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Research paper

Volatility spillover in crude oil market using Heston switching Clayton model

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Abstract:
The purpose of this study is to investigate the effects and risk spillover from the global crude oil market on Tehran Stock Exchange Oil Group. For this purpose, we used a combination of copula models and switching models in this research. First, we will examine marginal models and examine Heston switching and Markov switching models in this market. Then we create the multivariate distribution function using Clayton's copula. The data analyzed in this research are related to the global crude oil markets and the Tehran Stock Exchange Oil Group from December 2011 to January 2023. This time period was chosen due to the examination of different regimes in the above markets and also the selection of the appropriate marginal model for these markets. The results show the crude oil market has influenced on Tehran Stock Exchange and also the Tehran Stock Exchange Oil Group indices. Volatility in this global market cause turbulence in the Tehran stock market and this market is affected by the global crude oil market. This is due to the influence of the global crude oil market on total prices in these markets. Heston switching model and its combination with copula models including Clayton copula can bring good results. This is confirmed by comparing this model with other models such as copula Markov switching models.

Keywords: Heston switching copula, Clayton copula, Spillover, Energy markets, Oil shocks
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1 Introduction

Crude oil has become one of the most important materials in human life since the 50s. Crude oil is divided into three categories, commercially: Brent crude, which is extracted in Europe, WTI crude that is extracted in America and OPEC, which is extracted and supplied by OPEC member countries. Iran is a member of OPEC as the fourth country with oil reserves and also the sixth oil exporting country and it supplies oil to the world markets in interaction with the member such as wars, riots and social events such as the Corona virus have affected the crude oil price (Ding et al., 2023). countries of this organization. In addition to supply and demand, the crude oil price in the world is also related to markets such as the global gold market and the value of the US dollar, and these factors affect its price. Of course, we should not ignore political and social events and factors. Political events Further studies have shown that global oil prices volatility can have direct and indirect effects on the stock market and stock market industries. Companies stock prices varies based on the effects of oil in different industries and this factor can change the index of the whole stock market. Examining the affiliation structure between different markets can be important, including the affiliation structure of local markets on the global oil market. Affiliation structure of financial markets in forming the portfolio of financial assets of economic actors in order to gain more efficiency has been the subject of many financial studies in this field in recent years. So that experimental studies with different methods seek to identify the affiliation structure in the financial markets and its effect on the formation of asset portfolios with the least risk and high efficiency. Investigating structural correlation patterns in financial returns, considering the change in behavior in financial markets on a daily basis and their instability, gives economic activists in this field a special ability to predict and discover relationships between markets which plays an important role in achieving financial goals and achieving profitability. Examining the behavior pattern in the crude oil market is very important due to the tremendous impact of the oil market on the stock markets (Jin, 2015; Awartani, 2013). Researches have been conducted regarding the relationship between the crude oil markets and the stock market which have investigated the impact of this market volatilities on the stock markets around the world (Sarwar, 2020; Bouri and Demirer, 2016; Kang et al., 2015; Hamma et al., 2014; Narayan, 2012). In general, economists emphasize volatility spillover to understand the interdependence between financial markets. The relationship between these markets is classified into two categories: parallel (mainly affecting the cost price) and competitive relationships. Discover the relationships between different markets gives this ability and characteristic to the activists in the financial and economic fields to succeed in achieving their business goals and gain profitability. These relationships check will be possible by examining correlation patterns between these markets. Crude oil is considered as the driving engine of the world. So, the economy all over the world depends on it and oil prices volatility can affect the capital market. Crude oil price changes can affect the growth and development of the economy in a positive or negative way, considering that in the modern era, crude oil is known as an essential factor for production in any economy. It is noted that the increase in oil prices leads to an increase in production costs and an increase in the cost price, which naturally causes an increase in the inflation rate and a high inflation rate in the economy has an adverse effect on economic growth. According to Ran Ma (2019), oil price is an important element for the economy, so oil prices changes and volatility can adversely affect stock prices in stock markets. These effects are much greater in the Iranian stock market. The government's oil revenues and the share of these revenues in Iran's economy and consequently its impact on inflation make the importance of this market more colorful and show its commercial link with the stock market. In this research, we are looking to investigate the impact of the global crude oil market on the stock market index and also its impact on the oil group of this market.The purpose of the present research is to combine the marginal models of Markov Switching and Heston Switching models with Clayton's copula model and finally introduce Heston Switching Clayton model. Despite their many strengths, Markov Switching models suffer from weaknesses. For example, we can point to the weakness of these models in short time intervals. On the other hand, one of the basic assumptions of this model is the normal distribution of the input data, which is not possible in reality and can be accepted with a suitable approximation in large time intervals. It is also assumed in this model that the model outputs are independent from each other which this assumption is not correct. The Markov Switching model will perform poorly in cluster volatility. Many of these weaknesses will be overcome by the Heston Switching model. This model shows good performance in short time intervals. The problem of cluster volatilities can be solved in this model due to the existence of a stochastic differential equations. On the other hand, this model does not require that the data follow a certain distribution (Ganti 2021). These features create high flexibility for Heston models, which can lead to better performance. So far, researchers have been interested in using the Markov switching model as a marginal model and combining it with various copula models and many researches have been done in this regard. In this research, we will use the Heston Switching model as the marginal model, considering the Markov Switching model weaknesses and also to solve these weaknesses. Finally, we introduce the Heston Switching Clayton model by combining the Heston Switching model and the copula Clayton model.

