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Research paper

Deep Learning-Based Option Pricing Under the SVSI Model: Incorporating Stochastic Volatility and Interest Rates

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Abstract:
The SVSI model is an advanced financial framework that simultaneously accounts for stochastic volatility and stochastic interest rates in the pricing of financial instruments such as options. The primary objective of this model is to enhance forecasting accuracy and risk assessment. In this study, an optimized deep learning model is proposed for option pricing under the SVSI framework, considering both dependence and independence between the underlying asset and the interest rate. The Monte Carlo method, combined with the conditional Monte Carlo variance reduction technique, is employed to generate optimal pricing data for options under the SVSI model. The performance of the proposed deep learning model is evaluated by comparing it with the generated data. Additionally, Bitcoin option pricing is examined using the proposed model.

Keywords: Option pricing, SVSI model, Conditional Monte Carlo, Deep learning, Bitcoin option data.
MSC2020 Classification: 91G20; 91G60.

1 Introduction

Option pricing remains one of the fundamental topics in financial economics because of its direct applications in asset valuation, risk management, and investment decision-making. Since the introduction of the Black–Scholes (BS) model [2], analytical pricing frameworks have become essential tools in derivatives markets. Despite its theoretical importance and broad practical use, the BS framework relies on simplifying assumptions that are often inconsistent with actual market behavior, particularly the assumptions of constant volatility and deterministic interest rates. Empirical observations indicate that financial markets exhibit volatility persistence, changing interest-rate structures, and nonlinear pricing behavior across maturities and market conditions. These limitations have motivated extensive research aimed

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at developing more realistic pricing frameworks capable of better representing market uncertainty.

One major line of research has focused on relaxing the assumption of deterministic interest rates. Amin and Jarrow [1] extended derivative valuation by introducing stochastic interest-rate dynamics into pricing and hedging environments, while empirical studies of short-term interest-rate behavior further supported the importance of modeling interest-rate uncertainty [15, 16]. A parallel research direction introduced stochastic volatility while maintaining fixed interest-rate assumptions. Influential contributions include alternative stochastic process models proposed by Cox and Ross [3], equilibrium-based approaches developed by Bailey and Stulz [4], and the closed-form stochastic volatility formulation introduced by Heston [5]. Subsequent extensions incorporating jump components further improved the ability of option pricing models to capture market discontinuities and asymmetric return distributions [6]. Comparative empirical analysis by Bakshi et al. [22] demonstrated that models incorporating additional stochastic components, including stochastic volatility and stochastic interest rates, outperform traditional constant-parameter formulations in explaining observed option market behavior.

Building upon these developments, the Stochastic Volatility with Stochastic Interest (SVSI) framework emerged as a more comprehensive approach that simultaneously captures uncertainty in asset prices, volatility dynamics, and interest-rate evolution [7, 8]. By allowing both volatility and discounting mechanisms to evolve over time, SVSI models improve valuation flexibility and enhance derivative pricing performance, particularly for long-maturity and interest-sensitive contracts. Within this framework, the interaction between stochastic drivers plays an important role in determining analytical tractability and computational efficiency. Previous studies have shown that simplified assumptions may permit closed-form pricing solutions [9], whereas more realistic settings generally require numerical procedures such as Monte Carlo and Conditional Monte Carlo simulation methods [10, 11].

Recent advances have expanded SVSI models beyond conventional Markovian diffusion structures toward richer stochastic environments. New developments increasingly incorporate rough volatility, fractional behavior, memory effects, stochastic intensity, regime-switching mechanisms, liquidity considerations, and multifactor interest-rate dynamics. Abi Jaber et al. [23] proposed the Volterra Stein–Stein framework with stochastic interest rates, introducing correlated Gaussian Volterra processes to jointly describe volatility and interest-rate dynamics while preserving computational tractability. Mittal and Selvamuthu [24] further expanded hybrid option pricing structures by integrating Heston stochastic volatility, Hull–White stochastic interest rates, stochastic intensity, and Markov regime switching, demonstrating improved pricing flexibility under changing market regimes. Additional extensions introduced liquidity and credit-sensitive features into SVSI environments. He and Lin [25] showed that liquidity conditions can materially affect derivative valuation under stochastic volatility settings, whereas Jeon and Kim [26] developed analytical pricing solutions for vulnerable options under Heston volatility combined with two-factor stochastic interest-rate processes. More recently, unified frameworks integrating stochastic volatility, jump processes, and stochastic interest-rate structures have been proposed to improve pricing performance across different horizons and market conditions [27].

Alongside these theoretical developments, advances in artificial intelligence and machine learning have created new opportunities for financial pricing applications. Machine learning methods have increasingly been adopted to approximate complex pricing functions and reduce the computational burden associated with simulation-based valuation approaches [12, 13]. Deep learning architectures, in particular, have demonstrated strong performance in identifying nonlinear relationships and approximating high-dimensional option pricing functions [19]. These developments suggest that data-driven approaches may complement traditional financial models in environments characterized by multiple interacting stochastic factors. Recent attention toward artificial intelligence in financial applications also reflects broader technological trends accelerating across industries [18].

Motivated by these developments, this study investigates option pricing under the SVSI framework by integrating simulation-based valuation methods with deep learning techniques. Specifically, the Black–Scholes model is employed as a benchmark, while Standard Monte Carlo and Conditional Monte Carlo approaches are used to generate valuation data for training and evaluating neural network architectures. The objective is to develop an efficient predictive framework capable of approximating option prices under jointly stochastic volatility and stochastic interest-rate dynamics while maintaining pricing accuracy across diverse market conditions.

The remainder of this article is as follows: after a brief introduction, Section 2 examines the pricing of European call options under the SVSI model in two cases: with and without correlation between the interest rate process and the underlying asset price process. Section 3 explores machine learning concepts in finance and Section 4 presents a deep learning based option pricing prediction model. The results of the deep learning prediction model are discussed in Section 5. In Section 6, the methodologies have been effectively showcased through an in-depth analysis of Bitcoin option data, highlighting their practical utility in real-world scenarios, and finally, Section 7 summarizes the findings and draws conclusions.

2 Asset Model and Option Pricing

Contemporary research has extended the classical option pricing framework by incorporating stochastic interest rates and stochastic volatility into asset pricing models. These extensions provide a more flexible representation of financial markets and improve the ability to capture observed market dynamics. In particular, the stochastic volatility–stochastic interest rate (SVSI) framework allows option values to depend jointly on the evolution of asset prices, volatility, and interest rates.

