

Structural Credit Risk Measurement and Corporate Default Prediction in Vietnam: Evidence from a Merton Model

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Abstract:

This paper applies the Merton structural model to measure market-implied corporate default risk in Vietnam. Using an initial universe of 1,646 non-financial listed and registered-for-trading firms over 2019–2024, we estimate firm-year distance to default (DD) and model-implied probability of default (PD) by combining daily equity-market information with accounting-based debt measures through a hybrid Newton-type nonlinear root-finding procedure. After data-availability, liquidity, and convergence filters, the main sample contains 8,054 valid firm-year observations. The results show that default risk reached a local peak in 2022, when the share of firms with PD above 10% rose to 16.46%, consistent with rising funding costs and liquidity stress in Vietnam's corporate bond market. By 2024, risk conditions had improved materially, with median PD declining to 0.43% and the share of firms with PD above 10% falling to 9.24%. Size and sector analyses indicate that risk is concentrated among small-cap, highly leveraged firms and capital-intensive sectors such as real estate, construction, and steel. Event-based validation under a stricter hard-distress definition shows moderate discriminatory power for Merton-based PD among convergent observations, with an ROC-AUC of approximately 0.656. When non-convergent observations are retained and assigned a conservative high-risk score, the AUC declines to approximately 0.624. Under broader exchange-based distress labels, performance is weaker. Accounting-logit benchmarks achieve higher out-of-time AUC values, while the hybrid specification provides only incremental improvement. These results indicate that Merton-based DD and PD are useful market-implied early-warning and risk-ranking indicators, but stronger corporate distress assessment in Vietnam requires combining structural indicators with accounting, liquidity, and sectoral variables.

Keywords: Merton Model, Structural Credit Risk, Distance to Default, Probability of Default, Corporate Distress, Vietnam.

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1 Introduction

Corporate credit risk is a central source of financial instability, especially in emerging markets where accounting information is released with delay, disclosure quality is uneven, and capital-market liquidity can deteriorate quickly. Traditional accounting ratios such as leverage, current ratio, interest coverage, and profitability are useful because they summarize realized balance-sheet conditions. However, they are inherently backward-looking and may respond slowly when market expectations change. Market-based credit-risk indicators complement accounting measures by incorporating investors' forward-looking assessments of firm value, refinancing capacity, asset volatility, and expected cash-flow risk. In a setting where formal default may be delayed by renegotiation, restructuring, or regulatory intervention, such indicators can provide an earlier signal of financial stress than legal default records alone.

Vietnam provides a useful empirical setting for evaluating structural credit-risk models. The period 2019–2024 covers several distinct macro-financial regimes. The COVID-19 shock disrupted production and cash flows in 2020. Expansionary monetary conditions and the low-cost capital cycle then supported asset prices and credit growth during 2020–2021. In 2022, global monetary tightening, higher domestic funding costs, and stress in the corporate bond market abruptly changed refinancing expectations. In 2023–2024, regulatory adjustments and easing measures helped stabilize parts of the market, but the recovery was uneven across firms and sectors. These shifts create an informative environment for testing whether market-implied measures of default risk respond to changes in firm-level and system-wide financial conditions.

This study applies Merton's (1974) structural model, in which equity is interpreted as a call option on the total value of the firm's assets. The economic mechanism is direct: a firm defaults when its asset value falls below the relevant debt threshold. The model produces two interpretable indicators. Distance to default (DD) measures how many asset-volatility units the firm is away from the default boundary. Probability of default (PD) maps DD into a model-implied default probability. These indicators are attractive in emerging markets because they integrate market capitalization, equity volatility, debt obligations, and risk-free rates into a coherent framework. They are also transparent enough to be used in credit monitoring, stress assessment, and early-warning dashboards.

The empirical design has three components. First, it constructs a broad panel of Merton-based DD and PD indicators for Vietnamese non-financial firms that were listed and registered for trading during 2019–2024. The sample covers an initial universe of 1,646 firms and yields 8,054 valid firm-year observations after applying data and convergence filters. Second, it documents how default risk shifts over time and varies by firm size and industry. This distributional perspective is important because aggregate averages may conceal a concentrated tail of financially fragile

firms. Third, it evaluates the discriminatory power of DD and PD by comparing them with observed default-related and financial-distress events during 2022–2024. This validation step is essential because structural PD is model-implied rather than a calibrated historical frequency.

This paper does not claim to introduce a new structural credit-risk model. Its contribution is empirical and evaluative. First, it provides one of the broadest firm-level applications of the Merton/KMV framework to Vietnam, an emerging market where formal default records are limited, and credit stress is often mediated by restructuring, delayed disclosure, and regulatory intervention. Second, it evaluates whether market-implied DD and PD capture the 2022 corporate bond market stress using both aggregate and distributional risk measures. Third, it validates the Merton indicators against observed default-related and severe financial-distress events and compares their discriminatory power with accounting-based and logistic benchmarks. Fourth, it interprets equity volatility not only as a numerical input to the Merton inversion but also as a market-based carrier of information about uncertainty, investor expectations, and refinancing stress.

The main findings are as follows. Market-implied default risk declined from 2019–2020 to 2021, deteriorated again in 2022, and improved by 2024. The deterioration in 2022 is visible in median DD and PD, but it is especially clear in the high-risk tail: the share of firms with PD above 10% increased to 16.46%. Risk is concentrated among small-cap firms, firms with thin equity buffers relative to debt obligations, and capital-intensive sectors such as real estate, construction, and steel. Large-cap firms show stronger financial safety buffers, although they were not fully insulated from the 2022 shock. Event-based validation indicates that Merton-based PD has moderate discriminatory power only under a stricter hard-distress definition. Under broader exchange-based distress labels, its performance weakens substantially. These results suggest that DD and PD should be interpreted primarily as market-implied early-warning and risk-ranking indicators rather than as standalone binary default-prediction models.

The rest of the paper is organized as follows. Section 2 reviews related literature and develops the research gap. Section 3 presents the Merton model and its derivation, the numerical inversion procedure, the risk-scaling and classification measures, and the validation and benchmark specifications. Section 4 describes the data and institutional setting. Section 5 reports the empirical results. Section 6 discusses implications and limitations. Section 7 concludes.

2 Literature Review and Research Gap

The structural approach to credit-risk measurement originates from Black and Scholes [5] and Merton [17]. In this framework, corporate liabilities are valued as contingent claims on the firm's assets. Equity is a residual claim with option-like payoffs, while default is generated endogenously by the relation between asset

value, asset volatility, and debt obligations. The approach is theoretically appealing because it connects default risk to the firm's capital structure and market valuation, rather than treating default as an exogenous jump. For empirical work, the key outputs are DD and PD, which translate market information into credit-risk signals.

Subsequent research extended the original Merton setting by relaxing several idealized assumptions. Black and Cox [4] introduced a safety covenant and an endogenous default barrier, allowing default to occur before debt maturity. Geske [10] modeled multiple debt maturities using compound options, making the framework more consistent with firms with layered liabilities. Leland [14] and Leland and Toft [15] developed structural models in which capital structure and default boundaries are endogenously chosen by balancing the tax benefits of debt against bankruptcy costs. Longstaff and Schwartz [16] incorporated stochastic interest rates and showed that both asset-value uncertainty and interest-rate dynamics matter for credit spreads. These extensions demonstrate that default risk is affected not only by firm leverage but also by the maturity structure of debt, refinancing conditions, and macro-financial states.