2 Literature review

The markets spillover from each other is considered as one of the interested topics to researchers. As mentioned earlier, spillover between markets is more important than in the past by releasing information and increasing transactions between different markets. Researchers usually believe that risk spillover can occur due to various factors. Factors such as investors' behaviors and expectations (Mihai,Maria, 2020), market efficiency and the amount of transfer and freedom to information dissemination in the market (Ajaya et al., 2021), legal supervision of the market by various institutions including governments (Fan et al., 2020) and also, maturity level of financial systems (Alberto et al., 2019), for example, can affect the spillover level. There were limited studies on the relationship between markets, including the stock market, with other markets before the mid-1950s. Attentions to the volatility spillover increased by addressing researchers to issues in the field of investment portfolio and the need to create diversity in it as well as the integration of international financial markets, from the 1960s onwards. At first, these relationships investigation was focused on the long-term relationships of these markets, but short-term interactions were also investigated over time. Most of the conducted researches have used statistical models such as simple linear regression, quartile regressions, vector autoregression. Among these researches, longin and solnik (1995) investigated the spread of extreme volatility in the markets with the help of regression models. The results of their study showed that big shocks in a market tend to spread faster. Mc Queen et al. (1996) investigated the shocks effects on the Asian financial market using regression models. Non-linear approaches to investigate the markets spillover as well as the financial markets affiliation have attracted the attention of researchers since the 90s. Copula models were introduced considering the weaknesses of regression models. Copula functions have been examined by different researchers in the affiliation structures between markets, among which we can refer to researches of Costinot et al., (2000); Patton (2006); Canela (2006) (Bargkar and Sohrabi, 2020). A lot of research has been done regarding the investigation of the relationship between the markets with the help of copula models so that the dynamic affiliations between the markets can be determined with the help of these models. Huang et al. (2009) used Clayton copula to determine the asymmetric risk correlation. Chang (2017) used Gamble and Clayton copula to investigate the correlation between crude oil market and futures transactions. Li and Hussain (2018) investigated the relationship between the Chinese market and the Asian, European and United States markets. The copula models described are suitable in the case of static relationships. It is suggested to use suitable marginal functions in order to add dynamics to these models. Switching models can create this dynamic. Tansuchat et al. (2017); Selmi et al. (2019); Tiwari et al. (2020); and Mwamba and Mwambi (2021) have added this dynamic to copula models using the Markov switching model as a marginal model. According to the nature of Markov switching models, they will not perform well in cases with short time intervals, cluster fluctuations, and also in cases where the data distribution is not normal. Heston switching model can be used according to these limitations and to solve them. The relationship between the global oil market and global stock markets has been the subject of many researches in the world among which we can refer to the studies of

Degiannakis et al. (2013); Broadstock and Filis (2014); Bouri et al. (2017); Mensi et al. (2017); Antonakakis et al. (2018); Tiwari et al. (2018); and Ran Ma et al. (2020). Most of these researchers believed that in addition to its direct impact on the stock markets, the oil market also has an indirect impact due to its impact on the cost of products. In addition to the psychological effect, this indirect effect is due to the effect on the government's macro variables and naturally its effect on interest rates (Arouri et al. 2011). Another view of the relationship between the oil market and the stock market has been introduced by Zhang (2018). He has proposed the concept of financialization in his article. This concept implies that the financial markets are closer to each other than before. Crude oil price changes in recent years have had an increasing impact on financial markets, including local stock markets (Mensi et al., 2017; Peng et al., 2018). The introduction of new financial instruments and derivatives in oil markets (Ji and Zhang, 2019) is another sign of the convergence of oil markets and global financial markets. Currently, some investors are present in oil markets to diversify their assets. The closer connection of these markets strengthens the fact that these markets are influenced by each other and as a result risk spillover occurs in these markets. Market instability is closely related to risk spillover in related markets (Antonakakis et al., 2018). The instability caused by extreme volatilities also makes it difficult for investors. The purpose of this research is the investigating the relationship between the crude oil market and the Iranian stock market, especially the oil group of this market. For this purpose, we will use the combination of Heston switching models and Clayton copula model.