The standard SVSI valuation framework characterizes option prices as functions of three state variables: the underlying asset price, instantaneous variance, and short-term interest rate. Both volatility and interest rates are modeled as mean-reverting stochastic processes under Markovian dynamics. The model contains seven parameters, which may be estimated using the Generalized Method of Moments (GMM) introduced by Hansen (1982) [14] and employed in Andersen and Lund (1997) [15] and Chan et al. (1992) [16]. Following [9], option valuation is performed under a stochastic volatility and stochastic interest rate setting.

Consider a European call option written on an underlying asset with price $S(t)$, strike price K, and maturity T. Let $C(t)$ denote the option value at time t.

To formulate the pricing framework rigorously, let

$$(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{Q})$$

denote a filtered probability space satisfying the usual conditions, where $\mathbb{Q}$ is the equivalent martingale (risk-neutral) measure and $\{\mathcal{F}_t\}$ is the filtration jointly generated by the stochastic processes $S(t)$, $V(t)$, and $R(t)$. Under the measure $\mathbb{Q}$, discounted asset prices evolve as martingales.

Since (S, R, V) forms a joint Markov process, the option price $C(t)$ is represented as a function of the state variables $S(t)$, $V(t)$, and $R(t)$. The dynamics of the stock price, stochastic volatility, and stochastic interest rate are governed by the following stochastic differential equations (SDEs) under the risk-neutral measure [9]:

$$\begin{aligned} \frac{dS(t)}{S(t)} &= R(t)\,dt + \sqrt{V(t)}\,dW_S^{\mathbb{Q}}(t) \\ dR(t) &= [\theta_R - \bar{k}_R R(t)]dt + \sigma_R\sqrt{R(t)}\,dW_R^{\mathbb{Q}}(t) \\ dV(t) &= [\theta_V - \bar{k}_V V(t)]dt + \sigma_V\sqrt{V(t)}\,dW_V^{\mathbb{Q}}(t) \end{aligned} \tag{1}$$

where $W_S^{\mathbb{Q}}(t)$, $W_R^{\mathbb{Q}}(t)$, and $W_V^{\mathbb{Q}}(t)$ are standard Brownian motions under the probability measure $\mathbb{Q}$.

The instantaneous correlation structure among the Brownian motions is specified through

$$dW_i^{\mathbb{Q}}(t)dW_j^{\mathbb{Q}}(t) = \rho_{ij}\,dt,$$

with correlation matrix

$$\Sigma = \begin{bmatrix} 1 & \rho_{S,R} & \rho_{S,V} \\ \rho_{S,R} & 1 & 0 \\ \rho_{S,V} & 0 & 1 \end{bmatrix},$$

where $\rho_{S,R} \in [-1, 1]$ represents the correlation between asset return and interest-rate shocks, $\rho_{S,V} \in [-1, 1]$ captures the dependence between asset return and volatility shocks, and $\rho_{R,V} = 0$ assumes independence between interest-rate and volatility innovations.

The model parameters are defined as follows: k_V denotes the mean-reversion rate of variance toward its long-run level, θ_V denotes the long-run variance, and σ_V represents volatility of volatility. Similarly, k_R denotes the mean-reversion rate of interest rates, θ_R denotes the long-run interest-rate level, and σ_R represents interest-rate volatility.

Furthermore,

$$k_R = \bar{k}_R + \lambda_R, \qquad k_V = \bar{k}_V + \lambda_V,$$

where λ_R and λ_V denote market prices of interest-rate and volatility risk, respectively. Parameters $\bar{k}_R$ and $\bar{k}_V$ determine the speed of adjustment toward long-run equilibrium levels given by

$$\frac{\theta_R}{\bar{k}_R}, \qquad \frac{\theta_V}{\bar{k}_V}.$$

Under the risk-neutral measure, contingent claims are valued as discounted expectations of future cash flows. Therefore, the price at time t of a zero-coupon bond paying one unit at maturity $t+\tau$ is given by

$$B(t,\tau) = \mathbb{E}_{\mathbb{Q}}\left[\exp\left(-\int_t^{t+\tau} R(s)\,ds\right)\right].$$

The expectation is conditional on the filtration generated by $R(t)$ and $V(t)$. Evaluating the conditional expectation yields the following affine bond pricing expression:

$$B(t,\tau) = \exp\left[-\phi(\tau) - \rho(\tau)R(t)\right]. \tag{2}$$

where,

$$\begin{aligned}
\phi(\tau) &= \frac{\theta_R}{\sigma_R}\left\{(\zeta - k_R)\,\tau + 2\ln\left[1 - \frac{\left(1 - e^{-\zeta\tau}\right)(\zeta - k_R)}{2\zeta}\right]\right\}, \\
\rho(\tau) &= \frac{2(1 - e^{-\zeta\tau})}{2\zeta(1 - e^{-\zeta\tau})(\zeta - k_R)}, \\
\zeta &= \sqrt{k_R^2 + 2\sigma_R^2}\,.
\end{aligned}$$

The European call option valuation under the SVSI framework admits a closed-form solution when the correlation between asset returns and interest rates vanishes ($\rho_{S,R} = 0$). This analytic expression takes the following form:

$$C(t,\tau) = S(t)\Pi_1(t,\tau) + KB(t,\tau)\Pi_2(t,\tau), \tag{3}$$

where $\Pi_1(t,\tau)$ and $\Pi_2(t,\tau)$ are the risk-neutral probabilities obtained by inverting characteristic functions [9]. For $j = 1, 2$,

$$\Pi_j(t,\tau) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \Re\left[\frac{e^{-i\phi\ln(K)} f_j(t,\tau)}{i\phi}\right] d\phi\,,$$

