The Effect of Volatility Temporal Changes on the Predictability and Return of Optimal Portfolio Using the DMA Model

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Abstract:
According to most financial experts, it is not possible to study the predictability of stock prices without considering the risks affecting stock returns. On the other hand, identifying risks requires determining the share of risk in the total risk and the probability of risk occurrence in different regimes. Accordingly, different DMA models with full dynamics compared to TVP-BMA, BMA and TVP models have been used in the present study to provide this predictability. Findings showed that the DMA model is more efficient than other research models based on MAFE and MSFE indices. The present research was conducted in the period of 1-2003 to 12-2013 (including 144 periods) and was implemented in MATLAB 2014 software space. As the research results show, the bank interest rate coefficient in 45 periods, the first lag rate of the bank interest rate in 37 periods, the inflation rate coefficient in 17 periods, first lag coefficient of inflation rate in 26 periods, oil price coefficient in 78 periods, first lag rate of oil price in 85 periods, exchange rate coefficient in 64 periods and first lag rate of the exchange rate in 35 periods have a significant effect on stock returns. The final conclusion shows that the stock variables of oil price and the exchange rate had the highest impact on stock returns during the studied period.

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Introduction

The high sensitivity of the forecasting models of expected returns in different markets and conditions and their instability are among the problems of investors in using these models. Despite the possibility of finding evidence for the predictability of expected return models, the weakness of such evidences made investors unable to use them in practice based on the conducted research. There are several reasons for the poor performance of the out-of-sample standard approach. First, the very important characteristics of stock returns are not considered in the regression model. Particularly, since the return volatility changes over time, the constant volatility assumption strongly contrasts with the observed data. Failure to pay attention to this variability leads to optimal portfolios with a poor performance that varies over time only based on expected returns. Also, it is assumed in linear regression pattern that the relationship between x_t and $r_{(t+1)}$ is unchanged over time. In theory, specific asset pricing patterns like Menzly, Santos, and Veronesi (2004) or Santos and Veronesi (2006), show that the relationship between stock premium and xt changes over time. Paye and Timmerman (2006), Lettau and Van Nieuwerburgh (2008), Henkel, Martin and Nardari (2011), and Dangel and Halling (2012) found empirical evidence for changing the relationship between return on investment and common predictors over time. Overall, it may not be surprising that the out-of-sample standard approach works poorly. Therefore, the present paper aims to develop these features and out-of-sample performance re-evaluation. The present study is important because it seeks out the cause and makes the pricing models more predictable considering more realistic assumptions; it investigates the investor problem in developing an optimal portfolio by gaining knowledge about the investment portfolio over time. Considering a set of factors, in our opinion, leads to a significant improvement in out-of-sample performance. All important first-level features such as predictable expected returns, time-varying volatility and Parameter uncertainty must be taken into account for inaccurate modeling. Therefore, there is no "single final solution" to gain out-of-sample benefit.

Literature Review

The trade-off between risk and expected return is one of the principles of financial theory. By changing risk factors over time, the expected return can change and these changes of the shareholders expected return over time cause the prices to change out of a random walk. Therefore, according to many financial experts, it is not possible to study the predictability of stock prices without considering the risks ahead (Pesran and Timmerman, 1995). The relationship between risk and expected return underlies Markowitz's (1952) modern portfolio theory. Sharp (1964), Linter (1965), and Musin (1966) also developed the Capital Asset Pricing Model (CAPM), which is based on the hypotheses and findings of modern investment theory and Harry Markowitz's portfolio theory. As a sign of change in financial theories, we can refer to the development of the capital asset pricing model to select the appropriate model for evaluating performance in the investment portfolio. This model was first used by Black and Schulz in 1973 that is known as Black-Schulz model. Their research is the basis of many researches conducted in this field. They considered fixed the stock returns volatility used for buying options and used the concept of unconditional variance for calculations. The basis of these theories is that the risk is constant in a period, while the volatility is constantly changing. Accordingly, Engel (2003) argues that a Theory of dynamic volatilities is needed to correctly explain the relationship between risk and return in pricing models. There are various views on how to create these dynamic turbulences, which can be presented from Fisher's theory view. Fisher's basic theory is one of the important theories to create a theoretical framework for the relationship between the stock price index and macro variables. According to the Fisher's basic equation states, the real interest rate is obtained by the difference between the nominal interest rate and the inflation rate such that:

$$R_t{}^T = R_t{}^n - INF_i \tag{1}$$

Where is the real interest rate, $R_t{}^n$ is the nominal interest rate and INF_i is the inflation rate. Fisher also provides a relationship for stock returns, such that:

$$RS_t{}^T = RS_t{}^n - INF_i \tag{2}$$

Where RS_t^r is the real stock return and $RS_t{}^n$ is the nominal stock return. Nominal return is also equal to the change rate in stock prices, so that $RS_t{}^n = dL_nPS_t$ and PS_t is the stock price. Given this equation, Fisher introduces the following econometric model and believes that the inflation rate affects stock returns.