3 Research methodology

In this research, we will examine the combination of switching models and copula models. In terms of introducing the Heston switching copula model, this research is included in the basic research category, but it is also included in the category of applied research due to the fact that the results of this research can be used to create various portfolios. This research is part of the correlation research category given that its purpose is to examine the impact of the global crude oil market on the Tehran Stock Exchange and specifically the oil group of this market. And finally, according to the type of data, it is placed in the category of quantitative research. The basic assumption of this research is that the performance of Clayton Heston switching model will be more suitable than Clayton Markov switching model in predicting spillover. For this purpose, we reviewed the data obtained from the global crude oil market as well as the index of the Tehran Stock Exchange and the oil group of this market in the period from December 2011 to January 2023. Finally, spillover is checked from the global crude oil market to this market according to Clayton switching models. In this research, we will first examine the marginal model and then examine the combination of this model with the copula model. As we know, the task of connecting marginal distributions with each other is the responsibility of copula functions, regardless of any classification. Marginal distributions and then the multivariate model are examined together to check the risk spillover between different markets. Markov switching and Heston switching models are among the marginal models that are used to fit the existing data. We know, the parameters in switching models will not be the same in the whole model. The parameters will be different depending on the regime they are in. Switching models allow the economy to be in a regime with any number of constraints at any point in time. The regime has a great influence on the dynamic behavior of series. Regime can occur at any point in time. The transfer function will occur based on the Markov process in both Heston switching and Markov switching models. The probability of transition from regime i to regime j at time t is defined as $P_{ij} = Pr\left(S_t = i|S_{t-1} = j\right)$.

$$\Pr\left(S_t = K|S_{t-1} = j\right) = 1 - \sum_{i=1}^{i=K-1} \Pr\left(S_t = i|S_{t-1} = j\right) \quad (3.1)$$

A Markov switching model like (2) allows to change the volatility between regimes, that is, it allows σ to change and fluctuate between two regimes (Neisy , Salimi nasab, 2013).

$$\Delta r_t = a_i + b_i r_{t-1} + \sigma_i r_{t-1}^{\gamma} \varepsilon_t \quad (3.2)$$

Where $i \in \{1,\ 2\}$ is an input to the regime at time t, γ is the distribution parameter, σ is the volatility parameter, and $a,\ b$ are constant parameters in each regime. ε_t is a white noise process that is independent of r_t. All $\varepsilon_t$are independent and have the same distribution with mean zero and variance σ^2. The Heston switching model will be another model that we consider as a marginal model. The basic Heston model was introduced by Heston in 1993. This model is defined based on Brownian motion. This basic model has a big weakness because it does not consider different regimes. This model can be useful if it is accompanied by the possibility of changing the regime. Consider that we have a probability space $(\Omega, \mathbb{F}\left(\mathcal{F}_t\right)_{[.,T]}, \mathbb{P})$ where $\mathbb{P}$ represents neutral risk. We define X_t as a Markov chain in the space $E\left\{1,\ \ldots,\ S\right\}$ with initial value . This Markov chain actually specifies the regime in the desired data. We denote the generator matrix X by , which we have $\Pi_{ij} \geq 0$ for all values$i,\ j \in E$, if $i \neq j$ and$\Pi_{ii} = \sum_{i \neq j} \Pi_{ij}$. We assume that the transition probability is constant from state $i \in E$ at t to state $j \in E$ at $t + h$. This point leads to the creation of a generator transfer function called . Let we now denote by $X(h)$ the defined transition probability matrix. This matrix is defined as follows:

$$X_{ij}(h) = \mathbb{P}\left(Z_h = j\ Z_0 = i\ \right) \quad (3.3)$$

Then for all $i,\ j \in E$ we will have:

$$\frac{d\ X_{ij}(h)}{d\ h} = \ \Pi_{ij} \quad for\ all \quad i,\ j \in E \quad (3.4)$$

Finally, let us represent the transition matrix as a Markov switching model by P. So, we will have:

$$P_{ij} = \begin{cases} \frac{\Pi_{ij}}{\sum_{i \neq j} \Pi_{ij}} & if\ i \neq j \\ 0 & In\ others \end{cases}$$