where,

$$\begin{aligned}
f_1(t,\tau) &= \exp(A_1 + B_1 + C_1 + D_1 + E_1),\\
A_1 &= -\frac{\theta_R}{\sigma_R^2}\left[2\ln\left(1 - \frac{(\xi_R - k_R)(1-e^{-\xi_R\tau})}{2\xi_R} + (\xi_R - k_R)\tau\right)\right],\\
B_1 &= -\frac{\theta_V}{\sigma_V^2}\left[2\ln\left(1 - \frac{(\xi_V - k_V + (1+i\phi)\rho\sigma_V)(1-e^{-\xi_V\tau})}{2\xi_V}\right)\right.\\
&\quad \left. + (\xi_V - k_V + (1+i\phi)\rho\sigma_V)\tau\right],\\
C_1 &= i\phi\ln(S(t)),\\
D_1 &= \frac{2i\phi(1-e^{-\xi_R\tau})}{2\xi_R - (\xi_R - k_R)(1-e^{-\xi_R\tau})}R(t),\\
E_1 &= \frac{i\phi(i\phi+1)(1-e^{-\xi_V\tau})}{2\xi_V - (\xi_V - k_V + (1+i\phi)\rho\sigma_V)(1-e^{-\xi_V\tau})}V(t),\\
\xi_R &= \sqrt{k_R^2 - 2\sigma_R^2 i\phi},\\
\xi_V &= \sqrt{[k_V - (1+i\phi)\rho\sigma_V]^2},
\end{aligned}$$

$$\begin{aligned}
f_2(t,\tau) &= \exp(A_2 + B_2 + C_2 + D_2 + E_2),\\
A_2 &= -\frac{\theta_R}{\sigma_R^2}\left[2\ln\left(1 - \frac{(\xi_R^* - k_R)(1-e^{-\xi_R^*\tau})}{2\xi_R^*}\right) + (\xi_R^* - k_R)\tau\right],\\
B_2 &= -\frac{\theta_V}{\sigma_V^2}\left[2\ln\left(1 - \frac{(\xi_V^* + i\phi\rho\sigma_V)(1-e^{-\xi_V^*\tau})}{2\xi_V^*}\right)\right.\\
&\quad \left. + (\xi_V^* - k_V + i\phi\rho\sigma_V)\tau\right],\\
C_1 &= i\phi\ln(S) - \ln(B(t,\tau)),\\
D_2 &= \frac{2(i\phi-1)(1-e^{-\xi_R^*\tau})}{2\xi_R^* - (\xi_R^* - k_R)(1-e^{-\xi_R^*\tau})}R(t),\\
E_2 &= \frac{i\phi(i\phi-1)(1-e^{-\xi_V^*\tau})}{2\xi_V^* - (\xi_V^* - k_V + i\phi\rho\sigma_V)(1-e^{-\xi_V^*\tau})}V(t),\\
\xi_R^* &= \sqrt{k_R^2 - 2\sigma_R^2(i\phi - 1)},\\
\xi_V^* &= \sqrt{[k_V - i\phi\rho\sigma_V]^2 - i\phi(i\phi-1)\sigma_V^2}\,.
\end{aligned}$$

Let's assume the option pricing function at expiration T, denoted by $h(S(T))$. For example, for a European call option, $h(S(T)) = \max(S(T) - K, 0)$. Using the Martingale Pricing Theorem, the option price at time t, denoted by $C(t, S, V, R)$, can be expressed as

$$C(t, S, V, R) = \mathbb{E}\left[e^{-\int_t^T R(s)ds} h(S(T)) \,|\, S(t) = S, V(t) = V, R(t) = R\right], \qquad (4)$$

where expectation $\mathbb{E}[\cdot]$ is calculated under the risk-neutral measure. Given the initial asset price S_0, the variance V_0 and the interest rate R_0, the European option price at time zero is given by

$$C(0, S_0, V_0, R_0) = \mathbb{E}\left[e^{-\int_0^T R(s)ds} h(S(T))\right]. \qquad (5)$$

The overall computational procedure used for option valuation under the SVSI framework is illustrated in Figure 1. Based on Equation (5), Monte Carlo simulation is employed to estimate the option price.

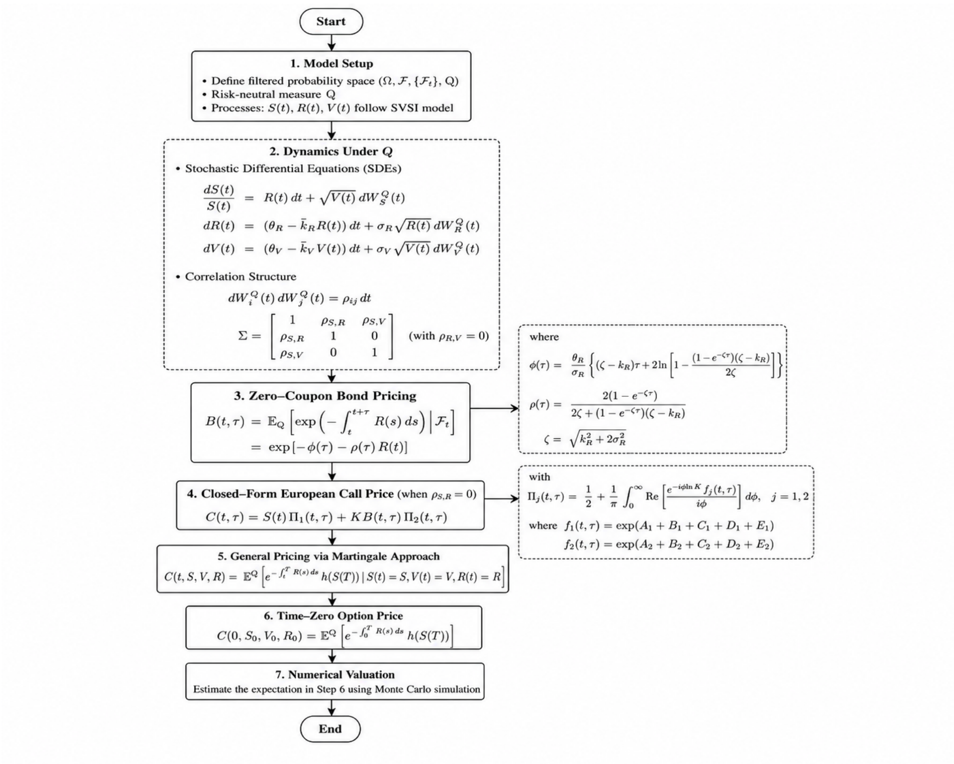

Figure 1: Workflow of the stochastic volatility–stochastic interest rate (SVSI) option pricing methodology. The process begins with parameter initialization and calibration, followed by specification of the risk-neutral dynamics, computation of bond prices and option valuation, and concludes with Monte Carlo estimation of the European option price.

2.1 Monte Carlo Simulation

Monte Carlo (MC) simulation is a probabilistic method that uses random sampling and statistical analysis to calculate results. It is widely used in option pricing. In this approach, a large number of simulated paths for the price of an underlying asset are generated and the option price is calculated for each path. The average option prices for the sampled paths are then calculated, giving us the current value of an option.