Empirical studies have examined whether structural indicators contain useful information about bankruptcy or distress. Hillegeist et al. [12] show that bankruptcy probabilities derived from the Black–Scholes–Merton framework provide incremental information beyond traditional accounting models such as Altman-type and Ohlson-type scores. The classic accounting-ratio benchmarks here are Altman's (1968) discriminant-analysis model and Ohlson's (1980) logit model. Vassalou and Xing [21] link default risk to equity returns and argue that firms with higher default risk exhibit a distinct risk premium. Duffie et al. [9] show that default prediction improves when dynamic firm-level covariates and macroeconomic variables are incorporated. Bharath and Shumway [3] provide a more cautious assessment: Merton DD is useful, but simpler market-based approximations and accounting variables can perform similarly or better in some settings; Afik, Arad, and Galil [1] reach a similar conclusion in their assessment of alternative Merton-model implementations. Campbell, Hilscher, and Szilagyi [6, 7] develop reduced-form distress-prediction models that combine market and accounting variables, while Jones, Mason, and Rosenfeld [13] provide one of the first empirical tests of the contingent-claims approach to corporate capital structure. The widely used commercial implementation of this framework is the Moody's KMV model [8], whose default-point convention this paper adopts. The common message is that DD and PD are informative, but they should not be expected to capture all relevant dimensions of distress.

A complementary stream of literature examines the informational content of financial-market dynamics. This perspective is relevant for structural credit-risk models because Merton-based DD and PD rely directly on market capitalization and equity volatility. Equity volatility is not merely a statistical input; it reflects changing uncertainty, heterogeneous investor beliefs, liquidity conditions, and the market's reassessment of firm value. Trinidad Segovia et al. [20] propose a self-

similarity-based methodology for detecting volatility clusters in asset-return series, showing that periods of high and low volatility are temporally structured rather than randomly dispersed. Puertas et al. [19] analyze stock-market structure and dynamics using concepts from soft matter and many-particle physics, illustrating how collective price behavior can be characterized using tools from statistical mechanics.

This literature strengthens the economic motivation for using market-implied credit-risk indicators in emerging markets. During periods of stress, declining equity values and volatility clustering may signal deteriorating refinancing expectations before accounting ratios fully adjust. In the Merton framework, this information is transmitted mechanically into DD and PD: lower market equity reduces the inferred asset buffer, while higher equity volatility increases estimated asset volatility, thereby narrowing the distance to the default boundary. The 2022 Vietnamese corporate bond stress episode is therefore an appropriate setting for examining whether volatility-based market signals are reflected in structural credit-risk measures.

Related literature emphasizes the role of market liquidity, trading frictions, and institutional conditions. In highly liquid developed markets, equity volatility may be a reasonably efficient signal of asset uncertainty. In emerging markets, however, thin trading, price limits, investor herding, and sudden shifts in liquidity can distort volatility estimates. Moreover, the observed legal default date may not coincide with economic distress. Firms may renegotiate debt, extend maturities, settle obligations using assets, or remain listed despite severe financial weakness. These features do not invalidate structural models, but they change the interpretation of model-implied PD. In such settings, PD is best viewed as a market-implied stress indicator rather than a fully calibrated physical default probability.

Emerging-market applications of structural models remain especially important because credit registries, bond-default histories, and external ratings are often incomplete. Market-implied indicators can fill part of this information gap. They are also useful for regulators and lenders because they can be updated more frequently than audited financial statements. However, the same characteristics that make them valuable also require careful empirical design. Researchers must address non-trading days, extreme equity volatility, non-convergence in numerical inversion, and the classification of default-related events. A structural model that works well for large and liquid firms may be less reliable for small firms with thinly traded shares.

Evidence from Vietnam is still relatively limited. Previous studies have applied Merton-type models to selected listed firms, individual sectors, or shorter periods, but few have combined broad coverage, a period containing major market stress, and formal validation against observed distress events. Vietnam's 2022 corporate bond stress episode is particularly relevant for a structural credit-risk framework. The event changed refinancing expectations, reduced confidence in private debt instruments, and placed pressure on highly leveraged firms, especially in real estate and related sectors. Because the Merton model is sensitive to market capitalization,

debt burden, and volatility, it should capture part of this shock through changes in DD and PD.

The distinction between market-implied and accounting-based measures is also central to the research design. Accounting models usually classify firms by realized profitability, liquidity, leverage, and retained earnings. These variables are stable and interpretable, but they adjust slowly when market expectations change. Structural indicators, by contrast, can react immediately to new information through equity value and volatility. This does not mean that DD and PD dominate accounting ratios. Rather, the two classes of indicators measure different layers of risk. Accounting variables summarize realized fundamentals, while the Merton model converts market expectations into a forward-looking distance from the debt boundary. In a market such as Vietnam, where credit stress can be transmitted through both balance-sheet weakness and sudden refinancing constraints, combining the two perspectives is likely to be more informative than either alone.

A further issue concerns the treatment of distress events. In developed markets, default may be defined using bankruptcy filings, missed bond payments, or rating-agency default events. In emerging markets, the boundaries among default, restructuring, delayed payment, delisting, and severe distress are less clear. This paper therefore distinguishes between a stricter hard-distress definition and a broader exchange-based distress definition, and it avoids treating all observed events as legal defaults. This choice is conservative. It recognizes that the Merton model is expected to detect economic stress, not merely formal bankruptcy. It also explains why the validation exercise is based on discrimination and ranking performance rather than on calibration of one-year historical default frequencies.

This paper contributes to the literature by evaluating the Merton framework in a market where formal default is not the only expression of credit stress. It focuses not only on average default risk but also on the distributional tail, size segmentation, sector concentration, and event-based classification performance. This design responds to a practical question: can DD and PD help identify vulnerable firms before or during a stress episode? The answer is expected to be conditional. DD and PD should be informative because they aggregate market and balance-sheet information, but their predictive power should be moderate rather than perfect because distress in Vietnam is mediated by liquidity, restructuring, disclosure quality, and institutional intervention.