Now consider $S = (S_t)$ to be a random process in our probability space. This space can exist in any of the markets that we conduct research on. Consider that $V = (V_t)$ is another random process that models the instantaneous variance of S. It is important to note that all the parameters of the random volatility process v and the correlation coefficient depend on the state X between the index of each market and its instantaneous variance v. In a simpler word, the parameters of each regime depend on that regime. Consider basic Heston model:

$$dS_t = \mu S_t + S_t \sqrt{v_t} dZ$$

$$dv_t = k(\theta - v_t)\, dt + \sigma \sqrt{v_t} dB$$

We know that Z and B are Brownian processes. v_t is the instantaneous variance, θ is the long-term variance and k represents the fluctuations of the variance. Z and B are independent of each other. Now we have:

$$\frac{dS_t}{S_t} = \mu + \sqrt{v_t} dZ \qquad (3.5)$$

And we also have:

$$\mu = rdt$$

Where r is the risk-free interest rate, so we have:

$$\frac{dS_t}{S_t} = rdt + \sqrt{v_t} dZ \qquad (3.6)$$

Now consider a situation where we are faced with different regimes. Then we will have:

$$X_t = \begin{cases} (1,0)^t, & Regime\ 1 \\ (0,1)^t, & Regime\ 2 \end{cases}$$

The transfer function between the two regimes follows the Poisson distribution. The transfer possibilities in this model are as follows:

$$P = Pr\left(S_t = 1 | S_{t-1} = 1\right)$$

$$q = Pr\left(S_t = 2 | S_{t-1} = 2\right)$$

For each regime, we will have:

$$dv_i = k_i(\theta_i - v_i)\, dt + \sigma_i \sqrt{v_i} dB_i \quad (3.7)$$

$$dZ_i\ dB_i = \rho_i dt \quad (3.8)$$

$$dZ_i\ dZ_j = \quad dB_i\ dB_j = 0 \quad (3.9)$$

Where ρ_i the correlation between two Brownian processes, θ is the long-term variance of volatility, k is the rate of return to the mean of volatility, and r is the interest rate. Now, after introducing marginal models, we will examine and introduce copula models. A copula function is a communication function for several marginal functions defined in the spaces $[0,1]^n$. For example, the joint distribution function with marginal distributions $F_x(x)$and $F_y(y)$ is defined as follows:

$$F_{XY}(x,y) = C\left(F_x(x), F_y(y)\right) \qquad (3.10)$$

If $F_x(x)$ and $F_y(y)$ are continuous and also these distributions are known, we will have:

$$C(u,v) = H\left(F^{-1}(u), F^{-1}(v)\right) \qquad (3.11)$$

In addition, their joint distribution will be as follows:

$$F_{XY}(x,y) = c(u,v)F_x(x)F_y(y) \qquad (3.12)$$

In such a way that $c(u,v) = \frac{\partial^2 C(u,v)}{\partial u \partial v}$ will be a copula density function and $F_x(x)$ and $F_y(y)$ are also marginal distributions. One can expand these equations and consider N marginal distributions connected to each other by a copula function and the joint distribution function will be created. As mentioned earlier, different joints can be used to create a multivariate distribution. Different types of joints have created a flexible space to specify the joint distribution. Clayton is one of the copula types that has a very good function in creating joint distributions. This copula can be used in 0, 90, 180 and 270 degree rotation modes. This joint is used in 0 and 180 degrees as well as 90 and 270 degrees to measure positive and negative affiliation, respectively. These two copula categories of Clayton are as follows(Liu, Peng , 2020):

$$C_1\left(u,v,\alpha_1,\alpha_3\right) = 0.5C_{c0}\left(u,v,\alpha_1\right) + 0.5C_{c180}\left(u,v,\alpha_3\right) \qquad (3.13)$$

$$C_2\left(u,v,\alpha_2,\alpha_4\right) = 0.5C_{c90}\left(u,v,\alpha_2\right) + 0.5C_{c270}\left(u,v,\alpha_4\right) \qquad (3.14)$$

