{if } t > 4. \end{cases}$$

Suppose the insured was 45 years old at the time of purchasing the whole life insurance policy. Table 1 shows the fuzzy premium for death benefits of 1000 monetary units for Iran and France life table.

Suppose a 65-year-old insured has combined skin cancer, for which the 5-year relative survival rate is 93%, according to [5]. That is, the probability of survival of the affected person is 93% of what is calculated based on the standard life table. Considering that in Iran life table, ${}_5P^*_{65} = 0.93 \times 0.9040$, the one-year mortality rate increase by this factor, which is represented by ρ_1 , by solving:

$$\prod_{t=0}^{4} [1 - (1 + \rho_1)q_{65+t}] = 0.93 \times 0.9040$$

The value of $\rho_1 = 0.70$ is obtained. Therefore, the intuitive estimation of β coefficient in relation (3) will be equal to $\beta = 1 + 0.70$.

α	Iran la	ife table	France	France life table			
	$\underline{\mathbf{P}}$	\overline{P}	<u>P</u>	\overline{P}			
1	7.18	7.18	10.02	10.02			
0.9	7.07	7.30	9.90	10.14			
0.8	6.97	7.41	9.79	10.26			
0.7	6.87	7.53	9.68	10.39			
0.6	6.76	7.65	9.57	10.52			
0.5	6.67	7.78	9.46	10.65			
0.4	6.57	7.91	9.36	10.79			
0.3	6.48	8.04	9.25	10.93			
0.2	6.39	8.18	9.15	11.08			
0.1	6.30	8.32	9.06	11.22			
0	6.22	8.46	8.96	11.38			

Table 1: Fuzzy annual premium for death benefits of 1000 monetary units for Iran and France life table

In Table 2, the coefficient of Adjustment of mortality probabilities β for insured persons with various cancers of the combined type according to the aforementioned explanations and based on the report of the American Cancer Society [5], which is based on the information of people who were diagnosed with cancer between 2012 and 2018 in the United States have been diagnosed with cancer and it has been calculated based on the article by Nemati et al. Considering that the purpose of Table 2 is to investigate the impact of 5-year survival rate on the adjustment multiplier of mortality probabilities, the life table for both is considered as Iran life table.

Table 2: Calculation of the Adjustment multiplier of mortality probabilities for insured persons with different cancers with 5-year survival rates for Iran and the United States

Type of cancer	Ire	an	United	United States		
	5-year	Adjustment	5-year	Adjustment		
	survival rate	multiplier	survival rate	multiplier		
Colon	55	6.5	63	5.3		
Skin (Melanoma)	48	7.7	93	1.7		
Bladder	70	4.4	77	3.5		

It is logical that the lower the survival rate, the higher adjustment multiplier of mortality probabilities, as you can see in the table. Also, in general, the survival probabilities for the types of cancers in Table 1 are lower in Iran than in the United States. This means that the adjusted multiplier of mortality for these cancers will be higher for Iran than for the United States. Differences in weather conditions, lifestyles, and stages of disease recovery, including access to medicine and required facilities, can be the reasons for the difference in survival rates and, as a result, the difference in the adjustment multiplier of mortality probabilities in these two countries. The fuzzy price of life settlement for two deterministic and probabilistic approaches and its stochastic price with a 95% confidence interval obtained from 1000 times of program implementation are presented in Tables 3 and 4. In these tables, the findings have been obtained using Iran 5-year survival rate. The calculations in Table 3 are based on Iran life table and in Table 4 based on France life table.

The adjustment multiplier of mortality probabilities for the insured with different cancers with Iran 5-year survival rate based on the France table for Colon, Melanoma, and Bladder cancers is 5.2, 6.2, and 3.6, respectively, which is used in the calculations of Table 4.

As it can be seen, for both Iranian and French life tables, the range obtained for the fuzzy probabilistic price and the fuzzy stochastic are closer to each other. So that in most cases, the range obtained for the fuzzy probabilistic price is a subset of the stochastic price obtained with a 95% confidence interval. The fuzzy price obtained based on the deterministic approach has a significant distance from the other two approaches in most cases. The remarkable thing about the intervals obtained for different is that in the case where $\alpha = 0$, all possible states are considered for solving the problem, And as becomes larger, the obtained interval becomes smaller and reaches a definite value at $\alpha = 1$.

Another noteworthy point is that for calculating the fuzzy price and the stochastic price of life settlement, the result obtained using the fuzzy deterministic approach for Iran life table is more than France life table. Meanwhile, in the case of the fuzzy probabilistic approach and the stochastic approach, the result is completely opposite. That is, the results obtained for Iran life table are lower than France life table. This point comes from the fact that in this article, the adjustment multiplier for the Iran and France life tables are calculated for each person separately and affect the mortality probabilities. Therefore, it is natural that the effect of the adjusted multiplier of mortality probabilities in different approaches shows itself in a different way.