$$RS_t^T = y_0 + y_1 INF_i + U_t \tag{3}$$

Fama (1981) stated that some macro monetary variables including liquidity and interest rates have been ignored in the Fisher equation. Fama used the money market equilibrium to prove his claim considering the relationship between the money market and the stock market. The money market balance is as follows:

$$\frac{M_t}{P_t} = M(y_1, R_1) \tag{4}$$

Where M_t is liquidity in the economy (notes and coins in the hands and sight and long-term deposits), P_t is the general level of prices, Y_t is the national income and R_t is the interest rate. So, Fama presents the money demand as follows:

$$Ln(\frac{M_t}{P_t}) = aLnY_t - bLnR_t \qquad a_1, a_2 > 0 \tag{5}$$

$$LnP_t = -a_1 LnY_t + a_2 LnR_t + LnM_t \tag{6}$$

That this relation is obtained by differentiating the above relation:

$$dLnP_t = -a_1 dLnY_t + a_2 dR_t + dLnM_t \tag{7}$$

$$INF_t = -a_1 dLnY_t + a_2 dLnM_t + U_t \tag{8}$$

By substituting this relation in Equation (3) and rewriting it, we have:

$$RS_t^T = \beta_0 + \beta_1 dLnY_t + \beta_2 dR_t + \beta_3 dLnM_t + U_t \tag{9}$$

so that $\beta_0 = y_0, \beta_1 = -y_1 a_1, \beta_2 = y_1 a_2, \beta_3 = y_1$
Using the relationship between nominal return and real stock returns, we

have:
$(RS_t^n = RS_t^r + INF_t)$
We write the above equation as follows:

$$RS_t^T = \beta_0 + \beta_1 dLnY_t + \beta_2 dR_t + \beta_3 dLnM_t + \beta_4 dLnIMF + U_T \quad (10)$$

Finally, this relation is expressed as follows for stock price:

$$LnRS_t^T = \beta_0 + \beta_1 LnY_t + \beta_2 R_t + \beta_3 LnM_t + \beta_4 P_t + U_t \quad (11)$$

The impact of macro factors on the return of the stock exchange will be studied in this section. The results of various researches in this regard can be divided into two general categories: one group agrees that macro factors affect stock returns, and the other group, disagrees. Table 1 provides the research of these two groups:

Table 1: Summary of research results

Research	
Proponents	**Opponents**
Daizly et al. (2014); Koro (2014); Hildhesy et al. (2013); Chetzi Antonio et al. (2013); Chang and Chena (2012); Chinsera (2011); Ali U (2011), Natalia Sizowa et al. (2011); Len Wang et al. 2011 Rosef (2010), Badukh and Richardson (2009), Torbek (2008), Goltkin (2008), Capriel and Jung (2007), Smith (2007), Madsen (2002), Christofergan et al. (2006), Feldstein (2006), Hump and Millian (2004), Koro (2003), Claudilin et al. (2003), Gregorio et al. (2003), Bernanki and Katner (2001), Mayasmai and Ke. (2000), Morindel and Abdollah (1997), Zhang (1995), Makhraji and Naka (1995), Hama (1988), Rol and Ross (1986), Selnik (1983), Fama and Short (1977)	Chen et al. (2007) Doli and Kerney (2005), Apostolos Serlitis (1993), Poon andTaylor (1991)

Table 1 shows that volatility in macroeconomic variables affect stock returns in most studies. As a result, it is necessary for investors to pay considerable attention to these indicators and the extent of their effectiveness when forming an optimal portfolio.