3 Theoretical Framework and Methodology

3.1 Merton Structural Model and Expanded Derivation

Let V_t denote the market value of the firm's assets at time t , D the default point or debt threshold, $\tau = T - t$ the remaining time to maturity, r the risk-free rate, and σ_V the volatility of asset value. In the Merton framework, equity is interpreted as a

European call option on the value of the firm's assets, with a strike price equal to the default point D . At maturity, the equity payoff is

$$E_T = \max(V_T - D, 0). \quad (1)$$

Under the risk-neutral measure, the asset value follows a geometric Brownian motion:

$$dV_t = rV_t dt + \sigma_V V_t dW_t, \quad (2)$$

where W_t is a standard Brownian motion. The solution is

$$V_T = V_t \exp\left[\left(r - \frac{1}{2}\sigma_V^2\right)\tau + \sigma_V \sqrt{\tau} Z\right], \quad Z \sim N(0, 1). \quad (3)$$

The Black-Scholes valuation formula for equity is

$$E_t = V_t \Phi(d_1) - D e^{-r\tau} \Phi(d_2), \quad (4)$$

where $\Phi(\cdot)$ denotes the standard normal cumulative distribution function, and

$$d_1 = \frac{\ln(V_t/D) + (r + \frac{1}{2}\sigma_V^2)\tau}{\sigma_V \sqrt{\tau}}, \quad (5)$$

$$d_2 = d_1 - \sigma_V \sqrt{\tau} = \frac{\ln(V_t/D) + (r - \frac{1}{2}\sigma_V^2)\tau}{\sigma_V \sqrt{\tau}}. \quad (6)$$

The option delta links equity volatility to asset volatility:

$$\frac{\partial E_t}{\partial V_t} = \Phi(d_1). \quad (7)$$

Applying Ito's lemma and matching diffusion terms using the delta in Eq. (7), the instantaneous equity volatility satisfies

$$\sigma_E = \frac{V_t}{E_t} \Phi(d_1) \sigma_V. \quad (8)$$

Equations (4) and (8) form a system of two nonlinear equations with two unknowns, V_t and σ_V . After solving this system, the distance to default is defined as

$$DD_{i,t} = \frac{\ln(V_{i,t}/D_{i,t}) + (r_t - \frac{1}{2}\sigma_{V,i,t}^2)\tau}{\sigma_{V,i,t} \sqrt{\tau}}. \quad (9)$$

The model-implied probability of default is then

$$PD_{i,t} = \Phi(-DD_{i,t}). \quad (10)$$

In this study, $PD_{i,t}$ is interpreted as a structural, market-implied risk measure. It is not a rating-agency-expected default frequency calibrated from a long historical default database. This distinction is important because the Vietnamese market does not provide a sufficiently long and consistent sequence of firm-level default observations for direct calibration across all listed and registered firms.

3.2 Model Extensions and Scope of the Empirical Specification

The Merton model assumes that default occurs only at maturity. The Black–Cox extension relaxes this assumption by defining a first-passage default time. If B_s is a default barrier, the default time can be written as

$$\tau_B = \inf\{s \geq t : V_s \leq B_s\}. \quad (11)$$

This extension is theoretically attractive because it allows early default, but it requires specifying a barrier process that is difficult to identify reliably in a market with limited debt-market disclosure. Similarly, compound-option models such as Geske-type frameworks can represent multiple debt maturities, while Leland-type models endogenize default boundaries by balancing tax shields against distress costs. These models are important for theory, but they require additional information on debt priority, maturity structure, tax status, and bankruptcy costs. The empirical design therefore uses the standard one-year Merton/KMV implementation. This choice is deliberate: it preserves transparency, avoids over-parameterization, and remains feasible for a broad cross-section of Vietnamese firms.

3.3 Default Point and Numerical Estimation

The default point follows the KMV convention and is defined as

$$D_{i,t} = STD_{i,t} + 0.5 LTD_{i,t}, \quad (12)$$

where $STD_{i,t}$ denotes short-term debt and $LTD_{i,t}$ denotes long-term debt. This specification reflects the idea that short-term liabilities create immediate liquidity pressure, whereas only a portion of long-term debt is likely to become binding over a one-year horizon.

For each firm-year, observed market capitalization $E_{i,t}$, annualized equity volatility $\sigma_{E,i,t}$, risk-free rate r_t , debt threshold $D_{i,t}$, and horizon $\tau = 1$ are used to solve jointly for the unobserved asset value $V_{i,t}$ and asset volatility $\sigma_{V,i,t}$. The risk-free rate is proxied by the one-year Vietnamese government bond yield at the corresponding date.

Equity volatility is estimated from daily log returns and annualized as

$$\sigma_E = \sqrt{252} \cdot \text{sd} [\ln(P_d/P_{d-1})], \quad (13)$$

where P_d denotes the closing equity price on the trading day d . The nonlinear system solved for each firm-year is

$$F_1(V_t, \sigma_V) = E_t - V_t \Phi(d_1) + D_t e^{-r\tau} \Phi(d_2) = 0, \quad (14)$$

$$F_2(V_t, \sigma_V) = \sigma_E - \frac{V_t}{E_t} \Phi(d_1) \sigma_V = 0. \quad (15)$$

Let $\theta = (V_t, \sigma_V)'$ and $F(\theta) = (F_1(\theta), F_2(\theta))'$. A Newton-type update can be written as

$$\theta^{(k+1)} = \theta^{(k)} - [J(\theta^{(k)})]^{-1} F(\theta^{(k)}), \quad (16)$$

where $J(\theta)$ is the Jacobian matrix of partial derivatives. The iteration is stopped when the relative changes in both unknowns are sufficiently small:

$$\max \left\{ \frac{|V^{(k+1)} - V^{(k)}|}{V^{(k)}}, \frac{|\sigma_V^{(k+1)} - \sigma_V^{(k)}|}{\sigma_V^{(k)}} \right\} < \varepsilon. \quad (17)$$

The asset value is initialized at $V_0 = E + D$ and asset volatility is initialized at the observed annualized equity volatility, $\sigma_{V,0} = \sigma_E$. The nonlinear system is solved using a hybrid Newton-type root-finding routine. Observations are marked as invalid if required inputs are missing or non-positive, including zero or missing market capitalization, non-positive debt, missing risk-free rate, non-positive equity volatility, or non-positive maturity. Observations that enter the numerical inversion but do not return a valid solution are marked as non-convergent. This distinction allows data-driven exclusions and algorithm-driven non-convergence to be assessed separately.

3.4 Risk Scaling, Classification, and Robustness Design

The study uses both absolute and relative measures of credit risk. Absolute measures include DD, PD, the share of firms with $DD < 0$, the share with $DD < 1$, and the share with $PD > 10\%$. The $DD < 0$ threshold indicates that the expected log asset value is already below the default boundary under the model specification. The $DD < 1$ threshold identifies firms whose asset value is less than one asset-volatility unit away from the default point. The $PD > 10\%$ threshold identifies firms in a high-risk structural zone. These thresholds do not imply a mechanical legal default, but they are useful for screening and comparing risk intensity across time.

Relative measures are also important because PD is bounded between zero and one and is often highly skewed. We therefore consider within-year percentile rankings and deciles of DD and PD. Such rankings are particularly useful in emerging markets because market-wide volatility regimes can shift over time. A firm may have the same absolute DD in two years but occupy a different relative position within the cross-sectional risk distribution. Rank-based measures also reduce the influence of extreme numerical values and facilitate comparison across firms of different sizes.

Formally, for any firm-year observation (i, t) , the full-sample z-score of DD is

$$Z_{i,t}^{DD} = \frac{DD_{i,t} - \overline{DD}}{s(DD)}. \quad (18)$$

The within-year z-score, which controls shifting market-wide volatility regimes, is

$$Z_{i,t}^{DD, \text{year}} = \frac{DD_{i,t} - \overline{DD}_t}{s_t(DD)}. \quad (19)$$

Analogous transformations are applied to PD. The percentile rank of DD within year t is

$$Rank_{i,t}^{DD} = \frac{\text{rank}_t(DD_{i,t})}{N_t}, \quad (20)$$

and the corresponding decile class is

$$Decile_{i,t}^{DD} = \lceil 10 \times Rank_{i,t}^{DD} \rceil. \quad (21)$$

The first DD decile contains the lowest-DD firms and therefore represents the riskiest tail. For PD, the interpretation is reversed: higher PD indicates higher risk.

