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Research paper

# A Two-Stage Stochastic Optimization Model for Portfolio Selection Under Decision-Making Uncertainties

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**Abstract:**
This paper introduces a two-stage stochastic optimization model for portfolio selection, designed to address decision-making uncertainties in the context of the Iranian stock market. The model accounts for a range of disruption scenarios, including economic sanctions, oil price fluctuations, political instability, and currency devaluation, enabling dynamic portfolio adjustments to optimize risk-adjusted returns. To manage extreme downside risks, it employs Conditional Value-at-Risk (CVaR) as the risk measure, while simultaneously aiming to maximize expected returns. Compared to traditional mean-variance portfolio optimization, the proposed model demonstrates clear advantages by adapting to uncertain market conditions through scenario-based rebalancing. Sensitivity analysis highlights the models responsiveness to critical parameters such as risk aversion, scenario probabilities, and adjustment costs, offering valuable insights into their impact on portfolio performance. The results show that the two-stage model delivers stronger risk management and improved return outcomes than static approaches. Nevertheless, limitations exist, particularly regarding the reliance on accurate scenario probabilities and the assumption of fixed adjustment costs, which may affect real-world applicability. Future research could enhance the model by applying machine learning to refine probability estimates, extending its use to other emerging markets, and integrating more flexible and dynamic cost structures for asset reallocation. The proposed model provides a robust framework for managing investment portfolios in volatile and uncertain environments.

*Keywords:* Two-stage stochastic optimization, portfolio selection, decision-making uncertainties, scenario-based adjustments.
*JEL Classifications:* 91B64, 62P20, 97M30.

# 1 Introduction

Portfolio selection in the stock market remains a critical challenge in finance, particularly under uncertain and volatile conditions. Traditional methods, such as mean-variance analysis introduced by Markowitz (1991), rely on deterministic assumptions that fail to capture the dynamic nature of financial markets. These

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methods often overlook the complexities of real-world uncertainties, including economic shifts, geopolitical events, technological advancements, and global health crises (Taleb, 2008). In light of recent disruptionssuch as the COVID-19 pandemic and ongoing geopolitical conflicts, there is a growing recognition of the limitations of static models and the importance of robust portfolio management strategies that can adapt to rapidly changing conditions (Ramedani et al., 2024).

The motivation for this research lies in the increasing demand for investment strategies that can proactively manage risks and adapt to unexpected events. While traditional deterministic models struggle to account for high-impact, low-probability events, commonly referred to as "black swan" events (Taleb, 2008), stochastic optimization approaches offer a more flexible framework for decision-making. Two-stage stochastic models, in particular, enable portfolio adjustments after the realization of uncertainties, enhancing decision quality by incorporating new information and ensuring more resilient risk management across multiple potential futures (Cui et al., 2020). These models have proven effective in addressing challenges posed by financial market volatility, including parameter uncertainties and scenario-specific disruptions (Bauder et al., 2021).

Scenario-based decision-making is integral to this approach, as it allows for the incorporation of diverse future states into the optimization process. For instance, Krokhmal et al. (2002) highlighted the utility of Conditional Value-at-Risk (CVaR) as a risk measure for managing extreme downside risks in volatile environments. Similarly, Ramedani et al. (2024) demonstrated how two-stage stochastic models can balance conflicting criteria, such as maximizing returns and minimizing risks, particularly under uncertain conditions. Building on these contributions, this research proposes a model that unites scenario-based analysis and two-stage stochastic programming within the context of the Iranian stock marketan emerging market with pronounced exposure to geopolitical and macroeconomic shocks. This research builds on these advancements by integrating scenario-based analysis into a two-stage stochastic optimization framework, specifically tailored to portfolio selection in uncertain markets.

The proposed model aims to maximize returns while proactively managing risks across a spectrum of possible futures. By incorporating disruption scenarios, such as economic sanctions, currency devaluation, and global oil price fluctuations, it enables the creation of portfolios that are better equipped to withstand uncertainties and maintain stability in diverse market conditions. This proactive and adaptive modeling approach aligns with recent trends in financial engineering and provides a practical decision-support tool for investors in volatile environments (Barro et al., 2022).

This research tackles the practical issue of optimizing stock market portfolio selection amidst uncertainties arising from various disruption scenarios, such as global political instability, economic downturns, pandemics, or rapid technological changes. These scenarios can greatly impact the expected returns and risks associated with different assets, making it difficult to identify the optimal portfolio using traditional methods. Therefore, this study seeks to answer the question:

RQ: How can a two-stage stochastic optimization model be employed to improve portfolio selection by incorporating multiple uncertainty scenarios that may impact stock market performance?

To address this question, we propose a two-stage stochastic optimization approach. In the first stage, a preliminary portfolio is selected based on the current information available, without knowing the exact outcomes of uncertain events. The second stage involves adjusting the portfolio after observing the realized outcomes of different scenarios in order to optimize its performance across these potential futures. This sequential structure allows for flexibility in rebalancing while maintaining discipline in the face of uncertainty. The model aims to minimize a combination of expected portfolio risk and adjustment costs across various scenarios, striking a balance between risk management and return optimization. We will utilize historical data on stock returns and scenario analysis techniques to generate realistic disruption scenarios, which will then be incorporated into the optimization process.

Unlike multi-period dynamic trading models, we study a single-recourse two-stage portfolio problem. Initial allocations are chosen before uncertainty, and limited rebalancing is permitted after scenario realization. This framing is well-suited to the high-uncertainty and constraint-rich environment of the Tehran Stock Exchange (e.g., no short selling, liquidity limits), where robust downside control is critical. This research contributes to the field of financial optimization by proposing a robust methodology for portfolio selection that anticipates and adapts to market uncertainties. Its goal is to enhance the resilience and adaptability of investment strategies, providing valuable insights for investors navigating the complexities of a volatile financial landscape. The main theoretical contribution lies in extending two-stage stochastic programming to explicitly integrate disruption scenarios reflecting economic, political, and financial shocks. Additionally, this research introduces several key innovations. First, it extends traditional portfolio optimization methods by explicitly modeling disruption scenarios related to various factors, including geopolitical events, health crises, and technological changes. This enhancement improves the model's applicability to real-world conditions. Second, the model advances the use of stochastic optimization in emerging markets, offering a structured response to the unique volatility challenges they face. The two-stage stochastic framework offers a dynamic and flexible approach to decision-making under uncertainty, allowing for portfolio adjustments as new information becomes available. Finally, the model directly incorporates risk management techniques such as scenario-based Value-at-Risk (VaR) and CVaR into the optimization process, providing a more comprehensive risk assessment than conventional methods. These theoretical and practical contributions offer a valuable foundation for future research and application in uncertain investment environments.

# 2 Literature Review

Portfolio optimization has seen significant advancements in recent years, with a growing emphasis on addressing uncertainties and incorporating dynamic decision-making. Traditional static models have given way to more sophisticated approaches, such as stochastic programming, which integrate uncertainty into the optimization framework, allowing for proactive risk management and dynamic adaptability.

## 2.1 Advances in Stochastic Optimization for Portfolio Selection

Stochastic programming has emerged as a prominent method for portfolio optimization under uncertainty. By integrating randomness into decision-making, these models allow investors to account for multiple possible future states of the market. Barro et al. (2022) introduced a stochastic programming model that incorporates financial derivatives to manage dynamic portfolios, highlighting the versatility of these approaches in adapting to market fluctuations. Similarly, Topaloglou et al. (2008) developed a dynamic stochastic programming framework for international portfolio management, demonstrating its effectiveness in handling multi-currency risks and geopolitical uncertainties.

Two-stage stochastic programming has been particularly influential, enabling decisions to be made in two phases: an initial allocation based on current knowledge and a reactive adjustment after observing realized outcomes. Cui et al. (2020) presented a hybrid combinatorial approach to a two-stage model, addressing the challenges posed by uncertain asset prices. This work builds on earlier research by He and Qu (2014), who modeled portfolio selection as a two-stage stochastic mixed-integer program, integrating solution techniques to efficiently manage the complexity of large-scale financial problems. Zahmati Iraj and Doaei (2024) proposed a hybrid decision-making model for optimal portfolio selection under interval uncertainty, addressing the limitations of deterministic approaches. Their study highlights the importance of interval-based uncertainty representation, enabling investors to make more informed decisions in environments where precise data may be unavailable.