To illustrate the concept more simply, consider estimating the mean of a random variable X, denoted by $\mu = \mathbb{E}(X)$, using the Monte Carlo method. To do this, several samples are taken from the distribution of X and the mean of each sample is calculated. The mean of the samples is then calculated and used as an estimate of the expected value of X. The law of large numbers ensures that this estimate converges to the true expected value as the number of samples increases.

As with any numerical method, the accuracy of Monte Carlo simulation results is of great importance. Therefore, several criteria have been developed to measure and improve its accuracy. The technique of variance reduction plays an important role in improving the performance of Monte Carlo simulations and, in a sense, ensures optimal use of computational resources. The use of variance reduction techniques is equivalent to reducing the variation in simulation results, thereby increasing the reliability and speed of convergence.

Variance reduction techniques primarily operate through two distinct approaches: using model properties to adjust simulation outputs and reducing variation in simulation inputs. Common variance reduction techniques include control variate, antithetic variate, importance sampling and conditional Monte Carlo, each designed to manage different sources of variability in simulations.

In this study we use conditional Monte Carlo variance reduction techniques. If $\rho_{(S,R)} \neq 0$, then a closed-form formula for European option pricing is not available. Monte Carlo simulation techniques are therefore used.

A fundamental concept in asset pricing theory is that, under conditions of uncertainty, the price of derivative securities can be represented as an expected value. The pricing of derivatives, such as options, is therefore reduced to the calculation of a mathematical expectation. For this reason, the Monte Carlo (MC) method is considered a suitable approach for option pricing [17]. Given the parameters of a model and initial conditions, the dynamics of stock prices can be simulated. Thus, the equation (1) can be rewritten based on the discretization of the forward Euler method as follows

$$
\begin{aligned}
S_{n+1} &= S_n \exp\left((R_n - \frac{1}{2}V_n)\Delta t + \sqrt{V_n \Delta t}\Delta W_{S,n} \right) \\
R_{n+1} &= R_n + (\theta_R - \bar{k}_R R_n)\Delta t + \sigma_R \sqrt{R_n \Delta t}\Delta W_{R,n} \\
V_{n+1} &= V_n + (\theta_V - \bar{k}_V V_n)\Delta t + \sigma_V \sqrt{V_n \Delta t}\Delta W_{V,n} \,,
\end{aligned}
$$

where $n = 1, 2, \ldots, N$, and the vector $(W_{S,n}, W_{R,n}, W_{V,n})$ follows a k-dimensional

Gaussian distribution characterized by location parameter vector $\boldsymbol{\mu}$ and dispersion matrix $\boldsymbol{\Sigma}$, respectively, as follows:

$$\boldsymbol{\mu} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \text{and} \quad \boldsymbol{\Sigma} = \begin{pmatrix} \Delta t & \rho_{S,R}\Delta t & \rho_{S,V}\Delta t \\ \rho_{S,R}\Delta t & \Delta & 0 \\ \rho_{S,V}\Delta t & 0 & \Delta t \end{pmatrix}.$$

3 Machine Learning Concepts in Finance

Finance is the science of managing money under conditions of uncertainty, which offers a natural potential for the application of artificial intelligence in this field. Deep learning technology enables the discovery of complex patterns in data, helping both humans and machines to make better decisions [18].

Machine learning explores computational methods where systems autonomously develop task-execution capabilities through data-driven pattern recognition rather than predetermined instructions. These adaptive algorithms process informational inputs to generate corresponding outputs. Machine learning is the study of algorithmic and statistical models in which computer systems perform specific tasks without explicit programming. These algorithms operate on input data and produce output. The four primary categories of machine learning are supervised, unsupervised, semi-supervised, and reinforcement learning.

In supervised learning, machines are trained using "labelled" datasets that indicate the relationship between inputs and outputs. In this approach, the machine first learns from input-output pairs (training data) and then has to predict the output using test data. The primary goal of this technique is to model the relationship between the input variable (x) and the output variable (y).

Overfitting in machine learning refers to a situation where the model is over-fitted to the training data, leading to a reduction in its performance on real test data. This often occurs because the model is highly flexible to the training data and is unable to generalize properly to new and unseen data. Overfitting is a significant challenge in machine learning and various solutions have been proposed to prevent it [20].

Deep learning, a subset of machine learning, attempts to simulate some of the salient features of the human brain in neural models with three or more layers. Using deep and complex structures, these neural networks have the ability to "learn" from large amounts of data. While a single layer network may not provide accurate predictions, adding hidden layers to these networks helps to optimize and improve the accuracy of the model. These hidden layers, by providing additional dimensions and flexibility to the model, make it possible to improve its performance [19].

4 Options Pricing Prediction Model

Here we employ neural network techniques to derive European option valuations under the SVSI framework, utilizing computer-generated training datasets. The implementation incorporates three distinct computational approaches based on the characteristic function specifications detailed in Section 2. The first approach applies conventional Monte Carlo techniques for option valuation, while the second method implements conditional Monte Carlo estimation under the assumption of zero correlation between underlying asset returns and interest rate movements ($\rho_{S,R} = 0$). The third methodology combines both standard and conditional Monte Carlo simulations to handle scenarios where asset returns and interest rates exhibit non-zero correlation ($\rho_{S,R} \neq 0$). These three methods generate a total of five datasets - three datasets with $\rho_{S,R} = 0$ and two datasets with $\rho_{S,R} \neq 0$ - each with a size of 10,000, which serve as input data for model learning. To generate these datasets, parameter values were chosen within the range of parameters presented in Table 1. The generated data are then divided into training and test sets.

Table 1: Parameter Ranges

Parameter	**Range**
Stock Price (S_0)	\$50 – \$700
Strike Price (K)	\$10 – \$500
Maturity (T)	1 – 10 years
Initial Risk-Free Rate (R_0)	1% – 10%
Initial Volatility (V_0)	1% – 90%
Volatility of Volatility (σ_V)	0% – 90%
Volatility of Rate (σ_R)	0% – 90%
Mean Revert Rate of Volatility (k_V)	0 – 10
Mean Revert Rate of Rate (k_R)	0 – 10
Long-Term Volatility (θ_V)	0% – 90%
Long-Term Rate (θ_R)	0% – 90%
Correlation of Stock and Volatility ($\rho_{S,V}$)	$-0.9 - 0.9$
Correlation of Stock and Rate ($\rho_{S,R}$)	$-0.9 - 0.9$
Factor Risk Premiums for Volatility (λ_V)	0 – 3
Factor Risk Premiums for Rate (λ_R)	0 – 3

The valuation of European options possesses an intrinsic scaling property, as established in [13], whereby the premium V maintains proportional relationships with both the underlying security price S and exercise price K. This fundamental characteristic enables a dimensionless representation of market data through strike-price normalization - transforming both option values and equity prices into unitless ratios relative to K. Consistent with arbitrage-free pricing principles, we consequently adopt strike-adjusted parameterization, rescaling all relevant variables by the exercise price before their incorporation into our neural network architecture.