5 Conclusions

The field of pricing LSs is one of the research fields that is highly attractive due to its pristine and untouched nature. The present research is one of the few researches conducted in the field of pricing LSs, which has modeled the parameters of economic uncertainty, and by considering the fuzzy interest rate in the calculation of life

Type of cancer	α	Detern	ninistic	 Probabilistic		Stochastic
		VED	\overline{VED}	 $\underline{\text{VEP}}$	\overline{VEP}	95% confidence interval
	1	438.88	438.88	338.87	338.87	
	0.9	438.48	439.28	338.61	339.11	
	0.8	438.07	439.66	338.35	339.35	
	0.7	437.64	440.03	338.08	339.59	
	0.6	437.20	440.39	337.81	339.82	
Colon	0.5	436.75	440.74	337.52	340.04	[346.19, 370.67]
	0.4	436.29	441.08	337.23	340.25	
	0.3	435.81	441.42	336.93	340.46	
	0.2	435.32	441.74	336.62	340.67	
	0.1	434.82	442.05	336.30	340.87	
	0	434.30	442.36	335.97	341.06	
	1	498.65	498.65	364.36	364.36	
	0.9	498.30	498.99	364.13	364.58	
	0.8	497.94	499.32	363.89	364.80	
	0.7	497.58	499.64	363.65	365.01	
	0.6	497.20	499.95	363.40	365.22	
Melanoma	0.5	496.81	500.26	363.14	365.42	[358.60, 382.36]
	0.4	496.41	500.55	362.88	365.61	
	0.3	495.99	500.84	362.60	365.80	
	0.2	495.57	501.12	362.32	365.99	
	0.1	495.13	501.40	362.04	366.17	
	0	494.69	501.66	361.74	366.34	
	1	308.85	308.85	275.45	275.45	
	0.9	308.40	309.28	275.13	275.75	
	0.8	307.94	309.71	274.81	276.04	
	0.7	307.47	310.12	274.49	276.33	
	0.6	306.98	310.52	274.15	276.61	
Bladder	0.5	306.48	310.91	273.80	276.88	[264.26, 289.15]
	0.4	305.96	311.29	273.44	277.15	
	0.3	305.43	311.66	273.07	277.40	
	0.2	304.89	312.02	272.69	277.65	
	0.1	304.33	312.37	272.30	277.90	
	0	303.75	312.71	 271.90	278.14	

Table 3: Fuzzy and stochastic price of life settlement with Iran life table

insurance premiums, it has examined the effect of this uncertainty on the price of life settlement. The fuzzy results obtained from the deterministic and probabilistic approaches are compared with the stochastic approach and the results for the life tables of Iran and France have been analyzed.

In conducting any research, the researcher must pay attention to the nature of the problem and its limitations. One of the limitations we faced in this research was the lack of access to up-to-date data on patients with various types of cancer and the survival rate of these people in Iran. Although the aforementioned data for different types of cancer are updated every year on the website of the American

Type of cancer	α	Deterministic		Probab	pilistic	Stochastic
		VED	\overline{VED}	VEP	\overline{VEP}	95% confidence interval
	1	432.58	432.58	375.27	375.27	
	0.9	432.15	433.01	375.04	375.50	
	0.8	431.71	433.42	375.80	375.73	
	0.7	431.25	433.82	374.55	375.94	
	0.6	430.79	434.21	374.30	376.16	
Colon	0.5	430.31	434.60	374.04	376.36	[368.10, 392.56]
	0.4	429.82	434.97	373.77	376.57	
	0.3	429.31	435.33	373.50	376.76	
	0.2	428.80	435.69	372.22	376.96	
	0.1	428.27	436.04	372.93	377.14	
	0	427.72	436.37	372.64	377.33	
	1	493.70	493.70	401.08	401.08	
	0.9	493.33	494.07	400.87	401.28	
	0.8	492.92	494.42	400.66	401.48	
	0.7	492.55	494.77	400.44	401.67	
	0.6	492.14	495.11	400.22	401.86	
Melanoma	0.5	491.73	495.44	399.99	402.05	[410.21, 433.39]
	0.4	491.30	495.76	399.75	402.23	
	0.3	490.87	496.08	399.51	402.40	
	0.2	490.42	496.39	399.26	402.57	
	0.1	489.96	496.69	399.00	402.74	
	0	489.49	496.98	398.74	402.90	
	1	302.98	302.98	308.88	308.88	
	0.9	302.50	303.45	308.58	309.17	
	0.8	302.01	303.91	308.27	309.45	
	0.7	301.50	304.36	307.96	309.73	
	0.6	300.98	304.29	307.64	310.00	
Bladder	0.5	300.45	305.22	307.30	310.27	[299.31, 324.66]
	0.4	299.90	305.63	306.97	310.52	
	0.3	299.34	306.04	306.62	310.78	
	0.2	298.77	306.43	306.26	311.02	
	0.1	298.18	306.82	305.89	311.26	
	0	297.57	307.20	305.52	311.50	

Table 4: Fuzzy and stochastic price of life settlement with France life table

Cancer Society, no official source was found for the publication of these statistics in Iran. So, we used the statistics published in one of the articles about the survival probability of some cancers in Iran. Considering that the rate of contracting various diseases and the probability of survival of these people in different countries can be different based on the common lifestyle and type of climate. Therefore, access to this up-to-date information in Iran can help to make the calculations of the adjustment multiplier and the price of life settlement more accurate.

In addition, it is not possible to accurately determine the adjustment multiplier of mortality for some diseases such as high blood pressure or having diabetes due to the use of ambiguous terms. To solve this problem, the use of fuzzy adjustment multiplier has been suggested in some articles. For future researches, we will work on the simultaneous use of fuzzy parameters such as interest rate, internal rate of return and adjustment multipliers. In these conditions, a range including all possible states for the price of life settlement is obtained, which takes into account the uncertainty caused by the adjustment multiplier of mortality and the interest rate, and the investor can determine the adjustment factor based on his assessment of the insured health risk and the interest rate and the internal rate of return based on his assessment of the economic conditions and consider a number in this range as the price of the life settlement.

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