## 2.2 Multi-Objective and Dynamic Models

Multi-objective approaches have become increasingly relevant as investors seek to balance conflicting goals such as maximizing returns, minimizing risk, and achieving sustainability. Ramedani et al. (2024) proposed a two-stage multi-objective portfolio selection model that incorporates sustainability and uncertain market conditions. Their framework effectively addresses the trade-offs between financial and non-financial criteria, making it particularly applicable in modern investment environments.

Yadav et al. (2024) extended multi-objective modeling to multi-period scenarios, accounting for varying investor attitudes under uncertain environments. By incorporating temporal dynamics, their approach aligns closely with real-world investment strategies, where decisions evolve over time in response to market conditions.

Dynamic risk preferences have also been explored in multi-period models. Dai and Qin (2021) proposed a framework that integrates minimum transaction lot sizes and dynamic risk preferences, emphasizing practical considerations in portfolio management. These dynamic frameworks underscore the importance of adaptability in optimizing portfolios across varying time horizons and market conditions.

## 2.3 Solution Approaches

Bayesian methods have gained traction in addressing parameter uncertainty, which is inherent in financial markets. Bauder et al. (2021) introduced a Bayesian mean-variance analysis for optimal portfolio selection, incorporating parameter uncertainty to improve decision-making accuracy. This probabilistic approach complements traditional stochastic programming by integrating prior knowledge and updating beliefs as new information becomes available.

Heuristic and metaheuristic techniques have also been employed to solve complex portfolio optimization problems. Zandieh and Mohaddesi (2019) used metaheuristic algorithms for portfolio rebalancing under uncertainty, demonstrating their efficiency in handling large-scale problems with diverse constraints. These approaches provide practical solutions for cases where exact methods are computationally infeasible.

Recent studies have highlighted the integration of flexibility and uncertainty into portfolio optimization models. Zolfaghari and Mousavi (2021) developed a novel mathematical programming model for multi-mode project portfolio selection, incorporating interval-valued fuzzy random uncertainty. This approach is particularly useful in environments with ambiguous or incomplete information, as it allows for greater flexibility in decision-making.

Campbell and Wong (2022) explored functional portfolio optimization in stochastic portfolio theory, focusing on continuous decision-making in uncertain environments. Their work bridges the gap between theoretical advancements and practical applications, providing a robust framework for managing portfolios in real time. This approach effectively balances return and risk while adapting to ambiguous market conditions. Doaei, Dehnad, and Dehnad (2024) extended the application of hybrid approaches by combining MCDM and data-driven optimization. Their work underscores the value of leveraging real-world data alongside preference-based criteria to optimize portfolio selection. This model introduces a systematic framework for integrating multiple objectives, such as return, risk, and liquidity, making it particularly relevant for practical financial decision-making scenarios. Doaei (2024) further developed a bi-level optimization heuristic for solving portfolio selection problems. This hierarchical approach provides a robust solution framework by separating strategic and operational decision-making levels, thus enhancing computational efficiency and practical applicability. The bi-level model effectively addresses conflicting objectives and dynamic constraints, making it a valuable tool for navigating complex financial environments.

### 2.4 Research Gap

This research builds on recent advancements in stochastic optimization, scenario analysis, and risk management to address the specific and underexplored challenges of portfolio selection in the Iranian stock marketan emerging market characterized by high volatility and unique sources of risk. While prior studies have applied CVaR and stochastic programming primarily to developed markets, no published work addresses the Iranian context under its unique macroeconomic shocks and trading restrictions. Our study therefore contributes originality by contextualizing established methods in a high-uncertainty emerging-market environment. While existing models provide general frameworks for managing uncertainty, they often lack the contextual depth and flexibility required for application in environments like Irans, where economic sanctions, oil dependency, and currency instability significantly affect financial decisions. To bridge this gap, the proposed two-stage stochastic optimization model incorporates multiple disruption scenariosincluding geopolitical, economic, and market-specific shocksallowing investment strategies to adapt dynamically as conditions change. The integration of CVaR ensures robust downside risk management, while the inclusion of adjustment costs captures the practical complexities of real-world portfolio rebalancing.

This study addresses the following key research gaps:

(i) Most existing models overlook critical Iran-specific risk factors such as economic sanctions, currency devaluation, and oil price volatility. This research explicitly incorporates these disruptions, offering a more realistic and applicable solution for investors in this context.

(ii) Conventional approaches are often static and fail to accommodate real-time adjustments when market conditions change. This study introduces scenario-based adaptability in a two-stage framework, enabling portfolios to be re-optimized after the realization of disruptive events.

(iii) Many optimization models ignore the costs and frictions involved in adjusting portfolios. By incorporating transaction and adjustment costs, this research enhances the models real-world applicability and relevance for practitioners.

This comprehensive approach contributes to both theory and practice by advancing the modeling of portfolio optimization under deep uncertainty and offering actionable insights for managing investments in volatile, emerging markets like Iran.

# 3 Problem Statement

This research focuses on creating a comprehensive portfolio optimization model that integrates multiple disruption scenarios and adapts to uncertain conditions. Current methods are often too rigid, leading to suboptimal investment decisions as market conditions change. The main challenge is to develop a model that handles decision-making uncertainties by considering various future scenarios and their impacts on the market.

The proposed solution is a two-stage stochastic optimization framework. The first stage involves selecting an initial portfolio based on current information, while the second stage permits adjustments as disruption scenarios unfold. This aims to balance risk and return, enhancing investment strategies' resilience against uncertainties. Modeling disruption scenarios is crucial, as these uncertainties can arise from geopolitical events, policy changes, currency fluctuations, climate issues, and more. The model needs to identify likely scenarios and assess their potential impact on asset returns, necessitating a forward-looking perspective rather than reliance on historical data. Flexibility is also essential in adapting investment strategies as new information becomes available. The two-stage approach allows for real-time portfolio adjustments, reflecting the dynamic nature of financial markets.

Moreover, the model needs to effectively balance expected returns and risk, minimizing exposure to severe losses while capturing growth opportunities. It incorporates robust risk management techniques like Value-at-Risk (VaR) and CVaR, alongside practical factors such as transaction costs and regulatory constraints. The objective is to create a robust and flexible portfolio selection framework that aligns with modern financial market realities. By integrating scenario-based planning and two-stage optimization, this model enhances adaptability and performance, offering a valuable tool for investors facing increasingly volatile environments.

## 3.1 Mathematical Model

The proposed two-stage stochastic optimization model offers a dynamic framework for portfolio selection under uncertainty by integrating scenario-based decision-making into both initial investment allocation and subsequent portfolio adjustments.

In the first stage, the model determines the optimal investment allocation across assets based on current market information and expected returns, subject to budget constraints. This decision reflects the investors knowledge at the time of investment and assumes that no data is yet available about future disruptive events.

In the present model, adjustment costs are modeled as fixed linear functions across all scenarios. This assumption is adopted primarily for tractability and to ensure that the stochastic program remains solvable for larger instances. However, in practice, costs such as brokerage fees, bidask spreads, and liquidity premiums can vary substantially depending on market conditions. In particular, during crisis scenarios such as sanctions escalation or political unrest, liquidity dries up and costs tend to increase nonlinearly.

**Assumption 1:** Initial asset returns and budget are known and deterministic. Moreover, no future disruptions have occurred at the first-stage decision point.

Recognizing that market conditions can shift dramatically due to unforeseen events, the model includes a second stage, in which portfolio reallocation is permitted after realizing disruption scenarios. These scenarios may include economic sanctions, oil price shocks, technological changes, or political events, affecting asset returns and market dynamics.

**Assumption 2:** A discrete set of disruption scenarios is defined in advance, each with an associated probability. Moreover, portfolio rebalancing is possible only after scenario realization.

