Journal of Mathematics and Modeling in Finance (JMMF)
Vol. 3, No. 2, Summer & Autumn 2023
Research paper

An analysis of volatility and herd behavior among investors in the SP500 stock market index, Bitcoin, and gold markets

Mohammad Qezelbash[1], **Saeid Tajdini**[2], **Farzad Jafari**[3], **Majid Lotfi Ghahroud**[4], **Mohammad Farajnezhad**[5]

[1] Department of Mangement and Accounting, Allameh Tabataba'i University, Tehran, Iran.
ghezelbash@ice.ir

[2] Faculty of Economics, University of Tehran, Tehran, Iran
saeidtajdini@ut.ac.ir

[3] Telfer School of Management, University of Ottawa, Ottawa, Canada
farzadjafari.phd@gmail.com

[4] Department of Technology and Society, The State University of New York, Incheon, Korea
majidlotfigh@gmail.com

[5] Azman Hashim International Business School, Universiti Teknologi Malaysia, Johor, Malaysia
taban1010@gmail.com

Abstract:
In recent years, cryptocurrency has attracted more attention and is a new option in the economy and the financial sector. The purpose of this study is to the volatility and "herd behavior" of the cryptocurrency, gold, and stock markets in the US. This research is aimed at investor "herd behavior" and how it correlates with the volatility of three assets: the Standard & Poor's 500 indexes, Bitcoin, and gold. Also, A new formula by applying the conditional standard deviation (risk), maximum return, minimum return, and average return to quantify the herding bias is designed in this research. In this study, the generalized autoregressive conditional heteroscedasticity model (GARCH) and the autoregressive moving average model (ARMA) were both employed. Research results show that Bitcoin is 3.3 times as volatile as the S&P 500 and 4.6 times as volatile as gold. The results of this novel equation also show that the herding bias of Bitcoin is more than 26 times higher than the global average and 10 times higher than the S&P 500. Also, it's important to consider the energy consumption and sustainability of investments when evaluating their long-term viability and risk. In some cases, investments in companies with strong sustainability practices and low carbon footprints may be seen as lower risk. Since Bitcoin relies on a network of computers to validate transactions based on proof of work and it is an energy consumption consensus mechanism, investment in Bitcoin may be seen as a higher risk.

Keywords: Herd mentality bias, Volatility, Bitcoin, S&P500, Gold.
Classifications: MSC2010 or JEL Classifications: D90, D91, E70, E71.

[4]Corresponding author
Received: 29/08/2023 Accepted: 12/12/2023
https://doi.org/10.22054/JMMF.2024.75516.1103

1 Introduction

In uncertain times, many people find safety and solace in holding cash to preserve liquid assets. Currency depreciation due to inflation is a bad investment. Bonds, shares (stocks), and even real assets or commodities are all examples of investment options. Cryptocurrencies are a novel option that has developed in the last several years. Due to the unpredictability of the cryptocurrency market, those that invest in it tend to be more adventurous. Herd behavior is a common form of investor herd mentality, which can lead to poor investing decisions. Price swings in both markets have a mutual relationship. According to research conducted by Yussef in the year 2022, herding in the cryptocurrency market increases in tandem with increases in volatility, the S&P 500, and the dollar index. Rising trade volume, the price of gold, and the economic policy uncertainty index (EPU) all make it harder for people to herd in the cryptocurrency market (Youssef, 2022). Further, primary research shows that herding is more prominent in times of crisis than in everyday life. In addition, there are actual studies that show herd mentality is only present in rising markets. These results shed light on the importance of liquidity in current markets and provide a road map for making more rational investment decisions (Kyriazis, 2020). Considering this, the volatility of the S&P 500 stock market index, Bitcoin, and gold were compared and analyzed to determine the herding tendency of investors.

Volatility refers to the price changes of an asset over time, and it is often used as a measure of risk. In terms of volatility, Bitcoin is often considered the most volatile of the three. The price of Bitcoin can fluctuate dramatically within a short period of time, making it a high-risk investment for some. The S&P 500 and gold are generally considered to be less volatile, but the S&P 500 can still experience significant price swings, particularly during times of economic uncertainty.

As mentioned, Herd behavior refers to the tendency for investors to follow the actions of the majority, rather than making independent decisions (Spyrou, 2013). In terms of herd behavior, the stock market can be particularly prone to this phenomenon, as investors may follow the actions of large institutional investors, causing prices to rise or fall rapidly. In recent years, the cryptocurrency market, including Bitcoin, has also exhibited signs of herd behavior, as retail investors have flocked to the market, driving up prices. Gold, on the other hand, is often seen as a hedge against market volatility and is less subject to herd behavior. It's important to note that these are general observations, and individual investments can still be subject to a variety of risks, regardless of their level of volatility or herd behavior. As always, it's important to conduct thorough research and consult with a financial advisor before making any investment decisions.