In this way that

$$C_{c0}\left(u,v,\alpha_1\right) = \left(u^{-\alpha_1} + v^{-\alpha_1} - 1\right)^{-\frac{1}{\alpha_1}}$$

$$C_{c90}\left(u,v,\alpha_2\right) = u - \left(u^{-\alpha_2} + (1-v)^{-\alpha_2} - 1\right)^{-\frac{1}{\alpha_2}}$$

$$C_{c180}\left(u,v,\alpha_3\right) = u + v - 1 + \left((1-u)^{-\alpha_3} + (1-v)^{-\alpha_3} - 1\right)^{-\frac{1}{\alpha_3}}$$

$$C_{c270}\left(u,v,\alpha_4\right) = v - \left((1-u)^{-\alpha_4} + v^{-\alpha_4} - 1\right)^{-\frac{1}{\alpha_4}}$$

Therefore, a Clayton copula can be created based on the weight of C_1 and C_2:

$$C_{Clayton}(u,v,\theta) = \omega C_1\left(u,v,\alpha_1,\alpha_3\right) + (1-\omega)C_2\left(u,v,\alpha_2,\alpha_4\right) \qquad (3.15)$$

Where $\theta = (\alpha_1, \alpha_2, \alpha_3, \alpha_4) \in (0. + \infty)$. Note that larger values of α_1,α_2, α_3 and α_4 indicate stronger correlation. $\omega \in [0, 1]$ is used to estimate the of $C_1$and C_2.that financial markets are in a dynamic state, so the relations of these markets will not be static. Switching models can be used in order for our model to consider this dynamic. In fact, we can get good results by combining them with Clayton copula. So, we have:

$$C_{clayton\ SW}\left(u, v; \theta^{S_t}\right) = \omega^{S_t} C_1\left(u, v, \alpha_1^{S_t}, \alpha_3^{S_t}\right) + (1 - \omega^{S_t}) C_2\left(u, v, \alpha_2^{S_t}, \alpha_4^{S_t}\right) \quad (3.16)$$

Where S_t will represent the desired regime. As we mentioned, the possibility of regime change will occur based on the Markov process, so we have:

$$P_{11} = P(\ S_t = 1 |\, S_{t-1} = 1) = \frac{exp(\pi_1)}{1 + exp(\pi_1)}$$

$$P_{12} = P(\ S_t = 2 |\, S_{t-1} = 1) = 1 - P_{11}$$

$$P_{21} = P(\ S_t = 1 |\, S_{t-1} = 2) = \frac{1}{1 + exp(\pi_2)}$$

$$P_{22} = P(\ S_t = 2 |\, S_{t-1} = 2) = 1 - P_{21}$$

We can use the maximum likelihood function in order to estimate the parameters of the above models. Based on (6), the joint frequency function of Clayton switching model with x and y variables is as follows:

$$f_{XY}(x, y) = \sum_{S_t=1}^{2} f_X(x) f_Y(y) c\left(u, v, \theta^{S_t}\right) P\left(S_t\right) \quad (3.17)$$

Where $P(S_t)$ is the probability of being in regime S_t at time$t - 1$. Therefore, $P\left(S_t = 1\right)$and $P(S_t = 2)$are defined as follows:

$$\begin{aligned} P(S_t = 1) =& P_{11} \times \Big[\frac{c_{t-1}^1 P(S_{t-1} = 1)}{c_{t-1}^1(S_{t-1} = 1) + c_{t-1}^2 P(S_{t-1} = 2)}\Big] \\ &+ P_{21} \times \Big[\frac{c_{t-1}^2 P(S_{t-1} = 2}{c_{t-1}^1(S_{t-1} = 1) + c_{t-1}^2 P(S_{t-1} = 2)}\Big] \\ P(S_t = 2) =& 1 - P(S_t = 1) \end{aligned}$$

Where c_{t-1}^1 and c_{t-1}^2 are the joint probability density function in regimes 1 and 2, respectively. Based on the maximum likelihood function, we have:

$$\ln L = \sum_{t=1}^{T} \ln c(u, v, \theta^{S_t}) P\left(S_t\right) + \sum_{t=1}^{T} \ln f_X(x) + \sum_{t=1}^{T} \ln f_Y(y) \quad (3.18)$$

4 Empirical results

First, let us analyze the data of this research. In this research, we examined global crude oil markets as well as Tehran Stock Exchange Oil Group Index. Then the marginal distributions are examined after investigating these markets. These markets relationship investigated with the help of Clayton copula function. This research data was collected from December 2011 to January 2023. The statistical description of these markets is as follows:

Table 1: Statistical description of global crude oil market data and Tehran Stock Exchange oil group

	Tehran Stock Exchange oil group	global crude oil market
Num	2860	3333
Mean	1412641.	69.78408
Median	298481.5	65.07000
Max.	9644020.	124.1400
Min.	34084.00	12.82000
Std. Dev.	2091773.	23.22490
Skew.	1.610595	0.156187
Kurt.	4.379434	1.850327
Jarque-Bera	1463.236	197.1089