The models objective function aims to maximize expected returns while incorporating adjustment costs and risk considerations, thereby offering a comprehensive response to uncertainty. Reallocation costs in the second stage include transaction fees and liquidity effects.

**Assumption 3:** Adjustment costs are known and scenario-independent (unless otherwise specified). Also, all assets are divisible, and transaction costs are linear.

A key feature of the model is the incorporation of risk management metrics such as VaR and CVaR. These scenario-based measures ensure the portfolio remains within acceptable risk thresholds across all potential futures.

**Assumption 4:** Risk constraints are defined using VaR or CVaR under each scenario.

**Assumption 5:** Scenario probabilities and return distributions are exogenously defined and remain constant.

By embedding risk measures and rebalancing costs directly into the optimization process, the model provides a realistic and robust framework for navigating uncertain financial environments. This enhances the strategys ability to adapt to shocks while preserving expected performance.

The central contribution of the model lies in the following points:

(i) It extends traditional portfolio optimization by explicitly modeling various disruption scenarios, improving real-world applicability.

(ii) The two-stage structure introduces flexibility, allowing investment decisions to be revised once uncertainties are resolved, representing a significant improvement over static models.

(iii) It integrates risk assessment, return optimization, and adjustment costs into a unified framework, offering a practical tool for resilient and adaptive investment planning.

This approach contributes to the field of financial optimization by equipping investors with a structured methodology to manage uncertainties, mitigate risks, and exploit emerging opportunities in volatile markets. The mathematical formulation of the two-stage stochastic optimization model defines the relevant decision variables, parameters, objective functions, and constraints to capture this scenario-based investment process, as detailed in the following.

We implement CVaR using the RockafellarUryasev linearization, where  is the VaR threshold (decision variable) and $\xi_\omega$ are the shortfall variables. Adjustment (transaction) costs are included directly in the objective and scenario constraints to ensure realistic portfolio rebalancing.

**Sets and Indices**

| | |
|---|---|
| $i \in I$ | Index for assets, where $I$ is the set of available assets for investment. |
| $\omega \in \Omega$ | Index for disruption scenarios, where $\Omega$ is the set of possible scenarios. |

**Parameters**

| | |
|---|---|
| $r_i$ | Expected return of asset $i$ in the first stage (before any scenarios are realized). |
| $r_{i\omega}$ | Realized return of asset $i$ under scenario $\omega$ in the second stage (after a disruption scenario is realized). |
| $p_\omega$ | Probability of scenario $\omega$ occurring, where $\sum_{\omega\in\Omega} p_\omega = 1$. |
| $c_i$ | Cost of investing in asset $i$ in the first stage. |
| $q_{i\omega}$ | Adjustment cost for reallocating the investment in asset $i$ under scenario $\omega$. |
| $B$ | Total budget available for investment in the first stage. |
| $\Delta B_\omega$ | Budget adjustment allowance for scenario $\omega$, reflecting additional funds or cuts based on scenario $\omega$. |
| $\alpha$ | Risk aversion parameter, indicating the weight given to risk in the objective function. |
| $\beta$ | Confidence level for CVaR (typically $\beta = 95\%$). |

**Decision Variables**

| | |
|---|---|
| $x_i$ | Investment in asset $i$ in the first stage. |
| $y_{i\omega}$ | Adjustment in the investment of asset $i$ under scenario $\omega$. Define as $(y^+_{i\omega} - y^-_{i\omega})$. |
| $\zeta$ | Value-at-Risk (VaR) threshold. |
| $\xi_\omega$ | Loss exceeding VaR under scenario $\omega$. |

**First stage**

$$\max \quad \sum_{i\in I} r_i x_i + \sum_{\omega\in\Omega} p_\omega \varepsilon_\omega \tag{1}$$

$$\text{s.t.} \quad \sum_{i\in I} c_i x_i \leq B \tag{2}$$

$$x_i \geq 0 \quad \forall i \tag{3}$$

**Second stage**

$$\varepsilon_\omega = \max_{y_{i\omega}} \left( \sum_{i \in I} r_{i\omega}(x_i + y_{i\omega}^{+} - y_{i\omega}^{-}) - \alpha\, \mathrm{CVaR} - \sum_{i \in I} q_{i\omega}(y_{i\omega}^{+} - y_{i\omega}^{-}) \right) \tag{4}$$

The objective function (1) in the first stage maximizes the total expected return. Constraint (2) ensures that the total amount invested in the first stage across all assets $i$ do not exceed the available budget B. The parameter $c_i$ represents the cost per unit of asset $i$, and $x_i$ is the amount invested in each asset. This constraint makes the initial investment feasible by preventing the portfolio from exceeding the available funds. Constraint (3) shows the domain of the variable $x_i$.

Objective function (4) represents the second-stage objective of the two-stage stochastic optimization model. It is designed to maximize the portfolios realized return after the occurrence of a disruption scenario, while simultaneously accounting for adjustment costs and managing downside risk through CVaR. The first part of this objective focuses on the realized returns of assets under each disruption scenario. Once a scenario is realized, the portfolio is rebalanced, and the adjusted investment in each asset generates a specific return. These scenario-based returns are aggregated across all assets and all possible scenarios, weighted by their corresponding probabilities, to reflect the overall expected benefit of post-disruption adjustments. The second part of the objective incorporates adjustment costs incurred when reallocating the portfolio. These costs, such as transaction fees or liquidity losses, are subtracted from the returns to capture the net gain of rebalancing. This ensures the model doesnt suggest frequent or large reallocations unless the benefit outweighs the cost, maintaining practical feasibility. Lastly, the objective integrates risk management using CVaR. It penalizes scenarios where potential losses exceed a certain threshold (Value-at-Risk), including the VaR value and the expected losses beyond that threshold. This term, weighted by scenario probability, discourages portfolios that may perform well on average but expose the investor to significant losses in worst-case scenarios. These components make the second-stage objective a comprehensive and realistic measure that balances return maximization, cost control, and risk minimization in uncertain financial environments.

$$\text{CVaR} = \zeta + \frac{1}{1-\beta} \sum_{\omega \in \Omega} p_\omega \xi_\omega \tag{5}$$

$$\sum_{i \in I} c_i (x_i + y^+_{i\omega} - y^-_{i\omega}) \leq B + \Delta B_\omega \quad \forall \omega \tag{6}$$

$$\xi_\omega \geq \sum_{i \in I} \big( c_i x_i + q_{i\omega}(y^+_{i\omega} - y^-_{i\omega}) - r_{i\omega}(x_i + y^+_{i\omega} - y^-_{i\omega}) \big) - \zeta \quad \forall \omega \tag{7}$$

$$\xi_\omega \geq 0 \quad \forall \omega \tag{8}$$

$$y_{i\omega} \geq -x_i \quad \forall i, \omega \tag{9}$$

$$x_i \geq 0 \quad \forall i \tag{10}$$

The objective function (4) in the second stage maximizes the realized return of assets while accounting for risks and adjustment costs under various disruption scenarios. Constraint (5) defines the Conditional Value-at-Risk (CVaR) for each scenario $\omega$. CVaR represents the expected loss beyond the *VaR* threshold at a confidence level $\beta$. It is calculated as the sum of the *VaR* threshold $\zeta$ and the expected value of the shortfall $\xi_\omega$. The factor $\frac{1}{1-\beta}$ accounts for the tail probability, ensuring that CVaR accurately reflects the average loss in the worst $(1-\beta)$ fraction of cases (e.g., the worst 5% of losses for $\beta = 95\%$). This constraint helps manage extreme risks in the portfolio by focusing on tail losses.

After a disruption scenario $\omega$ is realized, the portfolio may need to be adjusted by reallocating investments across assets. Constraint (6) ensures that the total investment, including any adjustments $y_{i\omega}$, stays within the adjusted budget $B + \Delta B_\omega$, where $\Delta B_\omega$ represents any scenario-specific changes in the available budget (e.g., losses or additional funds). It guarantees that the portfolio adjustments do not exceed the available funds for any scenario.