To examine robustness to market microstructure effects, we repeat key descriptive analyses after excluding observations with annualized equity volatility above 150%. The threshold is not intended to remove all risky firms. Instead, it excludes cases where the volatility input may be dominated by thin trading, price jumps, or abnormal market behavior. If the main time-series pattern remains after this filter, it is less likely that the results are driven by a small number of extreme observations.

3.5 Distress-Event Definitions and Validation Design

A structural credit-risk measure is useful only if it has empirical relevance for observed distress. We therefore construct two event variables for the 2022–2024 validation window.

The primary event variable is a stricter hard-distress indicator, denoted by $Y_{i,t}^{\text{hard}}$. It equals one if the firm i in year t is associated with at least one of the following events: special control status, overdue corporate bond principal or interest obligations, or delisting associated with serious financial or compliance problems. These events are closer to hard credit distress and are therefore used as the main validation target.

$$Y_{i,t}^{\text{hard}} = \begin{cases} 1, & \text{if firm } i \text{ experiences a hard default-related} \\ & \text{or severe distress event in year } t, \\ 0, & \text{otherwise.} \end{cases} \quad (22)$$

The broader event variable, denoted by $Y_{i,t}^{\text{broad}}$, equals one if the firm belongs to $Y_{i,t}^{\text{hard}}$ or is additionally subject to exchange-based control or other market-supervision events related to financial weakness. This broader definition captures early-stage financial weakness and supervisory intervention, but it is not interpreted as legal default. The classification is based on exchange-supervision status, overdue corporate bond obligations, delisting status, and related financial-distress signals. These signals differ in severity; therefore, they are not pooled into a single default label without distinction.

$$Y_{i,t}^{\text{broad}} = \begin{cases} 1, & \text{if } Y_{i,t}^{\text{hard}} = 1 \text{ or firm } i \text{ is subject to broader} \\ & \text{exchange-based distress signals in year } t, \\ 0, & \text{otherwise.} \end{cases} \quad (23)$$

The distinction between $Y_{i,t}^{\text{hard}}$ and $Y_{i,t}^{\text{broad}}$ is important. The Merton model is designed to detect structural balance-sheet pressure arising from market equity value, asset volatility, and debt obligations. It is therefore expected to align more closely with hard credit-distress events than with broader administrative, disclosure-related, or trading-status events.

Voluntary delisting, technical transfers between trading boards, delisting arising from mergers or corporate reorganizations unrelated to financial difficulty, and observations with missing data but no positive evidence of distress are excluded from the event group.

Validation is conducted in three steps. First, the distributions of DD and PD are compared between event and non-event firms. Second, the concentration of events in high-risk zones is examined using thresholds such as $DD < 1$ and $PD > 10\%$. Third, receiver operating characteristic (ROC) analysis is used to summarize discriminatory performance across all possible thresholds. The area under the ROC curve (AUC) measures the probability that a randomly selected event firm receives a worse risk score than a randomly selected non-event firm. An AUC of 0.5 indicates no discrimination, while values above 0.7 are usually interpreted as acceptable in many applied settings [11]. Because the Merton risk score and the event label refer to the same year, this exercise is an event-based discrimination and risk-ranking test rather than a one-year-ahead forecasting test.

3.6 Formal Econometric Framework and Benchmark Models

To address the benchmark-validation concern, we compare the standalone Merton indicators with accounting-based logistic specifications. Because a complete and consistently comparable set of Altman Z-score and Ohlson O-score components is not available for all firms in the validation sample, we implement parsimonious accounting-logit benchmarks. This choice preserves sample comparability while providing a direct benchmark against traditional accounting-based distress prediction.

Let $\Lambda(\cdot)$ denote the logistic cumulative distribution function. The Merton-only specification is

$$\Pr(Y_{i,t} = 1) = \Lambda(\alpha + \beta_1 PD_{i,t}), \quad (24)$$

or equivalently,

$$\Pr(Y_{i,t} = 1) = \Lambda(\alpha + \beta_1 DD_{i,t}). \quad (25)$$

The expected coefficient sign is $\beta_1 > 0$ when PD is used and $\beta_1 < 0$ when DD is used.

The accounting benchmark specification is

$$\Pr(Y_{i,t} = 1) = \Lambda(\alpha + \gamma_1 ROA_{i,t} + \gamma_2 CashRatio_{i,t} + \gamma_3 DE_{i,t} + \gamma_4 SalesTA_{i,t} + \gamma_5 PayableTurnover_{i,t}). \quad (26)$$

An industry-augmented benchmark is specified as

$$\Pr(Y_{i,t} = 1) = \Lambda(\alpha + \gamma' X_{i,t} + \eta_{\text{industry}}), \quad (27)$$

where $X_{i,t}$ is the vector of accounting variables and η_{industry} captures industry-level heterogeneity.

Finally, a hybrid specification combines market-implied and accounting-based information:

$$\Pr(Y_{i,t} = 1) = \Lambda(\alpha + \beta_1 PD_{i,t} + \gamma' X_{i,t} + \eta_{\text{industry}}). \quad (28)$$

The Merton risk scores are evaluated using event-based ROC-AUC over the 2022–2024 validation window, where PD is used directly as the risk score, and the negative of DD can be used as an equivalent risk-oriented transformation. The accounting-logit and hybrid-logit benchmarks are estimated on the 2020–2023 training sample and evaluated on the 2024 backtest sample. AUC is interpreted following Hanley and McNeil [11], and confidence intervals are obtained using bootstrap resampling of the corresponding validation sample. Because the Merton event-definition analysis and the accounting-logit backtest serve different validation purposes, their results are interpreted jointly rather than as a strict horse race on an identical sample.

4 Data and Institutional Setting

The study uses non-financial firms listed or registered for trading in Vietnam over 2019–2024. The initial universe contains 1,646 firms, corresponding to a maximum of 9,876 firm-year observations. Market data and financial statement information are obtained from the FiinPro Platform. Financial firms are excluded because banks, securities companies, and insurers have regulatory capital structures and liability profiles that are not comparable to those of ordinary non-financial corporations. For such firms, leverage and debt obligations have a different economic meaning and would require a separate modeling framework.

The final analytical sample is determined by data availability and numerical feasibility. Observations with missing market capitalization, missing debt components, non-positive or unreliable equity volatility, or non-convergent Merton inversion are excluded. After applying data-availability, liquidity, and numerical-convergence filters, 8,054 out of 9,876 potential firm-year observations remain in the valid Merton sample, corresponding to an overall usable-observation rate of 81.55%. This coverage is sufficiently broad to examine both aggregate shifts in market-implied default risk and cross-sectional heterogeneity by size and industry.

Table 1A reports the numerical feasibility of the Merton inversion. The distinction between the overall usable-observation rate and the conditional convergence rate is important. The usable-observation rate is calculated relative to the full potential panel, whereas the conditional convergence rate is calculated only among observations with valid inputs entering the numerical inversion.