In terms of volatility, the S&P 500 has average historical volatility of approximately $15-20\%$. Bitcoin, on the other hand, has average historical volatility that is significantly higher, ranging from $60-80\%$. Gold is often considered to have lower volatility, with average historical volatility in the range of $10 - 15\%$. It's difficult to quantify the extent of herd behavior in financial markets, but there are metrics that attempt to measure it. For example, metrics such as the Herding Index or the EasleyO'Hara-S degrading herding measure can be used to quantify the degree of herding in a given market. However, these metrics are complex and subject to interpretation, and a single numerical value is unlikely to accurately capture the complex dynamics of herd behavior in financial markets. It's important to remember that these numbers are only rough estimates and that the level of volatility and herd behavior can vary significantly over time, depending on a variety of factors. Moreover, it's important to consider a variety of factors beyond just volatility and herd behavior when making investment decisions (Kurka, 2019).

On the other hand, Energy consumption is not directly related to the volatility or herd behavior of the S&P 500, Bitcoin, and gold. However, it is an important factor to consider when evaluating the sustainability and long-term viability of certain investments, particularly Bitcoin. Bitcoin, as a decentralized cryptocurrency, relies on a network of computers to validate transactions and maintain the integrity of the blockchain. This process, known as mining, requires a large amount of energy, as it involves solving complex mathematical problems (Proof of work). According to Talaiekhozani et al., 2021, the energy consumption of the Bitcoin network is on par with that of entire countries, making it a significant contributor to global carbon emissions. The energy consumption of the S&P 500 and gold, on the other hand, is largely indirect, as the companies in the S&P 500 and the production of gold both require energy to function. However, the energy consumption of these industries is not directly tied to the volatility or herd behavior of these investments. It's important to consider the energy consumption and sustainability of investments when evaluating their long-term viability and risk. In some cases, investments in companies with strong sustainability practices and low carbon footprints may be seen as lower risk, while investments in industries with high energy consumption and significant environmental impact may be seen as higher risk. However, this is a complex issue, and a variety of factors, beyond just energy consumption, should be considered when making investment decisions (Spaargaren et al., 2013). This research investigates the behavior of investors in the US financial markets, focusing on the S&P 500, Bitcoin, and gold. It introduces a novel formula to quantify herding bias, considering factors such as conditional standard deviation and return metrics. Applying GARCH and ARMA models, the study finds Bitcoin to be significantly more volatile than traditional assets and exhibits a substantial herding bias. The results underscore the need to consider sustainability and energy consumption, particularly in Bitcoin investments. This research provides valuable insights into the interplay of volatility and herd behavior, shedding light on the dynamics of cryptocurrency markets and their implications for investors.

1.1 Literature review

Herd behavior is broadly defined as investors' imitation of the other's behavior. Devenow and Welch (1996) highlight three reasons for herd behavior. The first reason is the payoff externality (the outcome of action increases with the number of agents performing it). For example, investors tend to trade simultaneously to benefit from increased liquidity (Admati and Pfleiderer, 1988; Dow, 2004). A second reason is the reputational concerns and issues associated with principal-agent theory (Scharfstein and Stein, 1990; Rajan, 1994; Graham, 1999). If a manager's performance is measured relative to a benchmark (i.e., the average performance of other managers or the performance of a market/industry index) then mimicking the benchmark is very attractive. By doing this, the manager sacrifices the potential for outperformance, but hedges against relative underperformance. The third explanation for rational herding is information externalities. Externalities are so strong that investors may voluntarily choose to ignore their information. In the most extreme cases, individual behavior no longer conveys information. This is because an individual's behavior is only the result of imitating the behavior of others. In this case, an information cascade occurs.

Based on traditional asset pricing models, inefficient markets investors can price financial assets correctly under rational expectations.

There are many assumptions behind the models that have been shown not to hold in reality:

1. security markets are very competitive and efficient.

2. security markets are dominated by risk-averse and rational investors. They seek to maximize satisfaction from returns on their investments.

Without market efficiency, however, some investors follow others' strategies instead of relying on their information and opinions a type of herd behavior (Huang et al., 2015). In addition, in the cryptocurrency market, almost everything is different, and digital or virtual currencies are secured by cryptography. This new asset class, known as cryptocurrencies, has emerged that has attracted investors of all kinds with the extraordinary increase in digitalization around the world. The uniqueness of this new asset class has led researchers to measure unusual trading patterns and behavioral faults withinside the crypto market (Shrotryia & Kalra, 2021). Cryptocurrencies have emerged as an alternative innovative category of an asset for investments traded by international investors in data-rich markets. Much attention has been dedicated to their pricing properties, but the academic literature on behavioral drivers has not been well-developed to date (Gurdgiev & O'Loughlin, 2020). Recent studies and examinations have made a significant help to researchers better explore the current research trends within a particular field of study known as herding behavior in cryptocurrency markets. Academic research on herd behavior in cryptocurrency markets has been analyzed in this study. Various articles were selected to advance the research about herd behavior and its appearance in crypto markets.

Herd behavior is the behavior of an individual who acts together without a central direction. This kind of behavior can happen in animals and humans in a variety of situations. In finance, herding is a tendency of individuals (or organizations) to imitate the behavior of others after interactively observing their actions (Hirshleifer & Hong Teoh, 2003). Herding happens when individuals track the behavior of others and ignore their signs and general market fundamentals (Erdenetsogt & Kallinterakis, 2016). A basic study by Scharfstein and Stein (1990) on herd behavior has targeted different periods, nations, financial markets, financial crises, and investor types. Several methods and models were used to illustrate this behavior. To better understand herding behavior, an important literature review of herding behavior presence during the first decade and the second decade was published.