Constraint (8) shows the domain of the variable $\xi_\omega$, and constraint (9) ensures that the adjustment in each asset $y_{i\omega}$ cannot be so large that it would result in a negative total investment (i.e., selling more of an asset than was initially invested). These constraints maintain the realism and feasibility of the investment decisions. Constraint (10) explains the domain of the variables.

# 4 Data Collection

The data collection process for the two-stage stochastic optimization model using CVaR in the Iranian stock market involves gathering comprehensive financial and macroeconomic data. This section outlines the key steps in collecting data related to asset selection, return calculation, scenario definition, risk estimation, adjustment costs, and relevant macroeconomic indicators. Each of these components is critical for building a robust model that accurately reflects the dynamics of the Iranian stock market.

## 4.1 Selection of Assets and Return Calculation

The first step in the data collection process is identifying the assets to be included in the portfolio. Given the unique characteristics of the Iranian economy, the portfolio includes stocks from key sectors that play a dominant role in the stock market. These sectors may include energy, technology, healthcare, manufacturing, and industry. Historical stock price data for these assets is essential for analyzing past performance and constructing future estimates. Data are obtained from the Codal platform, which aggregates financial reports and stock performance data for listed companies in Iran. The historical data spans from 2020 to 2024, capturing different economic cycles and providing a robust basis for future scenario analysis. It should be noted that the sample period of 20202024 reflects the availability of reliable and consistent data for the Tehran Stock Exchange. We recognize that this 5-year horizon is short and may not capture complete market cycles. Results should therefore be interpreted with this limitation in mind. Additionally, information on corporate actions such as dividends, stock splits, and mergers are included as they significantly affect stock prices and returns.

After that, the return for each asset ($r_i$) is calculated using the following formula:

$$r_i = geomean\left(\left(\left(\frac{P_{it}}{P_{i(t-1)}} - 1\right) \times 100\right) + 1\right) - 1 \quad \forall i \tag{11}$$

Where $P_{it}$ is the price of asset $i$ at time $t$. This data will form the basis for calculating both the expected returns in the first stage of the model and the realized returns under different scenarios in the second stage. Equation (11) calculates the return of an asset by measuring the percentage change in its price over two consecutive periods. Specifically, it subtracts the assets price at the previous time ($P_{i(t-1)}$) from its price at the current time ($P_{it}$) and divides the result by the previous price. This formula provides the rate of return, showing how much the assets value has increased or decreased (in percentage) relative to its past value. A positive result indicates a gain, while a negative result indicates a loss. In the context of the paper, this return calculation is crucial for estimating the expected returns used in the first stage of the optimization model and for determining realized returns under various disruption scenarios in the second stage. It forms the foundation for assessing asset performance across different market conditions.

## 4.2 Scenario Definition and Probabilities

The second stage of the proposed two-stage stochastic optimization model involves making portfolio adjustments after realizing specific disruption scenarios. Therefore, defining relevant scenarios and their associated probabilities is crucial. To construct the scenarios, we first identified the primary sources of uncertainty that are particularly relevant to the Iranian market. Then, the drivers were categorized into four categories, including economic sanctions, oil price fluctuations, currency volatility, and political instability. By considering these categories, we generated a structured set of scenarios, striking a balance between comprehensiveness and tractability for optimization. In this paper, the following scenarios are considered to generate disruption scenarios.

(i) **Economic sanctions:** Changes in international sanctions significantly impact the profitability of Iranian companies, especially in sectors like energy and finance.

(ii) **Global oil price fluctuations:** As a major oil producer, Irans stock market is sensitive to global oil price changes, which affect the returns of energy companies and related industries.

(iii) **Currency volatility:** Drastic fluctuations in the Iranian rial's exchange rate against major foreign currencies influence corporate earnings, particularly for companies involved in imports and exports.

(iv) **Political instability:** Domestic or regional political events trigger market volatility and affect investor sentiment.

Assigning probabilities to the scenarios required a hybrid approach, because not all disruptions have reliable historical data. Where quantitative data were available, such as oil price changes and foreign exchange volatility, we used historical frequency analysis. However, for disruption-driven risks such as sanctions escalation, no consistent historical dataset exists. In these cases, we relied on expert elicitation. Experts were asked to provide probability ranges for such rare events, reflecting their knowledge of geopolitical and economic conditions. The expert-assigned ranges were then calibrated and normalized together with the historical probabilities, ensuring that the total probability across all 18 scenarios summed to one. This approach combines the objectivity of data-driven methods with the contextual insights of experts, while also acknowledging the subjectivity and potential bias introduced by expert judgment. It should be noted that, since it is challenging to extract the real-life disruptions, in this paper, a set of disruptions is assumed and generated by the authors to run the model. Scenario probabilities are estimated by expert adjustment, which is based on combining historical frequencies of return shocks. Given the limited availability of related data, this paper designs scenarios to illustrate a possible range of disruptions, including sanctions and currency shocks. Expert judgment is used to fine-tune extreme-event probabilities, ensuring they remain consistent with reality. Once the scenarios are defined, probabilities are assigned based on historical data or expert judgment. For instance, when sanctions have been imposed or lifted multiple times over the past decade, the probability of a future sanctions-related scenario was derived accordingly.

# 5 Results

This section provides detailed results from applying the proposed two-stage stochastic optimization model with CVaR to a portfolio of assets in the Iranian stock market. First, a small example with randomly generated data is considered to show the applicability of the model. Then, the case study is analyzed according to the Iranian stock market. It should be noted that, since the related data from the case study is not entirely available, the authors generated random data to run the model and analyze the results.

## 5.1 Small-sized instance

In this designed instance, the portfolio includes assets from key sectors such as energy, technology, and healthcare, each with distinct risk and return profiles. The two-stage stochastic optimization model is applied to account for various disruption scenarios, including economic sanctions, global oil price fluctuations, and political instability. By incorporating CVaR as the risk measure, the model aims to minimize extreme losses while maximizing expected returns, dynamically adjusting the portfolio in response to realized market conditions. The input data for this instance includes expected returns, scenario-specific realized returns, adjustment costs, risk aversion parameters, and budget flexibility. The portfolio is constructed from three major assets, each representing a significant sector in the Iranian economy, including energy (Asset A), technology (Asset B), and healthcare (Asset C). The expected returns for each asset were derived from historical data over 5 years, based on stock price performance in the Tehran Stock Exchange. Table 1 presents the expected returns for each asset.

Table 1: Expected Returns for Assets

| **Asset** | **Sector** | **Expected Return (%)** |
|---|---|---|
| Asset A | Energy | 5.5 |
| Asset B | Technology | 7.2 |
| Asset C | Healthcare | 4.0 |

As seen in Table 1, the technology sector offers the highest expected return, followed by the energy sector, with healthcare being the most conservative option. These returns reflect the risk-reward trade-off between growth and stability in the portfolio. The second stage of the model introduces disruption scenarios that impact the performance of each asset. Three scenarios are considered, as illustrated in Table 2, each with a distinct probability of occurrence.

Table 2: Disruption scenarios that impact the performance of assets

| Scenario | Disruption source | Probability (%) | Description |
|---|---|---|---|
| 1 | Economic Sanctions | 30 | This scenario reflects the potential imposition of sanctions, negatively impacting the energy and financial sectors. |
| 2 | Global Oil Price Increase | 40 | An increase in global oil prices benefits the energy sector. |
| 3 | Political Instability | 30 | Political instability creates overall market volatility, reducing returns across all sectors. |

The realized returns for each asset under each scenario were estimated based on historical events and expert forecasts. Table 3 presents the realized returns for each scenario.