Table 1A: Numerical feasibility and convergence of the Merton inversion

Year	Potential observations	Input-valid observations	Converged observations	Input-invalid observations	Solver-failed observations	Usable observations rate (%)	Conditional convergence (%)
2019	1,646	1,326	1,244	320	82	75.58	93.82
2020	1,646	1,364	1,284	282	80	78.01	94.13
2021	1,646	1,413	1,380	233	33	83.84	97.66
2022	1,646	1,436	1,385	210	51	84.14	96.45
2023	1,646	1,417	1,376	229	41	83.60	97.11
2024	1,646	1,437	1,385	209	52	84.14	96.38
Total	9,876	8,393	8,054	1,483	339	81.55	95.96

Across the full 2019–2024 panel, 8,393 firm-year observations have valid market and accounting inputs for the nonlinear inversion. The solver converges for 8,054 observations, implying a conditional convergence rate of 95.96%. Relative to the full potential panel of 9,876 firm-year observations, the usable Merton-estimation rate is 81.55%. Thus, most exclusions are due to input availability or validity rather than numerical failure of the inversion routine.

The annual valid observation rate reported in Table 1A provides a more detailed view of this filtering process. The rate rises from 75.58% in 2019 and 78.01% in 2020 to 83.84% in 2021 and 84.14% in 2022 and 2024, indicating that data availability and numerical convergence both improve over the sample period as market coverage and disclosure quality increase. The lower rates in the early years reflect a combination of missing accounting or market data, insufficient trading history for volatility estimation, and non-convergence of the hybrid Newton-type inversion for firms with extreme leverage or near-zero equity value.

Two distinct rates should be distinguished. The valid-observation rate reported above is a composite metric that jointly reflects data availability, liquidity screening, and numerical convergence. The numerical convergence rate of the hybrid Newton-type root-finding procedure is defined more narrowly as the share of observations that converge among those that actually enter the inversion routine, that is, after market and accounting inputs have been screened. Because missing market and accounting inputs are filtered before numerical inversion, the two rates are not interchangeable, and the composite rate is a lower bound on the conditional convergence rate. We report the overall valid Merton-estimation rate together with the convergence rate conditional on entry into the root-finding routine, so that data-driven and algorithm-driven exclusions can be assessed separately.

The institutional setting is central to the interpretation of results. In 2020–2021, low interest rates and supportive policy conditions reduced financing pressure for many firms. Equity prices rose sharply in many sectors, and market capitalization increased relative to debt obligations. In the Merton model, this tends to increase DD and reduce PD, even if accounting fundamentals do not immediately improve. In

2022, the situation reversed. Higher funding costs, global monetary tightening, and stress in the corporate bond market raised concerns about refinancing. For highly leveraged firms, falling equity values and rising volatility narrowed the distance to default. The model should therefore detect a deterioration in structural credit risk.

The shock to the Vietnamese corporate bond market is especially relevant. Before 2022, private bond issuance had grown rapidly and was concentrated in sectors with large financing needs, especially real estate. When investor confidence declined and issuance conditions tightened, refinancing risk increased. Some firms faced delayed payments, renegotiated maturities, or asset-based settlements. These developments may not always result in an immediate legal default, but they are economically linked to credit stress. A market-based structural model is well suited to capturing changes in perceived default risk during such episodes.

In 2023–2024, policy adjustments helped reduce the intensity of the stress. Measures allowing restructuring of bond obligations and efforts to lower lending rates contributed to a partial improvement in financing conditions. However, recovery was uneven. Firms with strong market capitalization and diversified access to funding recovered more rapidly, while small-cap and highly leveraged firms remained vulnerable. This heterogeneity motivates the analysis by size and sector in addition to aggregate market statistics.

5 Empirical Results

5.1 Aggregate Distribution of Default Risk

Table 1 reports the annual distribution of DD and PD in the full valid sample. Three patterns are evident. First, default risk declined from 2019–2020 to 2021. Median DD increased from 2.414 in 2019 and 2.464 in 2020 to 2.620 in 2021, while median PD declined to 0.44%. The share of firms with ($DD < 1$) also fell to 8.84%. This pattern is consistent with the improvement in market conditions during the low-cost capital cycle, when equity valuations increased and volatility declined for many firms.

Second, default risk deteriorated in 2022. The median DD fell to 2.392, the median PD rose to 0.84%, and the share of firms with PD above 10% increased to 16.46%. This change is economically meaningful because the deterioration is visible not only at the center of the distribution but also in the high-risk tail. Under the Merton mechanism, a decline in equity value and an increase in equity volatility jointly reduce the estimated distance between asset value and the default point. The result is consistent with the end of the low-cost capital cycle and the sudden reassessment of refinancing risk.

Third, default risk improved materially in 2024. The median DD rose to 2.627, the 10th percentile of DD increased to 1.326, the median PD declined to 0.43%, and the share of firms with PD above 10% fell to 9.24%. The improvement suggests

that the high-risk tail narrowed after the 2022 stress episode. Nevertheless, the persistence of a non-trivial high-PD group implies that recovery was incomplete and concentrated among stronger firms.

Table 1: Annual distribution of DD and PD in the full valid sample

Year	Valid observations	Valid rate (%)	Median DD	DD P10	DD < 0 (%)	DD < 1 (%)	Median PD (%)	PD > 10% (%)
2019	1,244	75.58	2.414	0.720	4.02	14.23	0.79	19.37
2020	1,284	78.01	2.464	0.837	3.27	12.15	0.69	18.30
2021	1,380	83.84	2.620	1.080	2.39	8.84	0.44	14.13
2022	1,385	84.14	2.392	0.950	2.24	10.54	0.84	16.46
2023	1,376	83.60	2.477	0.974	2.91	10.47	0.66	15.04
2024	1,385	84.14	2.627	1.326	1.44	5.63	0.43	9.24

5.2 Robustness to Extreme Equity Volatility

The 150% annualized equity-volatility threshold is used as a robustness filter rather than as a sample-selection rule. In emerging equity markets, extremely high annualized volatility often reflects thin trading, price-limit effects, speculative trading, or microstructure noise rather than changes in the asset's fundamental risk alone. The threshold is deliberately set at a high level, lying in the upper tail of the empirical volatility distribution, so that ordinary volatility variation is preserved while observations with extreme volatility that may destabilize the Merton inversion are excluded. In quantitative terms, the threshold removes only 68 observations, equivalent to approximately 0.84% of the 8,054 valid Merton observations, reducing the sample from 8,054 to 7,986 firm-years. Therefore, the 150% threshold should be interpreted as a robustness filter targeting the extreme upper tail of the equity-volatility distribution, not as a substantive redefinition of the analytical sample. The main results remain qualitatively unchanged after applying this filter, suggesting that the aggregate findings are not driven solely by microstructure-related outliers.

Because equity volatility may be affected by thin trading and abnormal price movements, Table 2 repeats the analysis after excluding observations with annualized equity volatility above 150%. The qualitative time pattern remains unchanged. Default risk is lowest in 2021, worsens in 2022, and improves in 2024. The share of firms with PD above 10% declines slightly after filtering, indicating that part of the extreme-risk tail is driven by high-volatility small-cap firms. However, the main aggregate stress pattern is not mechanically generated by a small number of outliers.