According to information from the Web of Science, 65 articles that cited 1944 times were published between 1990 and 2007. In the next five years, there were 74 articles with 2913 citations. 168 articles were cited 10,155 times between 2014 and 2021. The growing attention to this topic and following publications has made it possible to conduct research in specific sub-areas. Therefore, the latest general view of herd behavior in financial markets is guaranteed. To achieve a better view of herd behavior in financial markets and especially crypto markets, a bibliometric analysis of herd behavior in financial markets is used. The result is a comprehensive and organized source of data that may be used by researchers as a reference (Bonilla et al., 2015). This enables an assessment of scientific activity, the impact of publications, and sources for guiding new research (Moreno and Rosselli, 2012).

The remarkable growth of research in this area happened and encouraged the progress of subareas of interest to researchers divided into five well-defined research groups. The first group focused on a deeper understanding of herd behavior. The second group focuses on evidence of the existence of herding behavior in multiple financial markets and the possible reasons or causes for this behavior. The third group focuses on this behavior during the financial crisis. The fourth focuses on how investor types affect herding, and the final group focuses on how herding appears and its potential impact on portfolio management.

The existence of herd behavior in the crypto market has been analyzed by Kaiser and Stöckl (2020). Statistically significant proof of it has been provided. Results are in contrast with existing empirical evidence on this subject, primarily because earlier studies suffered from sampling biases. Incorporating the concept of beta herding into the discussion adds even more robustness to the results. In addition, it proposes the concept of Bitcoin as a "transfer currency". It empirically indicates that transfer currency-centric herding measures represent the diversification of investor beliefs on the crypto market more accurately (Kaiser & Stöckl, 2020).

The existence of herd behavior in the crypto market has been examined by Bouri, Gupta, and Roubaud (2019). The latter is the result of mass relationship and imitation. The results of the static model show that there is no significant herd behavior. However, the existence of structural breaks and non-linearities in the data series indicates that the application of static models is not suitable. Therefore, rolling window analysis has been used and the results show significant herd behavior that changes over time. It has been indicated that herding behavior tends to occur as uncertainty increases by using logistic regression. Results lead to useful insights related to risk management and portfolio, market efficiency, and trading strategies (Bouri et al., 2019).

Herding and its possible causes in the cryptocurrency market have been studied by Kallinterakis, and Wang (2019). Herding seems to be significant (regardless of Bitcoin's presence and trends over time), highly asymmetric (more intense during up-markets, low volatility, high volume days), with smaller cryptocurrencies enhancing its scale. The results show that the crypto market has the potential for strong destabilization, the latter of which is particularly relevant to the authorities entrusted with its regulatory treatment (Kallinterakis & Wang, 2019).

Though the cryptocurrency market is highly unpredictable, the amount of research on herding in this market is very limited. Hence, studies can provide a clear idea of the herding nature of the cryptocurrency market. The nature of herding behavior in the cryptocurrency market has been analyzed by Mahmood Ali (2022). The first 200 crypto coin data ranked based on the market capitalization on January 1, 2020, has been used to develop an analysis. It describes the nature and strength of the herding behavior in crypto investors in different periods (using daily, weekly, and monthly frequency data) and in different states (high vs. low EPU state and high vs. low VIX state). The results show that the nature and magnitude of herding in the short run (daily frequency) are not the same as the nature and magnitude of herding in the long and medium run (monthly and weekly frequency). Also, it was shown that herding in the high EPU (Economic Policy Uncertainty) and VIX (CBOE Volatility Index) states is not the same as in low EPU and VIX states. It also shows the magnitude of the herding impact on the next day's market profits in the crypto market (Ali, 2022).

Testing the herd behavior in the crypto market herd using the CSAD method of Chang et al. (2000) has been examined by Ajaz and Kumar (2018). The daily returns of the six main cryptocurrencies and market index CCI30 for the period July 8, 2015, to January 18, 2018, have been used. The potential for herding behavior in up and down markets, and under high and low volatility is tested. Herding behavior is seen in the up-and-down movements of the market, showing excessive enthusiasm and overreaction. It is found that Market volatility has no significant effect on herd behavior, herding depends more on market activity than on market volatility (AJAZ & KUMAR, 2018).

Investigating the presence of herding behavior in the cryptocurrency market under certainty using return cross-sectional absolute deviation (CSAD) of returns, generalized autoregressive conditional heteroscedasticity (GARCH) methods, Ordinary Least Squares (OLS), and TimeVarying Markov-Switching (TV-MS) model for both sub-periods and overall sample which was determined based on the results Bai-Perron breakpoint tests is investigated by Coskun, Lau, and Kahyaoglu (2020). daily data of 14 major cryptocurrencies in terms of closing price, market capitalization, and trading volume has been used. Also, dummy variables have been used to investigate whether asymmetric behavior happened during the "up and down" market period. The overall sample results are related to each model's anti-herding behavior. However, the results of the TV-MS model show the presence of herd behavior in low volatility regimes during the third sub-period (2/28/2017-1/16/2018), an anti-herding behavior happened during the high volatility regime, and the impact of uncertainty was significant on the anti-herding behavior. Results indicate that there were no significant asymmetric behaviors during the "up and down" phase of the market (Coskun et al., 2020). The herd behavior in the cryptocurrency market during the normal, skewed, Bitcoin bubble and COVID-19 pandemic phases has been investigated by Vijay Kumar Shrotryia and Himanshi Kalra (2021). Also, the importance of Bitcoin in promoting herd behavior in the market has been examined. Results show that the crypto herd occurs during normal, bullish, and high volatility periods. Results indicated that the recent outbreak of COVID-19 subjects the crypto market to herding activity (Shrotryia & Kalra, 2021).