Research model and method

In order to explain the Dynamic Mechanical Analysis (DMA) method, assume that K subset models of the variables z_t exist as estimators and $z^{(k)}$ with represent$k = 1, 2, ..., K$ of the above subset model. Accordingly, the space-state model is described as follows assuming the existence of K subset model at each point in time:

$$y_t = z_t^{(k)}\Theta_t^{(k)} + \varepsilon_t^{(k)} \tag{12}$$

$$\Theta_{t+1}^{(k)} = \Theta_t^{(k)} + \mu_t^{(k)} \tag{13}$$

In these equations $\varepsilon_t^{(k)} \sim N(0, H_t^{(k)})$ and $\mu_t^{(k)} \sim (0, Q_t^{(k)})$ with $\vartheta_t = (Q_t^{(1)}, ..., Q_t^{(k)})$ show that each model of K subset model has a better performance in one of the time periods. The dynamic mean model method makes it possible to estimate a different model at any point in time (Koop and Korobilis, 2011). About the difference between dynamic DMA models in predicting a variable at time t based on $t-1$ information, it can be said that with$L_t \in \{1, 2, ..., K\}$, the DMA model includes the calculation of $p_r = (L_t = k|y^{t-1})$ and averaging model predictions are based on this probability; however, DMS includes selecting a model with the highest probability $p_r = (L_t = k|y^{t-1})$ and predicting the model with maximum probability

. The DMA method developed by Raftari et al. (2010) includes two parameters α and β, which are called forgetting factors. For fixed values H_t and Q_t, standard filtering results can be used to perform recursive estimation or prediction. Kalman filtering begins based the name of the forgetting factors is on the following relation:

$$\theta_{t-1}|y^{t-1} \sim N\left(\widehat{\theta}_{t-1}, \sum_{t-1|t-1}\right) \tag{14}$$

In relation (14), the calculation of $\widehat{theta}_{t-1}$ and $\sum_{t-1|t-1}$ has a standard method that is a function of H_t and Q_t. The Kalman filtering process continues with the following relation:

$$\theta_t|y^{t-1} \sim N\left(\widehat{\theta}_{t-1}, \sum_{t|t-1}\right) \tag{15}$$

As $\sum_{t-1|t-1} = \sum_{t-1|t-1} + Q_t$, Raftri et al. (2007) replace Equation $\sum_{t-1|t-1} = \frac{1}{\beta}\sum_{t-1|t-1}$ with Equation $\sum_{t-1|t-1} = \sum_{t-1|t-1} + Q_t$ in order

to simplification; and comma with $0 < \beta < 1, Q_t = (1 - \beta^{-1}) \sum_{t-1|t-1}$.
The forgetting factors approach in Econometrics was used in 1980 by Doan et al after the introduction of the TVP-sVAR method and because of its limited computational power in estimation. The name of the forgetting factors is selected based on the notion that the observations of the previous period have a weight of β^j. The value of β close to one indicates the more gradual changes in the coefficients, which is set at0.99 by Raftri et al. (2010). The above value for the quarterly statistical data of the last 5 years show that the weight of the observations of the last five years accounts for 80% of the weight of the last observations of the period. In case that β is equal to 95%, the above value indicates that the observations of the last five years account for 35% of the weight of the last observations of a given period. Accordingly, selecting β is very important, which is usually considered between 90 and 99%.
DMA theory has many potential advantages in predicting model-independent variables over other forecasting methods, including the possibility of changing model estimates over time. The most important advantage of this method is that some subsets of estimators provide cost-effective models with low input variables that avoid over-fitting problems in estimation if the DMA model considers more weight for them. The probabilities of DMA and DMS methods are mostly related to cost-effective models with only a few estimates. If $size_{k,t}$ is the number of independent variable estimators of the model at time t for model k (excluding intervals and fixed sentences), the following equation is used to calculate the expected average number of estimators used in the DMA model at time t:

$$E(size_t) = \sum_{k=1}^{K} \Pi_{t|t-1,k} size_{k,t} \tag{16}$$

Comparing the performance of the methods used in prediction is another objective of this research. The two standard indices of (MSFE) and (MAFE) are used in this study, which are as follows.

$$MSFE = \frac{\sum_{\tau=\tau_0}^{T} \left[y_\tau - E(y_\tau | Data_{\tau-h}) \right]^2}{T - \tau_0 + 1} \tag{17}$$

$$MSFE = \frac{\sum_{\tau=\tau_0+1}^{T} \left[y_\tau - E(y_\tau | Data_{\tau-h}) \right]^2}{T - \tau_0 + 1} \tag{18}$$

Where $Data_{\tau-h}$ the information obtained from the period $\tau - h, h$ is the prediction time horizon and $E(y_\tau|Data_{\tau-h})$ is the Forecast y_τ.