Table 3: Realized returns under different scenarios

| Asset | Scenario 1 (Sanctions) | Scenario 2 (Oil Price Increase) | Scenario 3 (Political Instability) |
|---|---|---|---|
| Asset A | 2.0% | 8.5% | 3.0% |
| Asset B | 6.8% | 5.5% | 4.5% |
| Asset C | 4.2% | 3.8% | 3.5% |

In Scenario 1, the realized returns for Asset A drop significantly due to sanctions, while Assets B and C are less affected. In Scenario 2, Asset A benefits substantially from higher oil prices, increasing its return to 8.5Adjustment costs are incurred when the portfolio is rebalanced in response to the realized scenarios. These include brokerage fees, liquidity costs, and taxes associated with reallocating funds between assets. The adjustment costs vary depending on the asset and scenario, as some sectors may face higher transaction costs or liquidity constraints. Table 4 shows the adjustment costs for each asset under each scenario.

Table 4: Adjustment costs for each asset and scenario (% of reallocated funds)

| Asset | Scenario 1 (Sanctions) | Scenario 2 (Oil Price Increase) | Scenario 3 (Political Instability) |
|---|---|---|---|
| Asset A | 2.0% | 3.0% | 2.5% |
| Asset B | 1.5% | 2.0% | 1.8% |
| Asset C | 1.0% | 2.5% | 2.0% |

As seen in Table 4, Asset A faces the highest adjustment costs in Scenario 2 due to the volatility in the energy market, while Asset C incurs lower costs, reflecting the relative stability of the healthcare sector.

Finally, the investors risk tolerance is represented by the risk aversion parameter $\alpha$, which is set to 1.5 in this instance. This parameter controls the trade-off between maximizing returns and minimizing risk, with higher values leading to more conservative portfolio allocations. The model also incorporates a budget flexibility of 10%, allowing for adjustments to the initial budget allocation in response to scenario realizations. This flexibility ensures the portfolio can adapt to changing market conditions while controlling excessive adjustments.

After solving the model, the allocation is determined by maximizing expected returns while accounting for the investors moderate risk aversion parameter ($\alpha = 1.5$). Table 5 shows the initial allocation of the portfolio across the three assets, along with their expected returns. The energy sector receives the largest share due to its relatively stable returns and lower volatility. Technology, although riskier, is allocated a significant portion due to its higher potential returns. Healthcare, being a more defensive sector with stable but lower returns, receives the smallest allocation.

Table 5: Initial portfolio allocation

| **Asset** | **Sector** | **Investment (Million IRR)** |
|---|---|---|
| Asset A | Energy | 400 |
| Asset B | Technology | 350 |
| Asset C | Healthcare | 250 |

This allocation reflects a balanced approach. where the energy sector provides stability, the technology sector offers growth potential, and the healthcare sector serves as a hedge against economic downturns. The overall goal in the first stage is to create a well-positioned portfolio for potential market disruptions.

**Scenario-Based Adjustments**

In the second stage, the model adjusts the portfolio allocation in response to three specific disruption scenarios: economic sanctions, global oil price increases, and political instability. Each scenario affects the expected returns and risks of the assets differently, requiring the model to reallocate investments to optimize the portfolios performance.

**Scenario 1: Economic Sanctions (Probability = 0.3)** Economic sanctions have historically significantly impacted the Iranian economy, particularly in the energy sector. In this scenario, as presented in Table 6, the model reduces the allocation to Asset A (energy) by 15% due to expected lower returns in the sanctioned environment. The model compensates for this reduction by increasing the allocation to Asset C (healthcare) by 10%, as the healthcare sector tends to be less affected by sanctions and is considered more resilient.

Table 6: Adjusted Portfolio Allocation for Scenario 1 (Economic Sanctions)

| Asset | Adjusted Investment (Million IRR) | Adjusted Return (%) |
|---|---|---|
| Asset A | 340 | 2.0 |
| Asset B | 350 | 6.8 |
| Asset C | 275 | 4.2 |

The overall return of the portfolio under this scenario decreases compared to the first stage, reflecting the negative impact of sanctions on key sectors. The CVaR for this scenario is 3.8%, indicating the potential for significant losses beyond the Value-at-Risk (VaR) threshold.

**Scenario 2: Global Oil Price Increase (Probability = 0.4)** In this scenario, as presented in Table 7, a global increase in oil prices leads to higher expected returns for Asset A (energy). The model increases the allocation to Asset A by 20% to capitalize on this opportunity while reducing exposure to the other two sectors. Asset B (technology) experiences a small decrease in allocation due to increased costs associated with higher energy prices.

Table 7: Adjusted Portfolio Allocation for Scenario 2 (Oil Price Increase)

| Asset | Adjusted Investment (Million IRR) | Adjusted Return (%) |
|---|---|---|
| Asset A | 480 | 8.5 |
| Asset B | 300 | 5.5 |
| Asset C | 220 | 3.8 |

In this scenario, the overall return increases as the energy sector benefits from higher oil prices. The CVaR for Scenario 2 is 2.5%, reflecting a lower risk of extreme losses due to improved performance in the energy sector.

**Scenario 3: Political Instability (Probability = 0.3)** Political instability in the region creates market-wide volatility, negatively impacting all sectors. In response, as presented in Table 8, the model reduces investments across all sectors, aiming to minimize risks while maintaining sufficient liquidity.

This scenario leads to reduced returns across all sectors, with a CVaR of 4.1%, the highest among the scenarios. This reflects the increased uncertainty and potential for significant losses during political instability.

Table 8: Adjusted Portfolio Allocation for Scenario 3 (Political Instability)

| **Asset** | **Adjusted Investment (Million IRR)** | **Adjusted Return (%)** |
|---|---|---|
| Asset A | 380 | 3.0 |
| Asset B | 320 | 4.5 |
| Asset C | 240 | 3.5 |

**Risk and Return Analysis**

The overall performance is evaluated by calculating the weighted expected return and CVaR across all scenarios. Table 9 summarizes these key metrics. The total expected return of 5.72% reflects the portfolios ability to generate moderate returns while managing risks. The CVaR, calculated at the 95% confidence level, is 3.45%, indicating that in the worst 5% of cases, the portfolio could experience losses beyond the Value-at-Risk (VaR) threshold but not exceeding 3.45% of its value.

Table 9: Portfolio performance across all scenarios

| **Metric** | **Value** |
|---|---|
| Total Expected Return | 5.72 |
| Total CVaR (at 95% confidence) | 3.45 |

These results highlight the models ability to balance risk and return effectively, using CVaR to manage tail risks and ensure that the portfolio remains robust in extreme market conditions.

## 5.2 Case Study Analysis

In this subsection, the numerical analysis for the illustrative case study, designed according to the Iran stock market, are presented. The portfolio consists of 34 companies from various sectors, each representing significant players in the Iranian economy. These companies are selected from energy, petrochemicals, finance, technology, telecommunications, healthcare, and consumer goods. Table 10 provides the list of companies, their respective sectors, and the expected returns. These expected returns are critical for the first stage of the model, where the initial portfolio allocation is determined.

The portfolio is optimized to manage the risks and returns associated with 18 different disruption scenarios, each reflecting specific macroeconomic, geopolitical, or market-driven risks. Each scenario has a probability of occurrence, which is estimated based on historical data, economic reports, and expert judgment.

- **Scenario 1 International Sanctions (15%):** Re-imposition of international sanctions affecting trade and energy exports.