In another study, herding behavior in the cryptocurrency market during the coronavirus pandemic has been analyzed (Yarovaya et al., 2021). Mixed quantitative methods are used to hourly prices of the four most traded cryptocurrency markets (USD, EUR, JPY, KRW) for the period January 1, 2019, to March 13, 2020. However theoretical reasons imply the "Black

Swan" effect on Cryptocurrency herding, but results show that the coronavirus has not increased the herding behavior in the cryptocurrency market. In all the markets surveyed, herding behavior continues to be possible on the rising or falling days of the market but does not become stronger during the coronavirus pandemic. These results can help crypto investors and regulators to gain a better understanding of the crypto market and the economic impact of the coronavirus pandemic.

The effect of interaction between the behavioral factors behind investor decisions and publicly available data flows on the cryptocurrency price dynamics has been investigated by Gurdgiev and O'Loughlin (2020). Sentiment analysis is used to model the impact of public sentiment on the asset market, especially cryptocurrencies, on the valuation of crypto assets. results indicate that investor sentiment can predict the direction of cryptocurrency prices, showing the direct effect of herd and anchor biases (Gurdgiev & O'Loughlin, 2020).

In another survey developed by Rubbaniy, Tee, and Iren (2022) daily data from 382 cryptocurrencies and a quantile-on-quantile regression (QQR) framework advanced by Sim and Zhou (2015) have been used to set up a relationship between investors' sentiment and herd behavior and provide support for the mood-as-information hypothesis withinside the crypto market. the influence of investors' sentiment on herd behavior is asymmetric, regimespecific with a (weaker)higher (anti)herding tendency towards sad(happy) quantiles of investors' sentiment, shown by the results of the quantile-on-quantile regression analysis (Rubbaniy et al., 2022); (Sim & Zhou, 2015).

Literature reviews show that there is no agreement on the cause of herd behavior in financial markets. Therefore, new questions and prospects arise that explain the continuation of research on herd behavior. In this study, three different investment opportunities are studied. As mentioned, it is possible to compare different investment opportunities, such as Bitcoin, the S&P 500, and gold, and their investors. Each of these assets has its own unique characteristics, risk-return profile, and investor base, which can be compared and evaluated. The S&P 500 is an index of 500 large publicly traded companies in the United States. It is considered a more traditional investment opportunity and is widely used as a benchmark for the overall performance of the US stock market. Investors in the S&P 500 tend to be more risk-averse and are often looking for long-term growth and stability (Kamalov et al. 2020). Farajnezhad et al (2020), keynesian believes that a nominal increase in money stock instead of a certain price level upsurge the real money supply. Therefore, the equilibrium interest rate is reduced, and investment and production will be increased. In addition, it is reasonable to infer that a nation boasting a robust economy tends to exhibit a sturdy currency, and conversely, a nation with a formidable currency often signals economic strength. This connection is founded on the intricate interplay between a country's economic vitality and the value of its currency. Economic robustness can bolster a currency's value, reflecting the nation's overall financial health. In contrast, a resilient currency may indicate a thriving economy (Tajdini et al., 2022).

Gold is a precious metal that has been used as a store of value for thousands of years. It is considered a safe-haven investment, as it is less affected by economic and political events than other assets. Investors in gold tend to be more conservative and are often looking for a hedge against inflation and market volatility (Bogdan et al., 2019). Bitcoin is a decentralized digital currency that operates independently of a central bank or government. It is considered a high-risk, high-reward investment due to its volatility and lack of regulation. Investors in Bitcoin tend to be younger and more tech-savvy and are often attracted to its potential for high returns and its status as a store of value. Also, Bitcoin is a form of virtual currency that emerged in the wake of the 2008 financial crisis. Furthermore, its widespread interest can be attributed to the fact that it is distributed through a blockchain. Bitcoin's extreme swings in value scare off most investors, who instead go to safer bets like stock indices. It's important to note that the investment landscape is constantly changing, and that past performance is not a guarantee of future returns. In addition, each investment opportunity has its own risks and benefits, and it's crucial to carefully consider investment goals and risk tolerance before making any investment decisions (Ainia et al., 2019).

Risk-taker investors prefer high-risk, high-reward investments, while risk-averse investors seek alternatives. Moreover, herd behavior is seen among traders in the stock market and the cryptocurrency market. In this study, the S&P 500 stock index, Bitcoin, and gold as symbols for these three markets were studied by employing conditional risk and related models such as ARMA, and GARCH. The autoregressive moving average model (ARMA) and the generalized autoregressive conditional heteroskedasticity model (GARCH) must be known before the conditional risk can be comprehended.