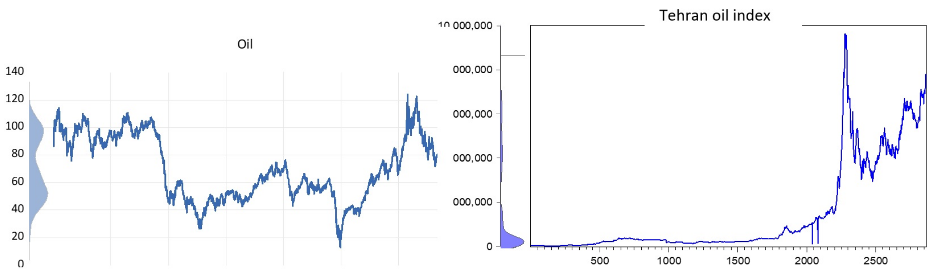

Figure 1: The global daily price of crude oil and the oil group and Tehran Stock Exchange Oil Group Index

Time series are one of the most important statistical data used in empirical analysis. Stationary is the main and primary assumption regarding time series. The results of most statistical tests will be doubtful if this assumption is not established. Lack of data stationary and time series causes problems such as dummy regression. The unit root test is used to check the time series stationary or a data set. Dickey Fuller test is one of the most important tests in this field. We have used Eviews software in order to perform the Dickey Fuller test. The test results can be seen in Table 2. If this test results show the data stationary, then these data will be used as input for the models introduced in this article. We use the differential series logarithm method to standardize the data and eliminate the unit root and the data stationary if the data has a unit root. The data is stationary and there is no risk of dummy regression using this method. Therefore, it can be used as a standard input data to copula stochastic volatility, copula Markov switching and copula Heston switching models.Consider the following stochastic process:

$$Y_i = \varphi Y_{i-1} + \varepsilon_i$$

$$\Delta Y_i = (\varphi - 1)Y_{i-1} + \varepsilon_i$$

This equation is in the form of a $AR(1)$ model, and $||\varphi| \leq 1$ and ε_i is also white noise. The process will not be stationary if $||\varphi| = 1$ and then we are facing unit root. Dickey Fuller test is a method to determine the unit root of the above process. In this test $\varphi = 1$ is null assumption. In this way, the data does not have a normal probability distribution and a standard shape. This test results in the Tehran Stock Exchange Oil Group Market are as follows:

Table 2: The result of unit root tests to Tehran Stock Exchange Oil Group Index

Dickey Fuller Test		t-Statistic	Prob.	Augmented Dickey-Fuller test statistic
Tehran Stock Exchange oil group	1% level	-3.961257	0.8439	-1.456956
	5% level	-3.411381	0.8439	-1.456956
	10% level	-3.127539	0.8439	-1.456956

According to the Dickey Fuller test, it can be concluded that the null hypothesis, which indicates the existence of a unit root, is not rejected and there will be a unit root. In this test we are looking to check the model stationary. Differential series logarithm can be used to eliminate the unit root. Now we will examine autocorrelation at different levels of data. Then we will have:

We have autocorrelation at all levels, according to table 3. So it can be concluded that small jumps and big jumps follow small and big jumps, respectively which fully confirms the hypothesis of autocorrelation. Now, according to the above evidence, we will perform the ARCH test and have:

The null assumption in this test is that we do not have an arch. According to the table, this assumption is rejected and we are facing the arch phenomenon and variance heterogeneity. We will examine and estimate the parameters of two switching models in this group of data after determining the GARCH model coefficients.

After estimating the above two models parameters, we can be concluded that both models performance is suitable in the oil group of Tehran Stock Exchange. Also, the Heston switching model works better than the Markov switching model in the crude oil market. According to the above results, marginal models can be considered in both Heston switching markets. In this way, its variance also follows the GARCH model. Now, we can proceed to estimating Clayton copula model after estimating the marginal models. We have examined this copula in two categories

Table 3: The result of the autocorrelation test in Tehran Stock Exchange Oil Group Index