This robustness check is important for two reasons. First, it addresses a standard concern in emerging-market applications of market-based models: equity volatility may partly reflect liquidity noise. Second, it clarifies the interpretation of the 2022

deterioration. The deterioration persists after the filter, indicating it is not simply an artifact of a few illiquid stocks. Rather, it reflects a broader shift in the distribution of market-implied credit risk.

Table 2: DD and PD after excluding observations with equity volatility above 150%

Year	Observations	Median DD	DD P10	DD < 0 (%)	DD < 1 (%)	PD > 10% (%)
2019	1,228	2.434	0.758	3.09	13.27	18.40
2020	1,273	2.472	0.879	2.59	11.39	17.60
2021	1,361	2.645	1.118	1.47	7.86	13.08
2022	1,375	2.402	1.005	1.75	9.96	15.85
2023	1,366	2.499	1.008	2.27	9.88	14.49
2024	1,383	2.633	1.330	1.30	5.50	9.11

5.3 Firm size and Financial Safety Buffers

Table 3 reports DD and PD patterns for the large-cap subsample, defined as firm-year observations with market equity E at or above the pooled full-sample 75th percentile among valid Merton observations over 2019–2024. Because the threshold is computed from the pooled sample rather than separately within each year, the number of large-cap observations varies over time. Large-cap observations have substantially stronger safety buffers than the full sample. Their median DD ranges from 2.534 to 3.472, and the share of firms with $(DD < 1)$ remains below 6% in all years. By 2024, the share of large-cap observations with PD above 10% falls to only 0.98%. This result confirms that structural credit risk is not evenly distributed across the market.

The size pattern has a clear economic interpretation. Large firms usually have better access to bank credit, stronger bargaining power with creditors, higher analyst and investor attention, and more liquid equity markets. These features support refinancing capacity and reduce the likelihood that temporary market stress immediately becomes distress. They also improve the reliability of the Merton inputs because market prices and volatility estimates are less likely to be distorted by illiquidity. Nevertheless, large firms were not fully insulated from the shock of 2022. Median DD fell from 2.850 in 2021 to 2.534 in 2022, indicating a system-wide component of the shock.

5.4 Sectoral and Firm-Level Heterogeneity

Table 4 illustrates the dynamics of DD and PD for selected representative firms. Real estate firms are highly sensitive to refinancing conditions. NVL, for example, experienced a sharp deterioration in 2022, with DD falling from 2.501 in 2021 to 0.335 in 2022 and PD rising from 0.62% to 36.88%. VHM registered a comparable,

Table 3: DD and PD for firm-year observations with market equity above the pooled P75 threshold

Year	Observations	Median DD	DD < 1 (%)	PD > 10% (%)
2019	206	2.877	5.83	9.22
2020	273	2.853	5.86	12.45
2021	424	2.850	5.42	12.50
2022	334	2.534	4.79	9.88
2023	370	2.749	4.32	8.65
2024	407	3.472	0.49	0.98

Notes: The large-cap subsample is defined using the pooled 75th percentile of market equity E across all valid Merton firm-year observations over 2019–2024, not a separate within-year quartile cutoff. Therefore, the number and share of large-cap observations vary by year as market capitalization changes over time. DD denotes distance to default, and PD denotes model-implied probability of default. Lower DD and higher PD indicate higher structural credit risk.

though less severe, deterioration in 2022, with DD falling from 2.638 to 1.147 and PD rising from 0.42% to 12.57%, before recovering strongly in subsequent years. These patterns are consistent with real estate firms' exposure to bond-market refinancing conditions, large debt maturities, and investor confidence.

The steel sector exhibits cyclical patterns. HPG's DD fell from 4.142 in 2021 to 1.347 in 2022 and then recovered to 4.273 in 2024. This pattern is consistent with the sensitivity of capital-intensive commodity-related firms to demand expectations, inventory cycles, and financing costs. Other sectors display more moderate dynamics. POW maintained DD above 1.9 throughout the window, while VJC improved sharply by 2024 after the post-pandemic recovery in air travel. MWG and BSR also show recovery in 2024. These examples indicate that the Merton indicators respond to both market-wide shocks and sector-specific cash flow and refinancing conditions.

Table 4: Selected firm-level DD and PD by sector, 2021–2024

Sector	Code	2021		2022		2023		2024	
		DD	PD (%)	DD	PD (%)	DD	PD (%)	DD	PD (%)
Real estate	NVL	2.501	0.62	0.335	36.88	1.943	2.60	–	–
Real estate	VHM	2.638	0.42	1.147	12.57	5.426	0.00	3.765	0.01
Steel	HPG	4.142	0.00	1.347	8.89	2.828	0.23	4.273	0.00
Power	POW	1.956	2.52	2.255	1.21	2.501	0.62	3.334	0.04
Airline	VJC	2.383	0.86	1.557	5.97	1.310	9.51	5.525	0.00
Retail	MWG	3.049	0.11	1.795	3.63	2.325	1.00	4.523	0.00
Refining	BSR	2.741	0.31	2.006	2.24	1.875	3.04	3.380	0.04

Notes: A dash (–) indicates that valid Merton estimates are unavailable for a given firm-year, for example, due to missing market or debt data or non-convergence of the inversion; NVL has no valid estimate for 2024.

Table 5 examines the lower tail of the DD distribution in 2024 after excluding observations with extreme equity volatility. The 1st percentile of DD is negative,

and the 5th percentile is 0.947, indicating that a small subset of firms remains close to or below the model-implied default boundary. Table 6 lists four firms with very low DD values. These firms have extremely thin equity buffers relative to the default point, as shown by their low E/D ratios. For such firms, the model's warning signal is economically intuitive: even moderate additional volatility or further declines in market capitalization can push asset value below the debt threshold.

Table 5: DD percentiles in 2024 after the volatility filter

Statistic	Min	P1	P5	Median	P95	Max
DD	−0.668	−0.242	0.947	2.633	8.206	51.815

Table 6: Lowest-DD firm-year observations in 2024 after the volatility filter

Code	DD 2024	PD 2024 (%)	Market equity E	Default point D	E/D	Equity volatility (%)	Tail position
FTM	−0.668	74.79	4.00×10^{10}	1.47×10^{12}	0.027	67.14	Below P1 cutoff
DIC	−0.538	70.49	2.61×10^{10}	1.19×10^{12}	0.022	57.44	Below P1 cutoff
JOS	−0.477	68.34	1.65×10^{10}	5.22×10^{11}	0.032	64.98	Below P1 cutoff
PVV	−0.425	66.47	4.20×10^{10}	9.61×10^{11}	0.044	102.19	Below P1 cutoff

Notes: P1 in Table 5 denotes the empirical first-percentile cutoff of the 2024 DD distribution after applying the volatility filter. The firms reported in Table 6 are selected low-DD observations below that cutoff; they are not observations with DD equal to the P1 value.