2 Research Methodology

2.1 Autoregressive Moving Average (ARMA)

The Autoregressive Moving Average (ARMA) model is a type of statistical time series model used in econometrics and finance to model and analyze univariate time series data. It is a combination of two statistical models: the Autoregressive (AR) model and the Moving Average (MA) model. The AR model considers past values of the time series to predict future values, assuming that the future value of a time series is a linear combination of past values. The MA model considers past residuals or errors in the time series to predict future values, assuming that the future value of a time series is a linear combination of past residuals or errors. The ARMA model combines these two models to consider both the past values and past residuals of the time series to make predictions about future values. The parameters of the ARMA model are estimated using maximum likelihood estimation or other statistical methods, and the resulting model can be used for forecasting, hypothesis testing, and other analysis tasks. In summary, the ARMA model is a flexible and widely used statistical tool for analyzing and predicting time series data, especially in the fields of economics and finance (Prado et al., 2020).

2.2 Generalized Auto Regressive Conditional Heteroskedasticity (GARCH)

It is a GARCH model, a variant of the ARMA model if the error variance is considered to follow an autoregressive conditional heteroskedasticity (ARCH) process (Bollerslev, 1986), which can be presented in Eq.(1):

$$a_t^2 = \alpha_0 + \sum_{i=1}^{i=u} \alpha_i a_{t-i}^2 + \sum_{j=1}^{j=v} \beta_j \sigma_{t-j}^2 \tag{1}$$

Integrated Generalized Autoregressive Conditional heteroskedasticity (IGARCH)

As shown in Eq. (2), Integrated (GARCH) is a limited version of the GARCH model, in which the continual parameters add up to one, and give the GARCH process a unit root.

$$\sum_{i=1}^{i=p} \beta_j + \sum_{i=1}^{i=q} \alpha_i = 1 \tag{2}$$

Glosten-Jagannathan-Runkle GARCH (GJR-GARCH)

In 1993, Glosten, Jagannathan, and Runkle first presented the GJR-GARCH model (Glosten et al., 1993). Specifically, the formula is Eq. (3):

$$\sigma_t^2 = \omega + \sum_{i=1}^{i=u} \alpha_i \sigma_{t-1}^2 + \sum_{j=1}^{j=v} \beta_j \sigma_{t-j}^2 + \gamma_i I_{t-i} \sigma_{t-i}^2 \tag{3}$$

If $\alpha_(t-i)$ is positive, the indicator function returns zero, and if it is negative, it returns one, where α, β, and γ are constant. This dummy variable thereby separates the positive and negative shocks, and the asymmetric effects are captured by γ (Cerovi Smolovi et al., 2017).

In this study, conditional risk was first measured using conditional risk formulas, and then the herd effect was measured using the innovative formula of this article.

Exponential General Autoregressive Conditional Heteroskedastic (EGARCH)

One variant of the GARCH model is the so-called exponential version (EGARCH). For the GARCH (p, q) models generated conditional variances to remain positive, the process parameters must be constrained to have nonnegative values (Bollerslev, 1986). However, Nelson and Cao demonstrate that these constraints should not be applied in estimation, and proposed the EGARCH model (Nelson Cao, 1992). The asymmetric impact of shocks on the conditional variance is verified if $\gamma \lneq 0$ is significant. Additionally, the model allows for the testing of leverage effects by assuming $\gamma < 0$.

$$\epsilon_t = \sigma_t z_t, \quad ln\sigma_t^2 = \omega + \sum_{i=1}^{i=p} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{j=q} \beta_j ln\sigma_{t-j}^2$$

The following specification also has been used in the financial literature (Dhamija Bhalla, 2010).

$$\epsilon_t = \sigma_t z_t, \ ln\sigma_t^2 = \omega + \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{j=q} \lambda_j ln(\sigma_{t-j}^2) + \sum_{i=1}^{i=p} \gamma_i \frac{\epsilon_{t-i}}{\sigma_{t-i}} - \sqrt{\frac{2}{\pi}}$$

Power Exponential General Autoregressive Conditional Heteroskedastic (PGARCH)

The basic GARCH model can also be extended to allow for leverage effects. This is made possible by treating the basic GARCH model as a special case of the power GARCH (PGARCH) model. This model by Taylor (1986) uses conditional standard deviation as a measure of volatility instead of conditional variance. PGARCH was generalized by Ding, Granger, and Engle (1993) using the PGARCH model as follows:

$$\sigma_t^\delta = \omega + \sum_{i=1}^{i=q} \alpha_i(|\mu_{t-i}| - \gamma\mu_{t-i})^\delta + \sum_{j=1}^{j=p} \beta_j \sigma_{t-j}^\delta \tag{4}$$

In this equation, γ denotes asymmetry. In the symmetric model, γ is zero for all values and the coefficient.

Zhang (2006) used to move average models, exponential moving averages, and various GARCH models to forecast the indices of China's Shanghai and Shenzhen Stock Exchanges. He concluded that there is no single model that performs best in all conditions. For example, asymmetric models such as his GJRGARCH and his EGARCH for the Shenzhen index performed better than other his GARCH models, but the asymmetric model was not suitable for conditional risk prediction of the Shanghai index.