Autocorrelation	Autocorrelation		AC	PAC	Q-Stat	Prob
|*******	|*******	1	0.998	0.998	2849.7	0.000
|*******	| |	2	0.995	0.011	5687.3	0.000
|*******	| |	3	0.993	-0.031	8512.2	0.000
|*******	| |	4	0.990	-0.030	11324.	0.000
|*******	| |	5	0.988	0.009	14122.	0.000
|*******	| |	6	0.985	-0.026	16906.	0.000
|*******	| |	7	0.983	-0.031	19676.	0.000
|*******	| |	8	0.980	-0.023	22431.	0.000
|*******	| |	9	0.977	0.001	25170.	0.000
|*******	| |	10	0.974	-0.031	27894.	0.000
|*******	| |	11	0.971	-0.005	30601.	0.000
|*******	| |	12	0.968	0.016	33293.	0.000
|*******	| |	13	0.965	-0.002	35970.	0.000
|*******	| |	14	0.962	0.004	38630.	0.000
|*******	| |	15	0.959	-0.024	41275.	0.000
|*******	| |	16	0.956	0.001	43904.	0.000
|*******	| |	17	0.953	0.003	46517.	0.000
|*******	| |	18	0.950	-0.015	49114.	0.000
|*******	| |	19	0.946	-0.008	51695.	0.000
|*******	| |	20	0.943	-0.007	54259.	0.000
|*******	| |	21	0.940	0.001	56807.	0.000
|*******	| |	22	0.937	0.007	59339.	0.000
|*******	| |	23	0.934	-0.016	61854.	0.000

of data, the crude oil market and stock market index, as well as the Tehran Stock Exchange Oil Group Index and the oil market.

Table 4: Arch test result of Tehran Stock Exchange Oil Group Index

	F-statistic	Prob.
Arch Test	855.4326	0.0000

Table 5: Estimation of marginal model parameters

<table>
<tr><td></td><td></td><td colspan="3">Heston Switching Model</td><td colspan="3">Markov Switching Model</td></tr>
<tr><td rowspan="5">Tehran Stock Exchange oil group</td><td rowspan="3">Conditional Mean Equation</td><td>Regime 1</td><td>Regime 2</td><td>Trans. Matrix</td><td>Regime 1</td><td>Regime 2</td><td>Trans. Matrix</td></tr>
<tr><td>$k = 0.0016$
$\theta = 0.0034$
$v = .00001$</td><td>$k = 0.0016$
$\theta = 0.0017$
$v = .0000$</td><td>$\begin{bmatrix} 0.03930 & 0.6070 \\ 0.599 & 0.4001 \end{bmatrix}$</td><td>$\alpha = -0.0001$
$\beta = -0.15$
$\sigma = 2.9018$</td><td>$\alpha = 0.0003$
$\beta = -0.162$
$\sigma = 2.8928$</td><td>$\begin{bmatrix} 0.03930 & 0.6070 \\ 0.599 & 0.4001 \end{bmatrix}$</td></tr>
<tr><td colspan="3">$MAE = 0.0072$
$RMSE = 2.0647$
$R^2 = 0.099$</td><td colspan="3">$MAE = 4.1512$
$RMSE = 9.22$
$R^2 = 0.099$</td></tr>
<tr><td rowspan="2">Conditional Variance Equation</td><td colspan="6">$c = 785228.4$
$RESID(-1)^2 = 0.479041$
$GARCH\ (-1) = 0.708026$</td></tr>
<tr><td colspan="6">$R^2 = 0.998495$
$S.E.\ of\ regression = 81179.98$
$GARCH = C(1) + C(2) \times RESID(-1)^2 + C(3) \times GARCH(-1)$</td></tr>
</table>

Table 6: Clayton copula parameters estimation

Copula		Oil Group - global crude oil market		Tehran Exchange- global crude oil market	
Clayton copula		Regime 1	Regime 2	Regime 1	Regime 2
	Copula parameter	5.0602	4.8106	-0.1913	0.5749
	Max likelihood	793.3967	464.4664	10.2904	27.9405
	AIC	$1.5888e + 03$	930.9328	22.5807	57.8809
	BIC	$1.5935e + 03$	935.2094	26.9591	61.7944

Clayton parameter has been determined in each of the regimes. Akaike's coefficients (AIC) can be a criterion for choosing the best joint. The criterion for choosing the best copula is the least amount of lost information.