Model Estimation

Introducing research data

Monthly data of Tehran Stock Exchange return variables, informal exchange rate changes as domestic market shock variables, one-year bank interest rates (monetary policy), oil price changes as external shock variables and inflation (public policy) for the periods of 2003 to 2013 was obtained from the Central Bank and the International Monetary Fund, respectively and used in this study. The logarithm of the price index ratio of Tehran Stock Exchange in each period is multiplied by 100% compared to the previous period and is considered as the stock return of Tehran Stock Exchange (Jammazi and Aloui, 2010).

$$Y_t = 100 \times ln\Big(\frac{TEPIX_t}{TEPIX_t - 1}\Big)$$

The variables used to forecast and estimate the cash return of the stock market using TVP, DMA-TVP models and their symbols in software computations are presented in Table (2).

Table 2: Model dependent variables and their symbols

Variable name	**Symbol**	**Variable name**	**Symbol**
Informal exchange rate	interest rate	constant	Constant
Bank interest rates	interest rate	First lag of returns	ARY_1
Oil price changes	oil price	Inflation	Inflation

Model Estimation

Table (3) shows the values of MAFE and MSFE from the estimates of different models of DMA, TVP-BMA, BMA and TVP in one and four-month forecast horizons:

Table 3: Comparison of different models

Forecast method	MAFE	MSFE	method Forecast	MAFE	MSFE
h=1			h=4		
DMA α =β = 0.99	39/7	97/28	DMAα = β = 0.99	8/64	113/98
DMA α =β =0.90	32/6	2/72	DMA α =β =0.90	24/6	53/60
DMA α= 0.99 ;β =0.90	41/6	01/70	DMA α = 0.99 ;β =0.90	19/6	69/59
DMAα = 0.90 ;β =0.99	22/6	25/71	DMA α= 0.90 ;β =0.99	23/7	21/83
BMA (DMA α =β = 1)	56/7	42/103	BMA (DMA α=β = 1)	64/8	79/117
TVP	46/7	20/100	TVP	98/8	87/127

Table (3) shows that the DMA model with $\alpha = \beta = 0.90$ has better prediction accuracy. Table (4) provides the results of estimating the best model with input parameters $\alpha = \beta = 0.90$ after estimation with the first lag of the model variables. With changing input variables over time, this model makes it possible to provide the best forecast of the cash return of the Iran Stock Exchang.

Table 4: Variables at any point in time in Best Mode

Time period	**Variable name**					
1-2003	constan	ARY_1	interest rate_ 0			
2-2003	constan	ARY_1	interest rate_0			
3-2003	constan	ARY_1	interest rate_0			
4-2003	constan	ARY_1	interest rate_0			
5-2003	constan	ARY_1	interest rate_0			
6-2003	constan	ARY_1	interest rate_0			
7-2003	constan	ARY_1	interest rate_0			
8-2003	constan	ARY_1	inflation_0	interest rate_0	exchange rate_1	interest rate_1
9-2003	constan	ARY_1	interest rate_0			
10-2003	constan	ARY_1	interest rate_0	exchange rate_1	interest rate_1	
11-2003	constan	ARY_1	interest rate_0	exchange rate_1		
12-2003	constan	ARY_1	exchange rate_0	interest rate_0	exchange rate_1	
Information between the years 1382 − 1383, which has been deleted for brevity,will be provided at the discretion of the reviewer						
1-2013	constant	ARY_1	exchange rate_0			
2-2013	constant	ARY_1	exchange rate_1			
3-2013	constant	ARY_1	exchange rate_0	exchange rate_1		
4-2013	constant	ARY_1	exchange rate_0	exchange rate_1		
5-2013	constant	ARY_1	oil price_0	oil price_1		
6-2013	constant	ARY_1	oil price_0	oil price_1		
7-2013	constant	ARY_1	oil price_0	oil price_1		
8-2013	constant	ARY_1				
9-2013	constant	ARY_1	oil price_0	oil price_1		
10-2013	constant	ARY_1	inflation_0	oil price_0		
11-2013	constant	ARY_1	inflation_0	interest rate_0	oil price_0	
12-2013	constant	inflation_0	interest rate_0	oil price_0		