Abdelaal (2011) studied the Egyptian stock market from 1998 to 2009. They realized that the EGARCH model predicts volatility better than other models. Liu and Hung (2009) tested the EGARCH, GARCH, ARCH, and GJR-GARCH models on the SP index and found that asymmetric models such as EGARCH and GJR-GARCH had a better performance in accurate prediction of volatility. Dritsaki (2017) investigated the daily returns of stocks on the Stockholm Stock Exchange and found that asymmetric GARCH models such as EGARCH with a Student distribution along with the ARIMA $(0, 0, 1)$ model provide accurate predictions of the GARCH models. Andreea-Cristina and Stelian (2017) investigated the volatility of the euro exchange rate against the Romanian currency and found that the asymmetric EGARCH and PGARCH models performed better in estimating risk and return than the symmetric GARCH model.

To quantify herd bias, we use the innovative equation including four factors: conditional risk, average return, maximum return, and minimum return as follows.

$$HerdBias = (|\max - Average| + |\min - Average|) \times \sigma_t$$

Where MAX is the Maximum return, MIN is the Minimum return, Average is the average return, and σ_t is the mean optimal conditional standard deviation.

In another word, in this research, we formulated the innovative coefficient including the sum of the absolute value of the difference between the maximum return and the average return and the absolute value of the difference between the minimum return and the average return as a conditional risk coefficient to measure herd bias. As it is clear in the inventive formula of conditional risk with index t and a maximum, a minimum and an average of the entire period, we have no need for index t.

2.3 Conditional risk

The ARMA-GARCH model is frequently employed to simulate the conditional mean and conditional variance dynamics of hazardous asset returns. Results from the real world indicate that these models are slowing down innovation and have positive extremum indices. Extreme value theory is used to figure out the top and bottom quantiles of the residuals. The process of estimating GARCH parameters has been around for quite some time. Consistency and asymptotic normality were observed across a range of scenarios (Hall & Yao, 2003; Jensen & Rahbek, 2004). On the other hand, there is a dearth of literature that addresses efficiency concerns when trying to estimate semiparametric GARCH models. Engel and Gonzalez-Rivera made the first attempt at this, whit some success in achieving efficiency through Monte Carlo simulation (Engle & Gonzalez-Rivera, 1991). Theoretical works by Linton, (1993) and Drost & Klaassen, (1997) explain the impossibility of perfectly adaptable estimators and demonstrate effective estimators through reparameterization. Adaptive estimation in nonstationary ARMA-GARCH models is further investigated by Ling & McAleer, (2003). Some studies have shown that conditional risk in GARCH models is better at predicting risk than unconditional risk (Liu & Hung, 2010; AbdElaal, 2011; Intaz et al., 2016; PETRIC & Stancu, 2017; Dritsaki, 2017; Guo, 2017; Guo, 2017a; Coffie et al., 2017; Tajdini et al., 2019; Mehrara & Tajdini, 2020; Tajdini et al., 2020). Also, Della Corte et al., (2009), Daniel et al., (2014), and Della Corte et al., (2021) predicted currency excess returns using construct mean-variance optimal portfolios and conditional risk factors.

3 Finding

To measure the risk, we measured the returns of Bitcoin and S&P500 and gold price with the GARCH family model which are shown in tables 1 and 2, and finally, the EGARCH model has been optimized after calculating the squared forecast errors. But due to the positive gamma coefficient, the EGARCH model cannot be considered for the global gold price index.

Table 1: EGARCH results

Title	α	β	Δ
S&P500	0.33*	0.94*	-0.14*
P-value	0.000	0.000	0.000
Bitcoin	0.24*	0.94*	-0.04*
P-value	0.000	0.000	0.000
GOLD	0.09	0.97	0.064
P-value	0.000	0.000	0.000

Anywhere coefficients α, β, and γ are significant. The symbol * denotes significance at the 5% levels.

Table 2: PGARCH results

	α	β	γ	Δ
S&P500	0.17*	0.81*	0.71*	0.54*
P-value	0.000	0.000	0.000	0.000
Bitcoin	0.12*	0.86*	0.19*	0.84*
P-value	0.000	0.000	0.000	0.000
GOLD	0.04	0.94	-0.59	1.7
P-value	0.000	0.000	0.000	0.000

Anywhere coefficients α, β, γ, and δ are significant. The symbol $*$ denotes significance at the 5% levels.

Table 3: Herd Bias in S&P500 Index and Bitcoin

Title	Average of Return	Optimal conditional S. D	MAX of Return	MIN of Return	Herd Bias
S&P500	0.0006	0.012	0.0897	-0.1276	0.0026
Bitcoin	0.0019	0.04	0.2251	-0.4647	0.026
GOLD	0.00025	0.0086	0.05	-0.061	0.001

In this study, data are compiled from data series of gold, S&P500, and Bitcoin markets. We collect stock-based data, e.g., closing price, maximum price, minimum price, maximum return, minimum return, average return, and mean optimal conditional standard deviation from market series during the sample period from 2015 to 2021.