4.1 Deep Learning Model Structure for Option Pricing

In this section, we design the architecture of the learning network, considering the parameters outlined in Table 1 as input data for the model. First, we use a simulated dataset obtained from the closed-form European option pricing formula under the SVSI model to construct and optimize a deep learning model for predicting European call option prices. We then apply the optimized learning model to four different option pricing datasets for further investigation.

First, we partition the dataset derived from the closed-form European option pricing formula under the SVSI model into training (80%) and test (20%) subsets. Then, 20% of the test set is designated as the validation set. Next, the training set is used to build the model. Regarding the neural network architecture, we systematically evaluated ten distinct models (Model-1 through Model-10), as presented in Table 2. The optimal architecture, Model-7, consists of five layers with 50, 100, 100, and 50 neurons in the hidden layers, followed by a single output neuron. The activation functions follow an alternating structure: ReLU, ELU, ReLU, and ELU in the hidden layers, while the output layer employs the Softplus activation function to ensure non-negative price estimates, which represents an important financial constraint.

Table 2: Neural Network Models

Model	**Number of layers**	**Neurons in each layer**	**Activator functions Activator functions**
Model-1	3	[20, 20, 1]	[ReLU, ReLU, Linear]
Model-2	3	[100, 50, 1]	[ReLU, ReLU, Linear]
Model-3	4	[30, 60, 60, 1]	[ReLU, ReLU, ReLU, Linear]
Model-4	5	[30, 60, 60, 30, 1]	[ReLU, ReLU, ReLU, ReLU, Linear]
Model-5	5	[50, 100, 100, 50, 1]	[ReLU, ReLU, ReLU, ReLU, Linear]
Model-6	5	[50, 100, 100, 50, 1]	[ReLU, ELU, ReLU, ELU, Linear]
Model-7	5	[50, 100, 100, 50, 1]	[ReLU, ELU, ReLU, ELU, Softplus]
Model-8	5	[50, 100, 100, 50, 1]	[ReLU, SELU, ReLU, SELU, Softplus]
Model-9	5	[50, 100, 100, 50, 1]	[LeakyReLU, SELU, ReLU, SELU, Softplus]
Model-10	5	[50, 100, 100, 50, 1]	[LeakyReLU, SELU, ReLU, SELU, Exp]

For optimization, mean squared error (MSE) is employed as the loss function together with the Adam optimizer, implemented using the TensorFlow framework. To mitigate overfitting and improve generalization performance, fifty independent experimental trials were conducted using different training–test partitions. In addition, each trial considered ten distinct initial parameter configurations, resulting in a total of 1,000 optimization runs performed over complete training epochs.

The predictive accuracy comparison presented in Figure 2 demonstrates Model-7's superior performance in estimating European option values within the SVSI framework, as evidenced by the error distribution analysis relative to benchmark models in 2. To enhance computational effectiveness, Figure 3 illustrates the neural architecture specifically designed for derivative valuation. This customized network processes fourteen input variables through four successive transformation layers, with neuron configurations progressing as 50-100-100-50 units respectively, applying nonlinear operations at each hierarchical level.

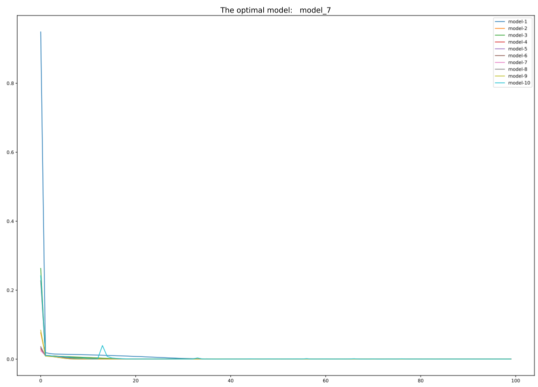

Figure 2: Error chart in test models

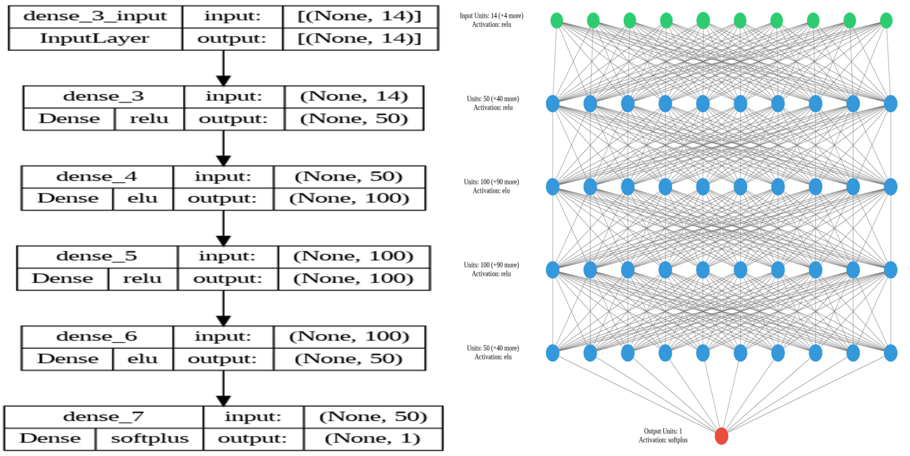

Figure 3: Structural representation and algorithmic process flow for Model-7's neural architecture performing options valuation under the SVSI model.

The neural architecture employs distinct nonlinear transformations across successive layers: rectified linear units initiate feature processing in the initial layer, followed by exponential linear units in the subsequent layer. This alternating pattern continues with ReLU activations reappearing in the third transformation stage and ELU functions concluding the hidden layer sequence. The output neuron utilizes a soft-plus transformation to ensure non-negative pricing estimates, maintaining financial consistency with derivative valuation principles. For network optimization, we implement a squared error minimization objective coupled with adaptive moment estimation, with all computations executed through the TensorFlow computational framework.