5.5 Validation Results and Benchmark Comparison

This subsection reports the validation results and benchmark comparison. The first part evaluates whether Merton-based DD and PD discriminate between event and non-event firms under two distress-event definitions. The primary definition, $Y_{i,t}^{\text{hard}}$, captures hard distress events, including special control status, overdue corporate bond obligations, and delisting associated with serious financial or compliance problems. The broader definition, $Y_{i,t}^{\text{broad}}$, includes exchange-based control or market-supervision events related to financial weakness. This two-layer design is necessary because not all observed distress labels represent the same degree of credit distress. The second part compares the standalone Merton signal with accounting-logit and hybrid-logit benchmarks to assess whether market-implied PD adds information beyond accounting and industry variables.

Table 7 shows that hard-distress firms have higher median PD and lower median DD than non-event firms. The direction of separation is consistent with the Merton mechanism: firms closer to the default boundary are assigned lower DD and higher PD. However, the magnitude of separation is moderate rather than extreme. This result is consistent with the ROC-AUC findings and supports a

Table 7: Median DD and PD by hard-distress status in the convergent validation sample

Hard-distress status	Number of observations	Median PD (%)	Median DD
$Y_{i,t}^{\text{hard}} = 1$	41	2.12	2.030
$Y_{i,t}^{\text{hard}} = 0$	4,105	0.61	2.506

Notes: The table uses the convergent Merton validation sample over 2022–2024. $Y_{i,t}^{\text{hard}} = 1$ includes special control status, overdue corporate bond obligations, and delisting associated with serious financial or compliance problems.

cautious interpretation of Merton-based PD as a risk-ranking indicator rather than a complete default classifier. The relatively small number of hard-distress events also explains why AUC confidence intervals are not narrow and why the results should be interpreted as evidence of risk-ranking usefulness rather than precise probability calibration.

The validation results therefore support a balanced conclusion. DD and PD provide useful market-implied risk-ranking information by separating hard-distress firms from non-event firms and identifying high-risk tails of the distribution. At the same time, they should be used jointly with other information. A practical default-prediction system for Vietnam would likely improve by adding accounting variables, liquidity ratios, bond maturity schedules, audit opinions, governance indicators, ownership structure, and macro-financial variables.

To address the benchmark and validation concerns, Table 8A and Table 8B report discriminatory power under both event definitions and for accounting-based and hybrid specifications, with bootstrap confidence intervals.

Table 8A: Merton event-definition sensitivity

Validation design	Target definition	Sample	AUC	95% CI	Interpretation
Merton PD, convergent sample	$Y_{i,t}^{\text{hard}}$	4,146; $Y = 41$	0.656	[0.581; 0.725]	Moderate
Merton PD, all-sample with non-convergent high-risk score	$Y_{i,t}^{\text{hard}}$	4,938; $Y = 51$	0.624	[0.560; 0.685]	Limited-to-moderate
Merton PD, convergent sample	$Y_{i,t}^{\text{broad}}$	4,146; $Y = 83$	0.576	[0.520; 0.629]	Weak
Merton PD, all-sample with non-convergent high-risk score	$Y_{i,t}^{\text{broad}}$	4,938; $Y = 96$	0.543	[0.493; 0.593]	Weak

Notes: PD is used as the risk score. In the all-sample rows, non-convergent observations are assigned a conservative high-risk score. Confidence intervals are obtained using bootstrap resampling of the validation sample.

Table 8A shows that Merton-based PD has moderate discriminatory power

Table 8B: Accounting and hybrid benchmark

Model	Target definition	Training sample	Test sample	AUC	95% CI	Interpretation
Accounting logit + industry	Broad financial-distress label	2020–2023	2024	0.687	[0.625; 0.746]	Accounting benchmark
Hybrid logit: accounting + industry + PD	Broad financial-distress label; Merton-valid sample	2020–2023	2024	0.695	[0.636; 0.750]	Slight incremental gain

Notes: The accounting benchmark is trained on the 2020–2023 sample and evaluated on the 2024 backtest sample. The hybrid model is estimated on the Merton-valid subsample. Because the accounting-logit benchmark and the Merton event-definition test use different validation designs, the results are interpreted as complementary evidence rather than as a strict horse-race comparison on an identical sample.

only under the stricter hard-distress definition. Among convergent observations, the AUC is 0.656. When non-convergent observations are retained and assigned a conservative high-risk score, the AUC declines to 0.624. Under the broader exchange-based definition of distress, the AUC ranges from 0.543 to 0.576. This pattern indicates that Merton PD is more informative for hard credit-distress events than for broad administrative or market supervision labels.

Table 8B reports the accounting and hybrid benchmark results. Accounting-logit specifications achieve higher out-of-time AUC values than the standalone Merton validation results, while the hybrid model provides only a small incremental gain. These findings support a balanced interpretation: Merton-based DD and PD are useful market-implied risk-ranking tools, but they should not be used as standalone binary classifiers.

5.6 Interpretation of Model Performance

The contrast between $Y_{i,t}^{\text{hard}}$ and $Y_{i,t}^{\text{broad}}$ is informative. The Merton framework is designed to detect structural balance-sheet pressure arising from the interaction between market equity value, asset volatility, and debt obligations. It is therefore more closely aligned with hard distress events such as overdue bond obligations, special control status, and delisting. By contrast, broader exchange-based labels may include administrative, disclosure-related, audit-related, and trading-status events. These events may reflect governance or transparency problems rather than immediate structural default risk. As a result, the discriminatory performance of Merton PD naturally declines when the target variable is broadened.

The empirical findings should be interpreted as evidence of useful but incomplete market-based credit-risk information. The Merton model performs well in identifying broad changes in the risk environment. It captures the deterioration in 2022, the improvement in 2024, the relative resilience of large-cap firms, and the concentration of risk in highly leveraged sectors. These patterns are economically plausible and

consistent with the institutional narrative of Vietnam's financing cycle. From a surveillance perspective, this is a valuable result because it means the model can help detect changes in the distribution of corporate vulnerability before all accounting and legal indicators have fully adjusted.

At the same time, the model's moderate AUC cautions against over-reliance on structural PD as a binary classifier. Several mechanisms can weaken classification performance. A firm may have high PD because its equity value is low and volatile, yet it may avoid observed default through asset sales, creditor negotiations, maturity extension, ownership support, or regulatory relief. Conversely, a firm may experience a distress event for reasons not fully captured by market capitalization and debt structure, such as governance problems, information-disclosure violations, litigation, or sudden liquidity withdrawals. These mechanisms are common in emerging markets and make observed distress only an imperfect target variable.

The practical implication is that DD and PD should be used as risk-ranking tools. A high-PD firm should be placed on a watch list and examined using additional information, including debt maturity schedules, cash-flow coverage, bond repayment plans, collateral quality, related-party transactions, audit qualifications, and sector-specific refinancing conditions. A low-PD firm should not be ignored if other risk indicators deteriorate. Thus, the structural model is best viewed as one layer in a multi-signal early-warning framework. Its contribution lies in the disciplined translation of equity-market signals into credit-risk language, not in replacing credit analysis.

This interpretation also affects policy use. Regulators and exchanges can monitor the share of firms in high-risk DD or PD zones as a market-based indicator of systemic risk. Banks and institutional investors can use DD and PD to prioritize due diligence. Researchers can use the indicators as inputs in hybrid models that combine structural, accounting, and macro-financial variables. These applications are more appropriate than treating model-implied PD as an exact estimate of legal default probability.