The Tehran Stock Exchange Oil Group market from the global oil market. Now we calculate VaR in the case where the confidence level is 95% and 5%. Volatility in these two levels is the reason for the spillover of the oil group market from the global crude oil market (Saghafi, 2018):

Table 7: Calculation of risk spillover from the global oil market to the Tehran Stock Exchange

	Negative domain	Positive domain
Risk spillover from the global crude oil market on the Tehran Stock Exchange	0.0115	0.2082
Risk spillover from the global crude oil market on the Tehran Stock Exchange Oil Group	0.0039	0.0728

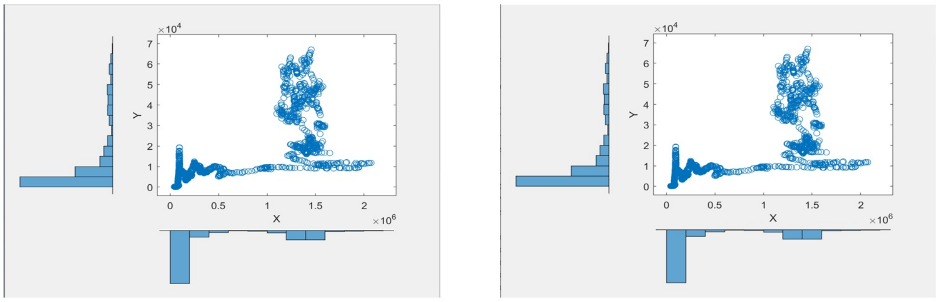

Figure 2: Clayton copula and the connection between the global crude oil market and the Tehran Stock Exchange market and the oil group of this market

As shown in Table 7, the global oil market affects both the entire stock market and specifically the oil group of this market. Increased risk spillover at the 95% confidence level compared to the 5% confidence level in the entire stock market index and in the oil group in particular is equal to 18 times and 18.6 times, respectively. This shows the tremendous impact of the crude oil market on these markets. The correlation coefficient between the oil group and the crude oil market is equal to 0.9316, which confirms the results of this research.

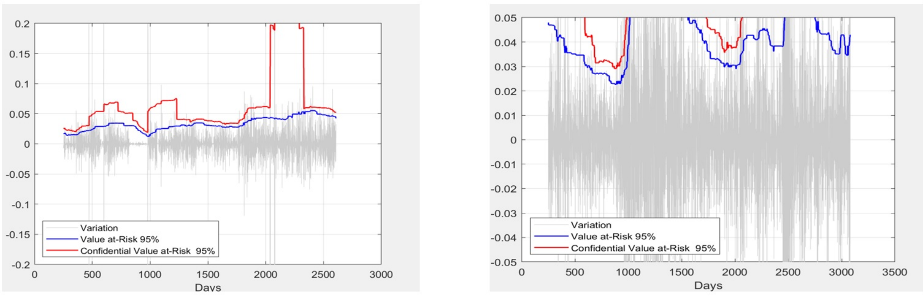

Figure 3: The spillover of the Tehran stock market from the global crude oil market

5 Conclusion

In this article, we investigated the relationship and risk spillover from the crude oil market to the Tehran stock market and specifically the oil group of this market. We used the combination of switching models as well as the copula model to investigate this relationship. First, we examined the margin models and tested Heston switching and Markov switching models in these markets. Considering the better performance of the Heston switching model, eventually we used this model as a marginal model. Then we estimated this model parameters after determining the regimes in these markets and the possibility of regime transfer. We used Clayton copula to determine the multivariate function. This copula can be a suitable model for creating a multivariable function considering that it measures rotations of 90, 180 and 270 degrees. We expect good results from combining this copula and switching models. Researchers such as Tansuchat et al. (2017), Selmi et al. (2019), Tiwari et al. (2020) and Mwamba and Mwambi (2021) had previously investigated the joint combination with Markov switching models, but Markov model limitations forced us to test the construction of the Heston switching model as a marginal model. It seems that using the Heston switching model as a marginal model and combining it with copula models such as Clayton can bring good results.

Markets proximity to each other and also the process of free circulation of information has strengthened the theory of financialization of the markets. Oil markets are no exception to this point. Paying attention to this market volatility can be important due to the close relationship between the global crude oil market and Tehran Stock Exchange Oil Group. Examining the factors affecting the price of crude oil is the most important point regarding the volatility of global oil prices. Among the factors affecting the price of crude oil are factors such as crude oil extraction process and related costs, crude oil market regulations that can be imposed by organizations such as OPEC and OPEC + or major governments as well as major crude oil exporters, oil supply and demand, and political and social factors. The war between Ukraine and Russia, for example, as a political factor, increased the price of crude oil in the world markets, on the other hand, Covid-19 and its spread throughout the world caused a decrease in crude oil demand and drop its price. According to the crude oil market volatility and affected by it, the Tehran Stock Exchange market, it is possible to reduce the investment risk to some extent by introducing oil derivatives in this market. On the other hand, strengthening the stock market and oil derivatives market as an independent market can greatly help investors in this sector. For future research, it is suggested to examine other marginal models and combine these models with copula models. It can also be interesting to examine economic policies and their impact on marginal models.

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