The computational framework employs the previously generated financial data to optimize a neural pricing engine via extensive retraining. Fifty independent experimental trials are performed, with each data partition (training-test division) enabling model refinement through ten different starting parameter configurations. Each configuration completes a full century of training epochs, amounting to 1,000 total optimization runs that collectively examine various weight initializations and data distributions.

Given the simulated data set generated by the closed-form formula for option pricing, and following the experiments conducted to establish the structure of a deep learning model for predicting option prices under the introduced model, we clearly demonstrate the power of deep learning.

5 Intuitive Results of the Deep Learning Prediction Model

Figures 4 and 5 present the neural network's performance across four distinct assessment metrics. The initial panel contrasts the model's price forecasts against simulated benchmark values derived from five SVSI valuation techniques. Subsequent panels analyze: (1) the dispersion of absolute prediction deviations, evaluating model accuracy; (2) the training progression through MSE evolution across both training and validation subsets; and (3) the squared error distribution within the test dataset, providing insight into generalization capability. As depicted in the figure, a significant portion of the errors is of a very small magnitude. The y-axis has been normalized to represent probabilities.

Figure 4 presents a tri-level comparative analysis of valuation methodologies: (i) analytical solutions computed via characteristic function inversion, (ii) conventional stochastic path simulations, and (iii) correlation-constrained conditional expectation estimates ($\rho_{(S,R)} = 0$) within the SVSI framework. In this figure, the performance of deep learning is demonstrated using four criteria for three data generation methods of European call option pricing under the SVSI model with $\rho_{(S,R)} = 0$. By analysing the plots in this figure, we obtain the following results:

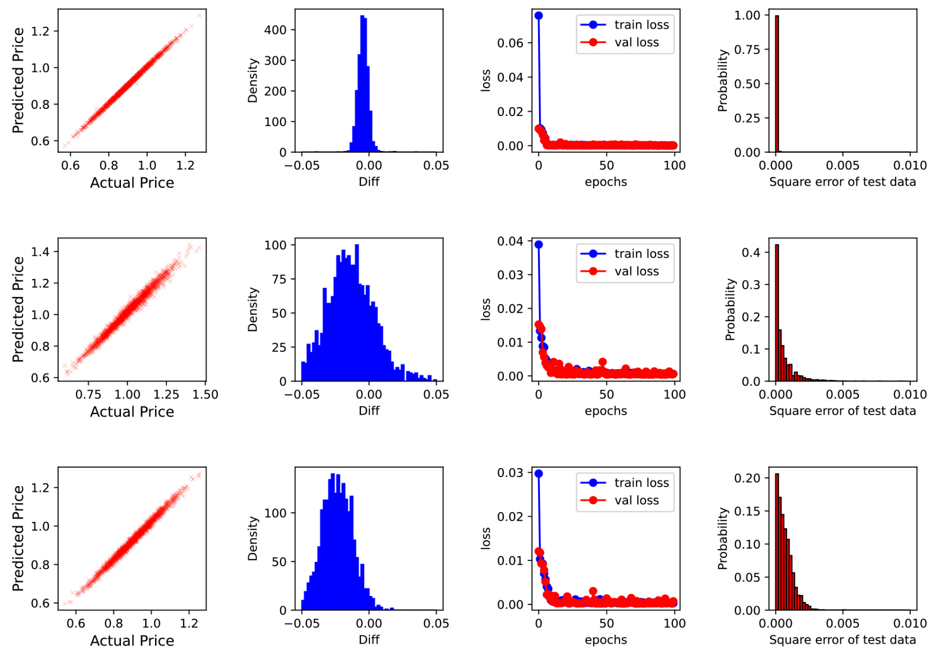

Figure 4: Intuitive results of deep learning for European call option pricing under the SVSI model with $\rho_{S,R} = 0$

- In the first column for each method in the SVSI model presented for the test set, almost all price pairs form a straight line with a slope of 1 and a very narrow width, indicating the closeness of the predicted prices to the option price data in the deep learning model.

- Evaluation results in the central display indicate that pricing inaccuracies for all SVSI computational techniques remain constrained to a ± 0.05 margin in the majority of test cases.

- The final column illustrates the optimization trajectory, documenting how prediction errors diminish during model training and validation across the three SVSI computational approaches. It can be seen that both curves converge closely after 60 epochs.

- In the fourth column, it is observed that the distribution of squared errors on the test dataset indicates that the majority of errors are relatively minor.

In conclusion, it can be inferred that the deep learning model for predicting European call option pricing under the SVSI model using three data generation methods for $\rho_{(S,R)} = 0$ has been well-trained.

Figure 5 presents the intuitive results of deep learning for SVSI model data with $\rho_{(S,R)} \neq 0$. This plot not only provides a comprehensive overview of the model's performance under specific conditions but also offers detailed explanations and interpretations comparable to those presented in Figure 5. In the third column, the deep learning curve for mean squared error (MSE) in the training set and validation datasets under all three methods in the SVSI model demonstrates a convergence close to the optimal value after 75 epochs.

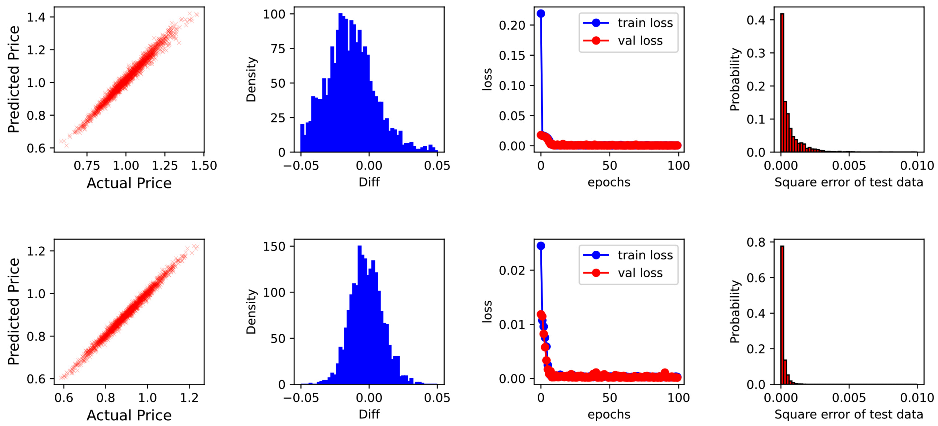

Figure 5: Intuitive results of deep learning for European call ption pricing under the SVSI model with $\rho_{S,R} \neq 0$

A comprehensive analysis of Figures 4 and 5 and their respective analyses leads to the conclusion that the designed deep learning model, utilizing the five-fold data generation methodology within the framework of the SVSI model, has undergone a comprehensive training process. The model is capable of accurately and efficiently predicting European call option prices.