Table 3 presents the descriptive analysis of the financial data of the gold, S&P500, and bitcoin markets. Column (5) reports the herding bias of $0.0026, 0.026$, and 0.001 with the optimal conditional standard deviation of $0.012, 0.04$, and 0.0086 , the minimum return of $0.1276, -0.4647$ and -0.061 , a maximum return of $0.0897, 0.2251$ and 0.05 , an average return of $0.0006, 0.0019$ and 0.00025 for the S&p500, Bitcoin market and gold, respectively.

Conclusions

In behavioral finance, herd mentality bias refers to investors' tendency to follow and copy what other investors are doing. They are largely influenced by emotion and instinct, rather than by their independent analysis. Numerous studies have been conducted on herding behavior, but the dispossession of a risk-based quantitative model for measuring herding bias has been observed. Most studies focus on stock characteristics to explain the herding behavior of individual or institutional investors. By introducing a new innovative formula that allows us to assess the herding behavior of a given investor over time, we can measure herd bias in financial markets such as the S&P500 and Bitcoin, and gold markets. In this study, we first examined conditional risk and then invented a new model to quantify herd behavior in three markets: low risks such as gold, medium risks such as the stock market, and high risks such as cryptocurrencies. In this study, we sought to invent a model to quantify herd behavior. Our results showed that the volatility of Bitcoin was 3.3 times that of the S&P500 and 7.6 times that of the gold, and based on our innovative equation, the herding bias of Bitcoin was 10 times that of the S&P500 and 26 times that of the gold market. Also, it's important to consider the energy consumption and sustainability of investments when evaluating their long-term viability and risk. As mentioned, investments in companies with strong sustainability practices and low carbon footprints may be seen as lower risk. Since Bitcoin relies on a network of computers to validate transactions based on proof of work consensus mechanism and it is an energy consumption process, investment in Bitcoin may be seen as a higher risk.

In summary, more empirical studies are needed in emerging markets where evidence suggests greater herd bias is likely to be seen. Information cascades and reputational herding are more likely to occur in these markets. The environment in these markets is relatively uncertain due to weak reporting requirements, low accounting standards, lax regulatory enforcement, and costly information gathering. Given the critical role of finance in economic sustainability, this study adds invaluable information to policymakers, and in particular, the findings will help potential investors in the stock, gold, and cryptocurrency markets. This study advances our understanding of the herding biases associated with investor investment decisions, which will enable corporate stakeholders, financial analysts, exchanges regulators, and crypto market regulators to devise their strategic and regulatory policies accordingly.

In conclusion, our study not only unveils the dynamics of herd mentality bias in financial markets but also introduces an innovative model for quantifying this bias. By examining conditional risk and applying our novel formula, we discerned distinct herd behaviors in lowrisk (gold), medium-risk (S&P500), and high-risk (Bitcoin) markets. The substantial difference in volatility among these assets highlights the varying degrees of risk investors face. Moreover, our findings underscore the importance of considering environmental sustainability in investment decisions, emphasizing the potential risks associated with Bitcoin's energy-intensive proof-of-work mechanism. As the financial landscape continues to evolve, our model provides a valuable tool for investors and researchers to navigate and understand the intricate interplay of herd behavior, risk, and sustainability in diverse markets.

Bibliography

[1] M. A. AbdElaal, *Modeling and forecasting time varying stock return volatility in the Egyptian stock market*, International Research Journal of Finance and Economics, 78.

[2] N. S. N. Ainia and L. Lutfi, *The influence of risk perception, risk tolerance, overconfidence, and loss aversion towards investment decision making*, Journal of Economics, Business, & Accountancy Ventura, 21(3), 401-413.

[3] T. Bollerslev, *Generalized autoregressive conditional heteroskedasticity*, Journal of Econometrics, 31(3), 307-327.

[4] S. Bogdan, S. Baresa, and S. Curcic, *Diversification Benefits of Gold and Other Precious Metals in an Investment Portfolio*, ICAMSS19, 21.

[5] J. Cerovi Smolovi, M. Lipovina-Boovi, and S. Vujoevi, *GARCH models in value at risk estimation: empirical evidence from the Montenegrin stock exchange*, Economic Research-Ekonomska Istraivanja, 30(1), 477-498.

[6] W. Coffie, G. Tackie, I. Bedi, and F. A. Otchere, *Alternative Models for the Conditional Heteroscedasticity and the Predictive Accuracy of Variance Models-Empirical Evidence from East and North Africa Stock Markets*, Journal of Accounting and Finance, 17(2), 100-116.

[7] K. Daniel, R. J. Hodrick, and Z. Lu, *The carry trade: Risks and drawdowns*, National Bureau of Economic Research.

[8] P. Della Corte, A. Jeanneret, and E. Patelli, *A credit-based theory of the currency risk premium*, Available at SSRN 3413785.

[9] P. Della Corte, L. Sarno, and I. Tsiakas, *An economic evaluation of empirical exchange rate models*, The Review of Financial Studies, 22(9), 3491-3530.

[10] A. K. Dhamija and V. K. Bhalla, *Financial time series forecasting: comparison of neural networks and ARCH models*, International Research Journal of Finance and Economics, 49, 185-202.

[11] C. Dritsaki,*An empirical evaluation in GARCH volatility modeling: Evidence from the Stockholm stock exchange*, Journal of Mathematical Finance, 7(2), 366-390.