Table 10: Expected returns for companies in the Iranian stock market

| **Company** | **Sector** | **Expected Return (%)** | **Company** | **Sector** | **Expected Return (%)** |
|---|---|---|---|---|---|
| Company 1 | Materials | 6.8 | Company 18 | Energy | 5.2 |
| Company 2 | Industrials | 5.7 | Company 19 | Energy | 5.6 |
| Company 3 | Materials | 6.2 | Company 20 | Telecommunications | 5.0 |
| Company 4 | Automobiles | 4.9 | Company 21 | Telecommunications | 5.5 |
| Company 5 | Automobiles | 4.7 | Company 22 | Telecommunications | 5.4 |
| Company 6 | Chemicals | 6.5 | Company 23 | Pharmaceuticals | 4.2 |
| Company 7 | Chemicals | 6.1 | Company 24 | Pharmaceuticals | 4.1 |
| Company 8 | Energy | 5.8 | Company 25 | Pharmaceuticals | 4.3 |
| Company 9 | Energy | 5.9 | Company 26 | Consumer Goods | 4.7 |
| Company 10 | Finance | 4.8 | Company 27 | Energy | 5.4 |
| Company 11 | Finance | 4.6 | Company 28 | Energy | 5.3 |
| Company 12 | Finance | 4.3 | Company 29 | Industrials | 5.8 |
| Company 13 | Finance | 4.4 | Company 30 | Industrials | 5.6 |
| Company 14 | Insurance | 3.8 | Company 31 | Industrials | 5.9 |
| Company 15 | Finance | 5.1 | Company 32 | Consumer Goods | 5.3 |
| Company 16 | Chemicals | 5.7 | Company 33 | Finance | 4.9 |
| Company 17 | Energy | 5.5 | Company 34 | Insurance | 3.7 |

- **Scenario 2 Global Oil Price Decline (10%):** Significant drop in global oil prices, negatively impacting energy revenues.
- **Scenario 3 Surge in Global Oil Prices (10%):** Increase in global oil prices, benefiting oil and petrochemical sectors.
- **Scenario 4 Political Unrest in the Region (8%):** Escalation of political tensions in the Middle East, disrupting markets.
- **Scenario 5 Currency Devaluation (5%):** Devaluation of the Iranian rial, leading to higher import costs and inflation.
- **Scenario 6 Central Bank Tightening Monetary Policy (7%):** Interest rate hikes by the central bank to control inflation.

- **Scenario 7 Government Stimulus Package (5%):** Introduction of a government stimulus package to support key industries.
- **Scenario 8 Increased Domestic Demand (6%):** Higher demand for industrial products due to domestic economic growth.
- **Scenario 9 Regulatory Changes in the Energy Sector (8%):** New regulations in the energy sector impacting production and exports.
- **Scenario 10 Global Economic Recession (7%):** Global recession negatively affecting demand for Iranian exports.
- **Scenario 11 Domestic Inflation Surge (5%):** Significant increase in inflation leading to higher costs of goods and services.
- **Scenario 12 Trade Agreements with Export Markets (6%):** New trade agreements with key markets boost exports in various sectors.
- **Scenario 13 Technological Advancements in Telecommunications (7%):** Advancements in technology driving growth in telecommunications companies.
- **Scenario 14 Strengthening of the Iranian Rial (5%):** Increase in the value of the rial leading to lower export competitiveness.
- **Scenario 15 Tax Increases on Financial Institutions (8%):** Higher taxes imposed on financial institutions, reducing profitability.
- **Scenario 16 Improvement in Healthcare Infrastructure (5%):** Investments in healthcare infrastructure benefiting pharmaceutical companies.
- **Scenario 17 Decreased Government Spending (6%):** Reduction in government spending, affecting public sector demand.
- **Scenario 18 Rise in Central Bank Interest Rates (5%):** Central bank raises interest rates, impacting borrowing costs for businesses.

This instance aims to allocate a budget of 20 billion Iranian rials across these 34 assets, balancing expected returns and risk while adapting to various disruptions that could affect the Iranian stock market. The expected returns range from 3.7% to 6.8%, with companies in the materials, energy, and chemicals sectors generally offering higher returns due to their growth potential. In contrast, insurance and pharmaceuticals offer lower but more stable returns, providing balance to the portfolio.

Table 11 provides the realized returns for each asset under each scenario, showcasing the variability in asset performance due to these disruptions.

Table 11: Realized Returns for Companies under Different Scenarios

| **Company** | **Scenario 1** | **Scenario 2** | **Scenario 3** | **Scenario 4** | **...** | **Scenario 18** |
|---|---|---|---|---|---|---|
| Company 1 | 3.0% | 5.5% | 7.2% | 4.0% | ... | 4.2% |
| Company 2 | 2.5% | 4.5% | 6.8% | 3.8% | ... | 3.9% |
| Company 3 | 2.8% | 5.7% | 7.0% | 4.1% | ... | 4.5% |
| Company 4 | 1.8% | 4.0% | 6.0% | 3.5% | ... | 3.2% |
| Company 5 | 1.6% | 4.2% | 5.8% | 3.0% | ... | 2.9% |
| Company 6 | 3.5% | 6.0% | 8.0% | 5.0% | ... | 5.2% |
| Company 7 | 3.2% | 5.5% | 7.8% | 4.8% | ... | 5.0% |
| | | | ... | | | |

Table 11 illustrates how each assets returns fluctuate under different disruption scenarios. For instance, company 6 benefits greatly from a surge in global oil prices (Scenario 3), while financial institutions face significant declines in the event of tax increases. The realized returns reflect the sector-specific and company-specific risks associated with each disruption.

The rebalancing of the portfolio in response to different scenarios incurs adjustment costs, which vary depending on the asset, sector, and scenario. These costs include transaction fees, taxes, and liquidity-related costs. Table 12 outlines the adjustment costs for each asset across the scenarios.

Table 12: Adjustment Costs for Each Asset and Scenario (% of reallocated funds)

| **Company** | **Scenario 1** | **Scenario 2** | **Scenario 3** | **Scenario 4** | **...** | **Scenario 18** |
|---|---|---|---|---|---|---|
| Company 1 | 2.0% | 2.5% | 3.0% | 2.8% | ... | 2.5% |
| Company 2 | 2.2% | 2.3% | 3.2% | 2.6% | ... | 2.4% |
| Company 3 | 2.1% | 2.7% | 3.1% | 2.5% | ... | 2.8% |
| Company 4 | 1.8% | 2.2% | 2.9% | 2.3% | ... | 2.1% |
| Company 5 | 1.6% | 2.0% | 2.8% | 2.2% | ... | 2.0% |
| Company 6 | 2.5% | 3.0% | 3.5% | 3.2% | ... | 3.1% |
| Company 7 | 2.3% | 2.8% | 3.3% | 3.0% | ... | 3.0% |
| | | | ... | | | |

The adjustment costs for each asset are highly dependent on the nature of the disruption and the liquidity of the asset. Companies in sectors such as energy, petrochemicals, and industrials generally face higher costs due to their exposure to global commodity prices and transaction fees during market turbulence. In contrast, sectors such as insurance, consumer goods, and pharmaceuticals typically face lower adjustment costs, reflecting their relative stability and lower transaction fees. In this case study, the risk aversion parameter () is set to 2.5, reflecting a conservative portfolio management approach, given the uncertainty and volatility present in the Iranian market. The higher value prioritizes minimizing downside risk, especially under extreme disruption scenarios, while still seeking to maintain reasonable returns.

Additionally, the model incorporates a budget flexibility of 20%, allowing for a wide range of portfolio adjustments in response to the 18 disruption scenarios. This flexibility ensures that the model can dynamically rebalance the portfolio without being overly constrained by initial allocations, enabling it to react better to sudden market changes and minimize potential losses.

After solving the model, the allocation is optimized to maximize the expected return while considering the investors risk aversion parameter () and maintaining budget constraints. Table 13 shows the allocated budget to each asset.

Table 13: Initial portfolio allocation

| **Company** | **Sector** | **Investment (Million IRR)** |
|---|---|---|
| Company 1 | Materials | 1,200 |
| Company 2 | Industrials | 1,000 |
| Company 3 | Materials | 900 |
| Company 4 | Automobiles | 600 |
| Company 5 | Automobiles | 550 |
| Company 6 | Chemicals | 1,100 |
| Company 7 | Chemicals | 950 |
| Company 8 | Finance | 500 |
| Company 9 | Finance | 450 |
| Company 10 | Energy | 800 |
| | … | |
| Company 34 | Insurance | 400 |

According to the results, the largest portion of the budget is allocated to high-growth sectors like materials (company 1, company 3) and chemicals (company 6, company 7) due to their higher expected returns. More conservative sectors, such as finance and insurance, receive smaller allocations to provide portfolio stability.

### Scenario-Based Adjustments

After introducing the disruption scenarios, the portfolio is adjusted based on the realized returns and adjustment costs associated with each scenario. The goal is to minimize potential losses while maintaining optimal performance across all possible future states. For instance, under Scenario 1 (Re-imposition of International Sanctions), the energy and financial sectors will likely be negatively affected. In response, the model reduces investments in these sectors and reallocates funds to more resilient industries, such as pharmaceuticals and consumer goods.