Figure 6 illustrates the distribution of relative errors in the predictions of the deep learning model, generated using five distinct data generation methods, for European call option pricing under the SVSI model. As anticipated, the graph illustrates that in the dataset generated by the closed-form formula and explicit option price, the distribution of relative prediction errors is superior. Furthermore, the distribution of relative prediction errors in the two datasets related to conditional Monte Carlo simulation methods for parameters $\rho_{(S,R)} \neq 0$ and $\rho_{(S,R)} = 0$ has improved after the dataset generated by the closed-form formula for option prices. This indicates a better distribution compared to datasets related to standard Monte Carlo simulation methods.

Finally, we assess the efficacy of the deep learning approach. Figure 7 provides a comparison of the computation time for calculating 1000 European call option prices using five different methods. These include a closed-form formula based on the

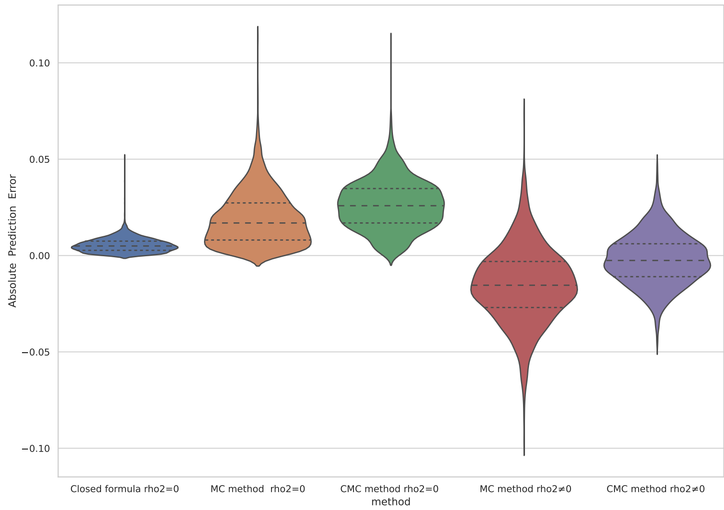

Figure 6: Distribution of relative prediction errors in the deep learning model across 5 data generation methods for european call option pricing under the SVSI model.

characteristic function, Monte Carlo simulation, conditional Monte Carlo simulation considering the SVSI model and $\rho_{(S,R)} = 0$, as well as Monte Carlo simulation and conditional Monte Carlo simulation considering the SVSI model and $\rho_{(S,R)} \neq 0$.

As illustrated in Figure 7, the time required by the deep learning model to compute 1000 option prices is less than one second. This speed is considerably faster than the accurate and explicit formula-based pricing method, which takes approximately 236 seconds to compute on an ordinary computer, not to mention simulation methods that take over 5000 seconds. These results demonstrate that the deep learning approach is a more effective method for option pricing computations than previous methods used in the financial industry. This is of great importance in the field of trade and industry, as experts must perform a significant number of calculations and experiments in a relatively short period of time.

To provide a comprehensive assessment of the capability of deep learning approaches for option pricing under different stochastic volatility frameworks, Table 3 presents a comparative evaluation across three widely used financial models: the Heston, Bates, and SVSI models (the latter incorporating stochastic interest rates). Model performance is assessed using Mean Squared Error (MSE) and the coefficient of determination (R^2) for training sample sizes of 10,000 and 20,000 observations. The results for the Heston and Bates models are adopted from Tables 2 and 3 in [28], whereas the SVSI model outcomes are generated using Monte Carlo simulation. This comparison enables an examination of how increasing structural complexity —

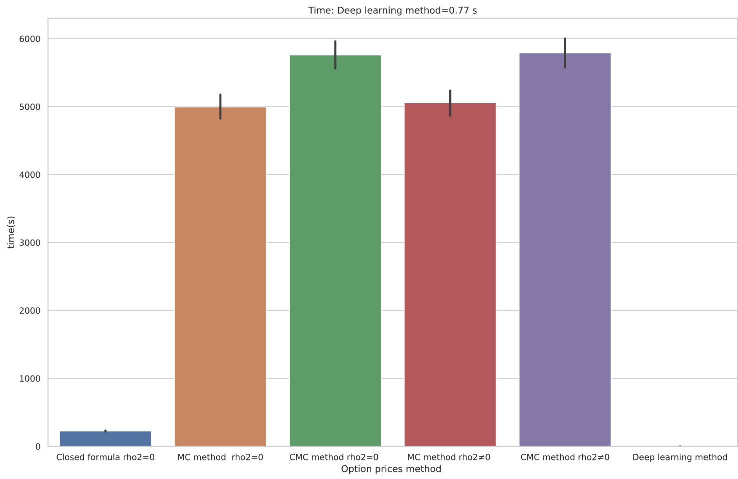

Figure 7: The calculation time for European call option prices under the SVSI model using the 5 methods described fo1 1000 option prices

particularly through the inclusion of stochastic interest rates — affects the predictive performance of deep neural networks.

Table 3: Comparison of deep learning performance across different stochastic volatility models

Methods	Sample Size	MSE	R^2
Heston model	10000	3.44×10^{-4}	0.9961
	20000	4.13×10^{-4}	0.9973
Bates model	10000	4.28×10^{-4}	0.9957
	20000	8.67×10^{-4}	0.9966
SVSI model	10000	2.59×10^{-5}	0.9968
	20000	3.17×10^{-5}	0.9976

The numerical results demonstrate that deep learning achieves consistently high predictive performance across all considered models. In particular, the R^2 values exceed 0.995 in every scenario, indicating an excellent agreement between the network outputs and the generated option price data. Nevertheless, noticeable differences emerge in terms of prediction error as measured by MSE. Among the three frameworks, the SVSI model exhibits the lowest error values, achieving MSEs of (2.59×10^{-5}) and (3.17×10^{-5}) for sample sizes of 10,000 and 20,000, respectively. These values outperform those obtained under the Heston (3.44×10^{-4}) and Bates (4.28×10^{-4}) models.