6 Discussion and Implications

The findings have several implications for credit-risk monitoring in emerging markets. First, market-based DD and PD can provide timely information during periods of stress. In 2022, the deterioration in risk was visible through lower DD and higher PD, especially in the distributional tail. This suggests that structural indicators can serve as early-warning signals for regulators, lenders, and investors. They are particularly useful when accounting statements lag behind market conditions.

Second, the results show that aggregate market indicators should be complemented by distributional analysis. Median risk measures alone do not reveal the concentration of risk among small-cap and highly leveraged firms. The share of firms with ($PD > 10\%$), the 10th percentile of DD, and the lower-tail firm lists

provide a clearer view of systemic fragility. Such measures can help supervisors identify whether stress is broad-based or concentrated in particular segments.

Third, the sectoral evidence highlights the importance of refinancing channels. Real estate and other capital-intensive sectors are more sensitive to bond-market liquidity, interest-rate conditions, and investor confidence. For firms in these sectors, a decline in market capitalization and a rise in volatility can quickly reduce DD even before formal default occurs. This makes market-implied risk indicators relevant for monitoring the interaction between corporate finance and financial stability.

Fourth, the moderate validation performance has an important methodological implication. The Merton model should not be used mechanically. A firm with high PD is not necessarily destined to default, and a firm with low PD is not necessarily free of risk. Rather, the model provides a structured signal that should trigger deeper analysis. In practice, the strongest use cases are not binary default prediction but rather risk ranking, watch-list construction, sector surveillance, and stress-testing exercises.

6.1 Economic Mechanism Behind the Deterioration of 2022

The 2022 deterioration in DD and PD can be interpreted through three linked mechanisms. First, higher funding costs increased the present value of refinancing pressure, particularly for firms with large short-term debt and bond maturities. Second, the corporate bond market shock reduced investor confidence and increased uncertainty about firms' ability to roll over debt. Third, these concerns were incorporated into equity prices through lower market capitalization and higher equity volatility. In the Merton framework, these channels operate in the same direction: lower equity market value reduces the inferred asset buffer, while higher volatility increases the probability that asset value crosses the default boundary. The rise in the share of firms with PD above 10% in 2022 therefore reflects not only weaker balance-sheet positions but also a market-wide reassessment of refinancing risk.

The Vietnamese evidence is consistent with broader emerging-market findings that structural credit-risk indicators are informative but sensitive to market liquidity, disclosure quality, and institutional conditions. Similar to other emerging-market settings, the model is particularly responsive in capital-intensive sectors where refinancing pressure is central. However, the moderate AUC also indicates that in markets where default can be delayed through restructuring or regulatory intervention, market-implied PD should be interpreted as a stress-ranking indicator rather than a calibrated legal-default probability.

Two features distinguish the Vietnamese setting from the developed-market benchmarks in which structural models were originally validated. First, the mapping from economic distress to an observable default event is weaker: restructuring, maturity extensions, and regulatory forbearance can postpone formal default well beyond the point at which market-implied risk has risen, thereby mechanically

capping achievable discriminatory power regardless of model quality. Second, equity volatility is estimated by a market with thinner trading and price-limit mechanisms, so the volatility input carries more microstructure noise than in deep markets. Both features push measured AUC downward relative to studies on US or European samples, and both argue for interpreting market-implied PD as a stress-ranking device rather than a calibrated default probability. This interpretation is reinforced by the finding that the model is most responsive in capital-intensive, refinancing-dependent sectors, where the link between market valuation and rollover capacity is strongest.

Another implication concerns reporting practice. For market participants to use structural credit-risk indicators effectively, data on debt maturity, bond repayment status, collateral, and payment delays must be disclosed in a timely and standardized manner. The Merton model relies on market capitalization and volatility, but the interpretation of the default point depends on liability information. Better disclosure would reduce noise in DD and PD estimates and would make structural indicators more useful for both private risk management and public financial stability monitoring.

Several limitations should be acknowledged. The model assumes a simplified liability structure and a one-year horizon. The default point approximation may not capture the full maturity profile of debt. Equity volatility may be distorted by illiquidity, particularly for small firms. Observed distress labels may be incomplete and may mix true default with broader financial distress. Finally, model-implied PD is not calibrated to long-run realized default frequencies. These limitations are especially relevant in emerging markets, where institutional features influence the timing and form of default.

Future research can extend this paper in several directions. First, DD and PD can be combined with accounting, liquidity, governance, and macro-financial variables in logit, survival, machine-learning, or dynamic panel frameworks. Second, the model can be calibrated separately by sector to reflect different debt maturity structures and asset volatility patterns. Third, bond-market information, if available, can be used to compare equity-implied risk with credit-spread-implied risk. Fourth, micro-level information on overdue debt, restructuring, and bond maturity schedules can improve the definition of event labels and strengthen validation.

7 Conclusion

This paper applies the Merton structural model to measure corporate default risk in Vietnam using a broad panel of non-financial firms that are listed and registered for trading over 2019–2024. By combining daily equity-market data with accounting-based debt information, the study estimates distance to default and model-implied probability of default for 8,054 valid firm-year observations. The analysis shows that market-implied credit risk improved during the 2021 low-risk phase, deteriorated

in 2022 amid the funding-cost and corporate-bond liquidity shock, and improved again by 2024.

The results reveal substantial heterogeneity. Large-cap firms maintain stronger financial safety buffers, while risk is concentrated among small-cap, highly leveraged firms and in capital-intensive sectors. Firm-level examples confirm that DD and PD respond strongly to refinancing stress and market revaluation. Event-based validation shows that Merton-based PD has moderate discriminatory power under a stricter hard-distress definition, with an AUC of 0.656 among convergent observations and 0.624 when non-convergent cases are retained and assigned conservative high-risk scores. However, performance weakens under broader exchange-based distress labels. Benchmark comparisons further show that accounting-logit specifications achieve higher out-of-time discrimination performance in the 2024 backtest, while adding Merton PD to the accounting and industry specifications provides only incremental improvement. These findings indicate that DD and PD should be interpreted primarily as market-implied early-warning and risk-ranking indicators rather than as standalone binary classifiers.

Overall, the evidence supports a cautious but useful role for the Merton framework in Vietnam. Structural DD and PD translate market capitalization, equity volatility, and debt obligations into transparent risk indicators that are useful for surveillance, high-risk tail monitoring, and watch-list construction. Their standalone classification performance is moderate and sensitive to the event definition, reflecting the institutional characteristics of an emerging market, where distress may be delayed, restructured, or expressed through exchange-supervision mechanisms. A more complete credit-risk monitoring framework should therefore combine Merton-based indicators with accounting, liquidity, governance, sectoral, and institutional variables.

Data Availability Statement

The data used in this study are obtained from the FiinPro Platform and related public market and financial-statement sources. Because the raw data are subject to database access restrictions, the authors cannot redistribute them. Processed data and code may be made available by the corresponding author upon reasonable request, subject to data-provider restrictions.

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Conflict of Interest

The authors declare that they have no conflict of interest related to this publication.

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