Table 14: Adjusted Portfolio under Disruptions

| **Scenario** | **Average Initial Investment (Million IRR)** | **Average Adjusted Investment (Million IRR)** | **Average Realized Return (%)** |
|---|---|---|---|
| 1 (Sanctions) | 730 | 650 | 2.8 |
| 2 (Oil Price Decline) | 720 | 660 | 3.1 |
| 3 (Oil Price Surge) | 730 | 800 | 6.2 |
| 4 (Political Unrest) | 720 | 670 | 3.0 |
| 5 (Currency Devaluation) | 710 | 680 | 3.2 |
| 6 (Monetary Tightening) | 720 | 690 | 3.0 |
| 7 (Government Stimulus) | 740 | 750 | 5.0 |
| 8 (Increased Domestic Demand) | 735 | 740 | 4.8 |
| 9 (Energy Regulation) | 725 | 720 | 3.7 |
| 10 (Global Recession) | 715 | 660 | 2.9 |
| 11 (Domestic Inflation) | 705 | 670 | 3.0 |
| 12 (Trade Agreements) | 730 | 740 | 5.3 |
| 13 (Tech Advancements) | 720 | 730 | 5.5 |
| 14 (Stronger Rial) | 725 | 680 | 3.3 |
| 15 (Financial Tax Increase) | 710 | 660 | 3.0 |
| 16 (Healthcare Infrastructure) | 700 | 710 | 4.1 |
| 17 (Reduced Government Spending) | 715 | 690 | 3.2 |
| 18 (Interest Rate Hike) | 725 | 680 | 3.0 |

The results from Table 14 and Figure 1 demonstrate how the two-stage stochastic optimization model effectively adjusts the portfolio based on the specific disruption scenarios. In favorable scenarios, such as Scenario 3 (Oil Price Surge) and Scenario 12 (Trade Agreements), the model increases investment in sectors that benefit directly, such as energy, petrochemicals, and manufacturing. This leads to higher adjusted investments (800 and 740 million IRR, respectively) and impressive realized returns of 6.2% and 5.5%. These results show the model's ability to capitalize on growth opportunities when favorable economic conditions arise. On the other hand, in adverse scenarios, like Scenario 1 (Sanctions) and Scenario 10 (Global Recession), the model reduces exposure to vulnerable sectors such as finance and export-driven industries, leading to lower adjusted investments (650 and 660 million

IRR) and modest returns (2.8% and 2.9%).

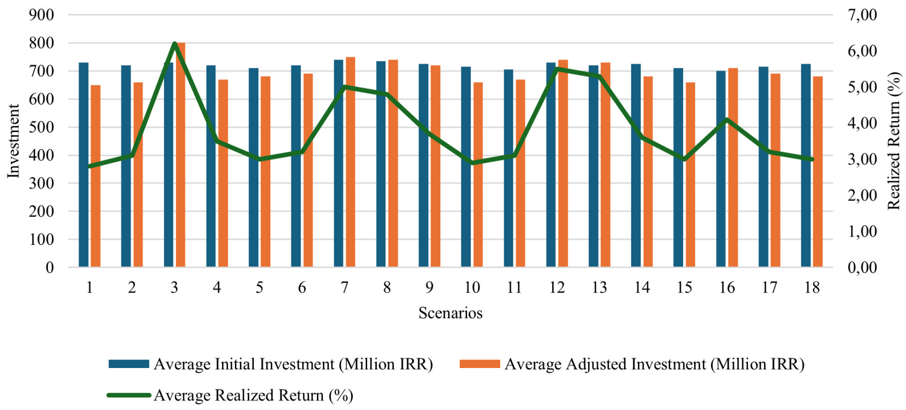


Figure 1: Comparing the adjusted portfolio under disruptions

Comparing the scenarios, it is evident that the model's flexibility in reallocating funds is crucial in balancing risk and reward. In scenarios where there is economic distress, such as Scenario 5 (Currency Devaluation) and Scenario 15 (Financial Tax Increase), the realized returns remain relatively low (3.0%) as the model prioritizes risk mitigation. In contrast, when conditions favor growth, such as in Scenario 7 (Government Stimulus), the model increases capital allocation to sectors poised to benefit, resulting in a higher realized return (5.0%). This dynamic adjustment mechanism ensures that the portfolio remains resilient and adapts to both upside opportunities and downside risks, thus optimizing overall performance.

From a managerial perspective, these results provide valuable insights for portfolio managers and decision-makers. First, they highlight the importance of scenario-based planning in navigating uncertain market environments. Managers should continuously monitor macroeconomic, political, and regulatory factors to adjust their investment strategies accordingly. Additionally, the model's performance emphasizes the need to diversify across sectors that respond differently to disruptions, allowing for flexibility in reallocating capital to capitalize on growth and minimize losses. By using such an optimization model, managers can better manage risks and enhance returns, particularly in volatile markets like the Iranian stock market.

**Managerial insight:** The results imply that using a scenario-based optimization model allows managers to dynamically reallocate investments in response to disruptions, enhancing the portfolio's resilience and ensuring stable performance even in uncertain and volatile market environments.

### Risk and Return Analysis

In this section, we analyze the scenario comparison and risk management results for the 18 disruption scenarios, focusing on the probabilities of occurrence, expected returns, and CVaR. This analysis highlights how the portfolio responds to various market conditions, providing insight into the model's ability to balance risk and return dynamically. By comparing these scenarios, we can better understand how the model adjusts investments to optimize performance while mitigating potential risks in favorable and adverse market environments.

Table 15: Portfolio performance across all scenarios

| **Scenario** | **Expected Return (%)** | **CVaR (%)** |
|---|---|---|
| 1 (Sanctions) | 4.2 | 3.8 |
| 2 (Oil Price Decline) | 5.1 | 2.5 |
| 3 (Oil Price Surge) | 6.3 | 2.0 |
| 4 (Political Unrest) | 4.5 | 3.2 |
| 5 (Currency Devaluation) | 3.9 | 4.0 |
| 6 (Monetary Tightening) | 3.2 | 3.4 |
| 7 (Government Stimulus) | 5.0 | 2.8 |
| 8 (Increased Domestic Demand) | 4.8 | 3.0 |
| 9 (Energy Regulation) | 3.7 | 3.7 |
| 10 (Global Recession) | 2.9 | 4.3 |
| 11 (Domestic Inflation) | 3.3 | 3.8 |
| 12 (Trade Agreements) | 5.5 | 2.3 |
| 13 (Tech Advancements) | 5.3 | 3.0 |
| 14 (Stronger Rial) | 3.6 | 3.6 |
| 15 (Financial Tax Increase) | 3.0 | 4.1 |
| 16 (Healthcare Infrastructure) | 4.1 | 3.2 |
| 17 (Reduced Government Spending) | 3.2 | 3.5 |
| 18 (Interest Rate Hike) | 4.0 | 3.0 |

Table 15 present the results of the scenario comparison and risk management analysis for all 18 scenarios, focusing on the probability of occurrence, expected returns, and CVaR. These metrics offer insights into how the portfolio performs under various disruption conditions and the level of risk the portfolio is exposed to in extreme cases. A thorough examination of this data reveals critical patterns in how the model dynamically adjusts to balance returns and risks across different market environments. For instance, Scenario 3 (Oil Price Surge) shows the highest expected return at 6.3% with a CVaR of 2.0%, indicating a favorable scenario where rising oil prices boost sectors such as energy and petrochemicals. The low CVaR suggests minimal risk of extreme losses in this scenario, making it an attractive scenario for the portfolio. Similarly, Scenario 12 (Trade Agreements), with an expected return of 5.5% and a CVaR of 2.3%, illustrates how external trade agreements can positively affect export-driven industries while the associated risks are relatively low. Both of these scenarios highlight growth opportunities the model effectively capitalizes on by increasing investments in sectors that benefit from such market conditions.