The superior performance observed for the SVSI framework suggests that increased model complexity does not necessarily hinder neural network learning. Although the SVSI model introduces additional mathematical complexity through the joint modeling of stochastic volatility and stochastic interest rates, the richer input structure appears to provide more informative representations of market dynamics. Consequently, the deep neural network is better able to capture nonlinear relationships and reduce pricing errors.

Furthermore, increasing the sample size from 10,000 to 20,000 observations results in a modest but consistent improvement in R^2 across all models, supporting the expectation that larger datasets contribute to enhanced generalization performance. Overall, these findings indicate that, in contrast to conventional analytical approaches — which often become computationally challenging or analytically intractable as model complexity increases — deep learning methods maintain robust predictive performance and can effectively exploit additional structural information to achieve highly accurate option pricing under realistic market conditions.

6 Application to Bitcoin Option Data

In this section, we analyze real-world calibration and parameter estimation for a stochastic interest rate and stochastic volatility model using Bitcoin option data. Following the calibration method outlined in [21], we extract key model parameters that describe the dynamics of Bitcoin option prices. These parameters are then used to train a deep learning model for modeling and forecasting Bitcoin's behavior under stochastic interest rate and stochastic volatility conditions. The dataset encompasses diverse market conditions, ensuring the robustness and practical relevance of the calibration and subsequent analysis.

The real-world dataset consists of Bitcoin option prices from Deribit, a leading cryptocurrency derivatives exchange. It spans a 24-month period from April 1, 2019, to April 1, 2021, covering various market conditions, including bullish, calm, and stressed periods (e.g., the COVID-19 market crash). The dataset includes European-style call and put options across a broad range of strikes and maturities, with a focus on options with maturities of 1 month, 3 months, and beyond.

This methodology ensures arbitrage-free option pricing and smooth volatility surfaces for calibration. By segmenting the data into bullish, calm, and stressed market regimes, we systematically evaluate model performance under different conditions. Anomalies such as stale prices or outliers are removed to improve calibration reliability.

To calibrate the stochastic interest rate and stochastic volatility model, we follow the approach in [21] and perform daily calibrations to ensure that the extracted parameters accurately reflect evolving market conditions. The calibrated parameters include the volatility of volatility (σ_V), the mean reversion rate of volatility (κ_V), the long-term volatility (θ_V), the risk premium for volatility (λ_V), the correlation between asset and volatility ($\rho_{S,V}$), the mean reversion rate of interest rate (κ_R), the volatility of the interest rate (σ_R), the long-term interest rate (θ_R), the risk premium for interest rate (λ_R), the correlation between asset and interest rate ($\rho_{S,R}$), and the correlation between volatility and interest rate ($\rho_{V,R}$) in the SVSI model.

We employ a Tikhonov L_2-regularized optimization to minimize the root-mean-square error (RMSE) between market option prices and those derived from the SVSI model, ensuring stability in parameter estimation, especially in the presence of noise. Through iterative parameter adjustments to align with observed option prices, the calibration process yields an optimal set of parameters that effectively capture market dynamics. These calibrated parameters are then used as input for training the deep learning model.

After estimating the model parameters for 721 European Bitcoin call options using the outlined calibration method, we normalized the real data based on the option price. The normalized data was then used as input for our deep learning model. Figure 8 illustrates the model's performance on the test dataset.

The scatter plot compares actual option prices (y-axis) with those predicted by the deep learning model (x-axis). The close alignment of points along the diagonal indicates a strong positive correlation, suggesting that the model's predictions closely track actual prices. The scatter around the diagonal represents prediction errors, which appear relatively small.

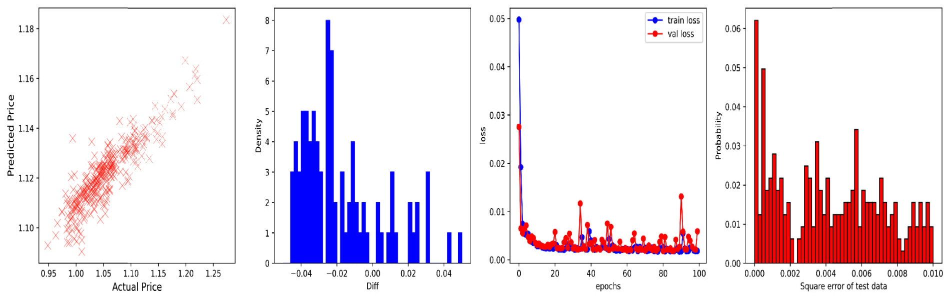

Figure 8: Model Performance Evaluation on Test Dataset: (a) Scatter plot of actual vs. predicted option prices showing strong correlation along the diagonal; (b) Error distribution histogram centered at zero, indicating unbiased predictions; (c) Training and validation learning curves demonstrating stable convergence with minimal overfitting; (d) Probability density of squared errors highlighting predominant small deviations and occasional larger discrepancies.

The histogram depicts the distribution of differences (Diff) between actual and predicted prices. This distribution is centered around zero, indicating that the model's predictions are generally unbiased, neither systematically overestimating nor underestimating actual prices. Most differences fall within a narrow range around zero, further supporting the model's accuracy.

The learning curves illustrate the model's performance during training. The blue line represents the training loss, while the red line denotes the validation loss. Both curves exhibit a downward trend, demonstrating a consistent reduction in error over the training epochs. Their close proximity suggests that the model generalizes well to unseen data without significant overfitting. A slight uptick in validation loss towards the end may warrant further investigation, but overall, the learning process appears stable.

The final histogram presents the probability distribution of the squared errors calculated from the test data, offering another perspective on the model's predictive performance. The concentration of probability density near zero indicates that most squared errors are small, reinforcing the accuracy of the model's predictions. However, the presence of some larger squared errors suggests areas for potential improvement.

7 Conclusion

The study examines the pricing of European options, utilizing the SVSI model in two different scenarios. These scenarios examine the correlation between the movements of the underlying Brownian asset and the stochastic interest rate dynamic model. In addition, a deep learning European option pricing model is presented that demonstrates superior performance in terms of both accuracy and computational efficiency. The research explores the impact of correlation on option pricing, shedding light on the complexity of asset and interest rate movements. It also presents a novel approach to pricing European options, using deep learning techniques to improve accuracy and computational speed. The results provide valuable insights into the dynamics of European option pricing under different scenarios, providing a comprehensive understanding of the effectiveness of the SVSI model. Furthermore, the integration of deep learning into option pricing shows promising results, highlighting its potential to improve pricing models in financial markets.

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