Note: the index zero shows the variable level and index one shows the

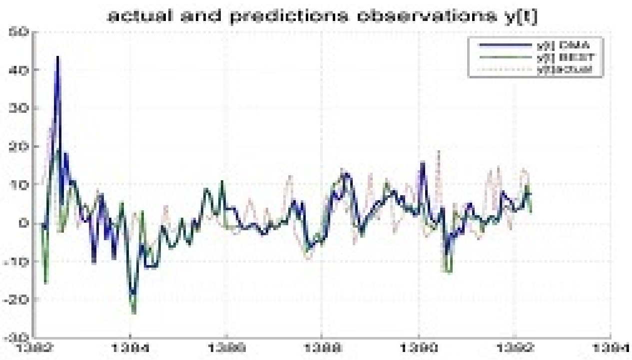

Figure 1: **The model actual and predicted value on the prediction horizon of h = 1 with** $\alpha = \beta = 0.90$

first lag of the research variables.

The results of Table 4 show the variables affecting the stock market return in each time period. For instance, in the period of 1-2003, stock returns and interest rates affect stock returns or the first lag in stock returns, inflation, interest rates, exchange rates and the first lag in interest rates have the highest effect on stock returns in the period 2003-8. All other periods can be analyzed in this way. The following results can be drawn from Table 4: The intercept coefficient and the coefficient of the first lag of stock returns in the whole period (144 periods) had a significant impact on stock returns. Bank interest rate coefficient has had

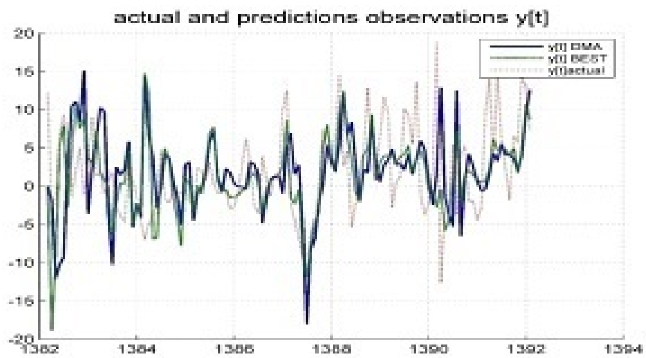

Figure 2: **The model actual and predicted value in the prediction horizon of four h = 4 with** $\alpha = \beta = 0.90$

a significant effect on stock returns in 45 periods. The first lag coefficient of bank interest rates has had a significant impact on stock returns in 37 periods. Inflation rate coefficient has had a significant effect on stock returns in 17 periods. The first lag of inflation rate has had a significant effect on stock returns in 26 periods. Oil price coefficient has had a significant impact on stock returns in 78 periods. The oil prices first lag coefficient has had a significant impact on stock returns in 85 periods. The exchange rate coefficient has had a significant effect on stock returns in 64 periods. The first lag of exchange rate has had a significant effect on stock returns in 35 periods. It is observed, in the final conclusion, that oil prices and exchange rates had the highest impact on stock returns during the period under review after the first lag in stock returns. The actual and predicted values of the forecast horizon model are h = 1 and h = 4 in the DMA model with$\alpha = \beta = 0.90$. Figures (1) and (2) shows these values.

As the diagrams show, the predictions accuracy is high based on the stocks calculated in the previous step and the best model; however, predictions in one-month periods based on these charts are much more accurate than forecasts over a four months period based on the charts. The different levels of accuracy are due to the long prediction period and is not a weakness for the model.

Conclusion

Based on what was mentioned in the third section, the degree of coefficients variability is determined by the values, so that the coefficients can change over time when they are equal to one. Also, the degree of dynamism of the model is determined by values, so that a value equal to one takes the model out of the dynamic state and only coefficients remain as time variables. Based on the results of the present study, dynamic models with time - variable parameters make a more accurate prediction of the stock market returns in the Iranian economy, so that different models of DMA and DMS with full dynamics have more levels of MAFE and MSFE than TVP-BMA and BMA-TVP models. Also, the variability of the coefficirnt of the TVP model variables cannot cause the return simulation accuracy of the stock exchange and the dynamism assumption of the input variables to the model is an important factor in increasing the modeling accuracy of the stock market return in the Iranian economy. Also, the estimates obtained with the DMS model show that the input variables of the model change over time and it shows that considering dynamic models in stock market performance modeling is more important than using the constant input variables assumption in the model.

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