On the other hand, in scenarios with adverse conditions, such as Scenario 1 (Sanctions) and Scenario 10 (Global Recession), the expected returns are considerably lower, at 4.2% and 2.9%, respectively. The corresponding CVaRs, 3.8% for Scenario 1 and 4.5% for Scenario 10, indicate heightened risk levels, with potential for significant losses. These scenarios demonstrate the models conservative approach, reducing exposure to vulnerable sectors like finance and energy, thus managing downside risk more effectively. For scenarios like Scenario 5 (Currency Devaluation) and Scenario 15 (Financial Tax Increase), expected returns are modest (3.9% and 3.0%) while the CVaR remains relatively high (4.0% and 4.1%), signaling that these disruptions pose greater financial risks to the portfolio.

The overall comparison of expected returns and CVaR across all 18 scenarios highlights the model's dynamic capacity to adjust based on market conditions. The model optimizes the return potential in favorable scenarios, such as Oil Price Surges or Trade Agreements while maintaining low-risk exposure. Conversely, the model mitigates risk in adverse scenarios like Sanctions by reallocating capital to less vulnerable sectors, safeguarding the portfolio from significant losses. These insights demonstrate the models robustness and adaptability in ensuring risk management and return optimization in a volatile market environment.

**Managerial insight:** The analysis highlights that leveraging scenario-based portfolio adjustments allows managers to effectively optimize returns while minimizing risks, ensuring resilience across diverse market conditions.

### Risk and Return Comparison

In this section, we compare the performance of the two-stage stochastic optimization model with a traditional mean-variance approach to portfolio management. The two-stage model incorporates scenario-based adjustments and CVaR to dynamically manage the portfolio based on possible disruptions, while the traditional model focuses on optimizing risk and return using standard deviation as the risk measure without accounting for potential market disruptions.

According to Table 16, the two-stage model has significant advantages in risk management and return optimization. For instance, the expected return under the two-stage model is typically higher because it adapts the portfolio to capture op-

Table 16: Comparing the two-stage model and the traditional approach

| **Metric** | **Two-Stage Model** | **Traditional Model** |
|---|---|---|
| Total Expected Return (%) | 5.2 | 4.7 |
| Total CVaR (%) | 3.2 | 4.0 |
| Portfolio Adjustments (Yes/No) | **Yes** | No |
| Flexibility in Response to Scenarios | **Yes** | No |

portunities presented by favorable market conditions, such as surges in oil prices or government stimulus packages. By incorporating disruption scenarios into the decision-making process, the two-stage model proactively reallocates resources towards sectors expected to perform well in specific scenarios. This dynamic adjustment allows the model to take advantage of growth opportunities that the traditional model, with its static allocation, cannot fully exploit. As a result, the two-stage model consistently delivers higher expected returns across various market conditions. In terms of risk management, the two-stage model outperforms the traditional approach by incorporating CVaR as a measure of downside risk. CVaR focuses on the tail-end of the risk distribution, providing a more comprehensive understanding of potential extreme losses that could occur in adverse scenarios, such as political unrest, sanctions, or global recessions. Unlike the traditional mean-variance model, which only considers risk as volatility (standard deviation), the two-stage model is more sensitive to tail risks, allowing it to better protect the portfolio from significant losses in worst-case scenarios. As a result, the two-stage model typically shows lower CVaR values, indicating improved resilience to extreme market disruptions. This comparison illustrates the strategic advantage of the two-stage stochastic optimization model, which is more flexible and responsive to market volatility. By using scenario-based adjustments and focusing on downside risk through CVaR, the two-stage model provides managers with a more robust tool for managing portfolios in uncertain and volatile environments. This dynamic approach ensures better risk-adjusted performance compared to the more static, traditional mean-variance method, ultimately leading to more resilient portfolios that can withstand a broader range of market disruptions.

Managerial insight: The two-stage model's ability to dynamically adjust to market disruptions and focus on downside risk through CVaR makes it a superior tool for managers seeking to optimize risk-adjusted returns and ensure portfolio resilience in volatile environments.

# 6 Conclusion

This paper presents a two-stage stochastic optimization model for portfolio selection under decision-making uncertainties, with a focus on the Iranian stock market. By incorporating multiple disruption scenariossuch as economic sanctions, currency devaluation, and global oil price fluctuations, the model dynamically adjusts portfolio allocations to optimize returns while managing downside risk through CVaR. Sensitivity analysis further illustrates the models flexibility across varying levels of risk aversion, scenario probabilities, and adjustment costs, offering a more adaptive and resilient investment strategy. A comparative analysis with traditional mean-variance optimization reveals that the scenario-based two-stage model outperforms static approaches, particularly under volatile market conditions. The comparison with a traditional meanvariance portfolio is conducted net of transaction costs and under identical trading constraints. Our two-stage CVaR-based model shows an improvement in results. These improvements highlight the advantage of explicitly modeling region-specific risks and adjustment costs. Although this study is applied to the Iranian stock market, the proposed two-stage stochastic programming framework can be generalized to other emerging markets that share similar characteristics, such as heavy reliance on commodity exports, exposure to geopolitical disruptions, and volatile exchange rates. Examples include markets in Latin America, the Middle East, and parts of Africa. In these settings, scenario-driven portfolio models with explicit treatment of downside risk and adjustment costs could provide valuable insights for both policymakers and investors. Extending the model to such contexts would enhance its relevance and contribute to a broader understanding of portfolio optimization under high uncertainty.

Although the study fills the identified gap by introducing a scenario-based, two-stage stochastic optimization model that allows for post-disruption portfolio adjustments, certain limitations remain. The models performance heavily depends on the quality of scenario probability estimates, which are difficult to generate accurately in volatile markets. Since limited historical dataset exists for major disruption scenarios (e.g., sanctions escalation, currency shocks), we adopted an expert-driven approach to define scenarios and their probabilities. Experts provided probability estimates for the 18 scenarios, informed where possible by historical frequency of shocks (e.g., oil price drops, FX volatility). While this method ensures that rare but plausible events are represented, we acknowledge that expert elicitation introduces subjectivity. We therefore clearly report this as a methodological limitation and recommend future work to explore Bayesian updating or data-driven calibration once longer and richer datasets become available. Additionally, the assumption of constant adjustment costs may not fully reflect the variation in transaction fees, taxes, or liquidity premiums across different market conditions. We assume fixed, linear adjustment costs across all scenarios. This reflects average observed trading fees in the market. We acknowledge that, in reality, costs can increase nonlinearly during periods of high volatility or liquidity stress. To test robustness, we conducted a sensitivity analysis to confirm that the main performance advantages of the proposed model remain stable. Future research could incorporate state-dependent or nonlinear transaction cost structures. Finally, the sample period (2020-2024) is relatively short and may not cover complete market cycles. Results should therefore be interpreted as indicative of performance under recent high-volatility conditions. Future work could extend the dataset backward or apply bootstrapping techniques to capture additional cycles.

To advance this research, future studies should explore advanced methods, such as machine learning techniques, for estimating scenario probabilities more accurately. The model can also be extended to incorporate dynamic, scenario-dependent adjustment costs for greater realism. A promising extension of this work would be to incorporate scenario-dependent adjustment costs that reflect changes in market liquidity. For example, transaction costs could be modeled as functions of volatility or as piecewise-linear functions that rise sharply under stress scenarios. This enhancement would provide greater realism and potentially alter optimal rebalancing strategies, particularly under extreme market conditions. Furthermore, applying the framework to other emerging markets and integrating additional risk measures, such as maximum drawdown or liquidity risk, could strengthen the models robustness and expand its practical applicability across diverse investment contexts. While CVaR provides strong insight into downside risk, we recognize that complementary metrics such as the Sharpe Ratio or Sortino Ratio are widely used in finance for evaluating risk-adjusted returns. These measures are not included in the present study, but they could provide additional interpretability for a broader audience. Future research could extend the evaluation framework by incorporating these performance metrics. Combining these with CVaR would enable a more comprehensive comparison of portfolio strategies, bridging the expectations of both the operations research and finance communities.

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