[12] F. C. Drost and C. A. J. Klaassen, *Efficient estimation in semiparametric GARCH models*, Journal of Econometrics, 81(1), 193-221.

[13] R. F. Engle and G. Gonzalez-Rivera, *Semiparametric ARCH models*, Journal of Business & Economic Statistics, 9(4), 345-359.

[14] M. Farajnezhad, S. Ramakrishnan, and M. Shehni Karam Zadeh, *Analyses the Effect of Monetary Policy Transmission on the Inequality in OECD Countries*, Journal of Environmental Treatment Techniques, Volume 8, Issue 2, Pages: 589-596.

[15] L. R. Glosten, R. Jagannathan, and D. Runkle, *On the relationship between GARCH and symmetric stable process: Finding the source of fat tails in data*, Journal of Finance, 48(5), 1779-1802.

[16] Z.-Y. Guo, *GARCH models with fat-tailed distributions and the Hong Kong stock market returns.*

[17] Z.-Y. Guo, *Models with short-term variations and long-term dynamics in risk management of commodity derivatives*, ZBW-Leibniz Information Centre for Economics.

[18] P. Hall and Q. Yao, *Inference in ARCH and GARCH models with heavy-tailed errors*, Econometrica, 71(1), 285-317.

[19] A. Intaz, D. Subhrabaran, and R. Niranjan, *Stock market volatility, firm size, and returns: A study of automobile sector of National Stock Exchange in India*, International Journal of Innovative Research & Development, 5(4), 272-281.

[20] F. Kamalov, L. Smail, and I. Gurrib, *Forecasting with deep learning: S&P 500 index*, In 2020 13th International Symposium on Computational Intelligence and Design (ISCID), 422-425.

[21] N. A. Kyriazis, *Herding behavior in digital currency markets: An integrated survey and empirical estimation*, Heliyon, 6(8), e04752.

[22] J. Kurka, *Do cryptocurrencies and traditional asset classes influence each other?*, Finance Research Letters, 31, 38-46.

[23] S. Ling and M. McAleer, *Asymptotic theory for a vector ARMA-GARCH model*, Econometric Theory, 19(2), 280-310.

[24] O. Linton, *Adaptive estimation in ARCH models*, Econometric Theory, 9(4), 539-569.

[25] H.-C. LIU AND J.-C. HUNG, *Forecasting S&P-100 stock index volatility: The role of volatility asymmetry and distributional assumption in GARCH models*, Expert Systems with Applications, 37(7), 4928-4934.

[26] M. MEHRARA AND S. TAJDINI, *Comparison of profitability of speculation in the foreign exchange market and investment in the Tehran Stock Exchange during Iran's currency crisis using conditional Sharpe ratio*, Advances in Mathematical Finance and Applications, 5(3), 271-284.

[27] D. B. NELSON AND C. Q. CAO, *Inequality constraints in the univariate GARCH model*, Journal of Business & Economic Statistics, 10(2), 229-235.

[28] A.-C. PETRIC AND S. STANCU, *Empirical Results of Modeling EUR/RON Exchange Rate using ARCH, GARCH, EGARCH, TARCH, and PARCH models*, Romanian Statistical Review, 1.

[29] F. PRADO, M. C. MINUTOLO, AND W. KRISTJANPOLLER, *Forecasting based on an ensemble autoregressive moving average-adaptive neuro-fuzzy inference system-neural network-genetic algorithm framework*, Energy, 197, 117159.

[30] G. SPAARGAREN AND A. P. MOL, *Carbon flows, carbon markets, and low-carbon lifestyles: reflecting on the role of markets in climate governance*, Environmental Politics, 22(1), 174-193.

[31] S. SPYROU, *Herding in financial markets: a review of the literature*, Review of Behavioral Finance, 5(2), 175-194.

[32] S. TAJDINI, M. MEHRARA, AND R. TEHRANI, *Double-sided balanced conditional Sharpe ratio*,Cogent Economics & Finance, 7(1), 1630931.

[33] S. TAJDINI, M. MEHRARA, AND R. TEHRANI, *Hybrid Balanced Justified Treynor ratio*,Managerial Finance.

[34] S. TAJDINI, A. HAMOONI, J. MAGHSOUDI, F. JAFARI, AND M. LOTFI GHAHROUD, *Trade War and the Balanced Trade-Monetary Theory*,Journal of Mathematics and Modeling in Finance, 1(2), 93-110.

[35] A. TALAIEKHOZANI, M. LOTFI GHAHROUD, AND S. REZANIA, *Estimation of carbon monoxide, sulfur oxides, nitrogen oxides, volatile organic compounds, and particulate matters emission due to cryptocurrency miners activity in Iran*,Earth, 2(3), 667-673.

[36] M. YOUSSEF, *What drives herding behavior in the cryptocurrency market?*,Journal of Behavioral Finance, 23(2), 230-239.

How to Cite: Mohammad Qezelbash[1], Saeid Tajdini[2], Farzad Jafari[3], Majid Lotfi Ghahroud[4], Mohammad Farajnezhad[5], *An analysis of volatility and herd behavior among investors in the SP500 stock market index, Bitcoin, and gold markets*, Journal of Mathematics and Modeling in Finance (JMMF), Vol. 3, No. 2, Pages:77–92, (2023).