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Research paper

Improving the accuracy of financial time series prediction using nonlinear exponential autoregressive models

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Abstract:
In recent years, precise analysis and prediction of financial time series data have received significant attention. While advanced linear models provide suitable predictions for short and medium-term periods, market studies have indicated that stock behavior adheres to nonlinear patterns and linear models capturing only a portion of the market's stock behavior. Nonlinear exponential autoregressive models have proven highly practical in solving financial problems. This article introduces a new nonlinear model that allocates coefficients to significant variables. To achieve this, existing exponential autoregressive models are analyzed, tests are conducted to validate data integrity and identify influential factors in data trends, and an appropriate model is determined. Subsequently, a novel coefficient allocation method for optimizing the nonlinear exponential Autoregressive model is proposed. The article then proves the ergodicity of the new model and determines its order using the Akaike Information Criterion (AIC). Model parameters are estimated using the nonlinear least squares method. To demonstrate the performance of the proposed model, numerical simulations of Kayson Corporation's stocks are analyzed using existing methods and the new approach. The numerical simulation results confirm the effectiveness and prediction accuracy of the proposed method compared to existing approaches.

Keywords: Financial time series, Nonlinear exponential autoregressive model, Prediction, Parameter estimation.
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1 Introduction

In recent years, the analysis of financial time series has garnered significant attention, particularly in relation to the evaluation of assets over time (Taylor, 2011 [27]). Both financial mathematics theory and empirical time series inherently involve elements of uncertainty. Prediction is a crucial application of time series analysis, making it vital to employ methods that enhance the accuracy of time series predictions (Cuthbertson & Nitzsche, 2005 [8]). Experts in financial mathematics utilize models from various sciences, including statistics, to optimize predictions for interest rates, stock market pricing, and more. Some time series provide valuable information, especially in financial markets, offering insights into stable long-term trends. However, these series may exhibit non-normality, presenting challenges in their analysis. Models can aid in extracting meaningful information from such data, providing optimal models for meaningful interpretations (Mills, 1991 [15]). In the realm of financial mathematics, autoregressive integrated moving average models, autoregressive models, and stochastic volatility models have been extensively examined, with a focus on their predictive capabilities (Chatfield, 2003 [6]). In many cases, time series data, particularly in financial markets, are subject to noise and uncertainty. This noise can result from various factors, such as market fluctuations and economic events, making the data analysis more complex. The organization of data points in financial time series is often influenced by the independence of observations, leading to increased complexity. Various types of data irregularities, arising from outliers and missing values, further complicate the analysis of financial time series. Recently, technological advancements and the adoption of innovative prediction methods, such as nonlinear time series models, neural networks, and fuzzy algorithms, have introduced new challenges to the field. Research indicates that if the data generation process of a variable, whether linear or nonlinear, can be determined, predicting that variable becomes more manageable with less error. While advanced linear models provide suitable predictions for short and medium-term periods, market studies have shown that stock behavior adheres to nonlinear patterns, and linear models only capture a portion of the market's stock behavior.

Researchers have extensively explored financial market predictions using various time series models. Stock and Watson (1998) [25] compared and studied nonlinear prediction models, demonstrating significant differences between linear and nonlinear models, with nonlinear models exhibiting superior performance. Ramsay (2004) [24] employed stochastic differential equations for modeling financial time series. Lamarche (2010) [13] compared multiple data panel models with single-variable models and extended the use of multiple models. Farnoosh et al. (2016) [9] utilized stochastic differential equations for simulating and predicting the time series of OPEC oil prices. Hjorth (2017) [11] investigated prediction for multiple time series models. Many events in the world are part of random processes. Random processes can be controlled or predicted using autoregressive and moving average time series models or a combination of the two, known as autoregressive moving average (ARMA) models (Box, Jenkins, & Reinsel, 2015 [4]). Raei, et al. (2020 [23]), considering non-uniformity and variance in the Iranian market, optimized investment portfolios. Rahimpour and colleagues (2020) performed modeling and prediction of gold and dollar prices using simulation-based robust estimation. Mohammadi and Nabati (2021) [16] addressed financial market modeling using the combination of a return-to-mean time series with Levy noise. Yazdani, et al. (2022) [33] identified optimal turning points in financial time series using graph-based methods. They introduced a new method by searching for the longest path in the graph structure to identify optimal turning points. Nabati and Hajrajabi (2022) [17] introduced a three factor mean reverting model for financial markets with stochastic drift term innovation.

Among financial time series, nonlinear autoregressive models have considerable flexibility. However, some processes exhibit nonlinear behavior. The study of nonlinear models began in the 1970s, covering a wide range of models. The exponential autoregressive (ExpAR) model is a highly practical nonlinear model first introduced by Ozaki and Oda (1977) [18]. Various studies demonstrate the application of nonlinear time series models in financial contexts, such as the research by Merzougui et al. (2016) [14] which examines periodicity tests in restrictive ExpAR models and their use in financial data analysis. The work by Chen et al. (2018) [7] on generalized exponential autoregressive models highlights the flexibility and capability of these models in handling nonlinear time series data. This includes discussions on the stationarity, estimation methods, and practical applications in finance. Research by Xu et al. (2021) [32] covers the modeling a nonlinear process using the exponential autoregressive time series model. Pan et al. (2023) [21] studied the gradient based parameter estimation for nonlinear ExpAR time series model using the multi-innovation. These sources collectively provide a thorough understanding of the methodologies, applications, and advancements in using nonlinear exponential autoregressive models for financial time series prediction. They offer valuable perspectives on the strengths and limitations of these models in capturing the nonlinear characteristics of financial data.

The basis of this article is the investigation of nonlinear behaviors of exponential autoregressive models, introducing a new nonlinear autoregressive model with a coefficient allocation method that enhances the prediction accuracy of time series. The article is organized as follows: the second section presents the research background, including a review of the EGARCH model. The third section focuses on data analysis, conducting tests to identify data validity and influential factors for data trends to determine a suitable model. The fourth section introduces the mathematical modeling, including the presentation of the new coefficient allocation method for nonlinear autoregressive time series. The fifth section involves the simulation of real data from Kayson Corporation using existing models and the proposed new model. Simulation results confirm the accuracy of the new method for prediction. The conclusion and suggestions for future work are provided in the final section.

2 Research Preliminaries

One of the indicators of a country's development on the international stage is the existence of an active and dynamic capital market. The capital market is considered a crucial economic platform for investment and financial support for companies and economic enterprises in most developed countries (Raei, Bajalan, & Ajam, 2021 [22]). Time series analysis is one of the advanced methods for data analysis. By analyzing data over time, one can study the quantitative behavior of observations and devise suitable patterns for data behavior or attempt to classify them. It is evident that recognizing a repetitive structure, a sequence, or a distribution shape of data will assist in a more accurate diagnosis of the model (Tabatabaei, Pakgohar, 2020 [26]). Below are some of the characteristics of these models that are further examined:

Definition 2.1. The nonlinear exponential autoregressive model of order p for a time series $\{x_1, x_2, \ldots, x_N\}$ is defined as follows:

$$x_t = \left\{c_1 + \pi_1 e^{-\gamma x_{t-1}^2}\right\} x_{t-1} + \ldots + \left\{c_p + \pi_p e^{-\gamma x_{t-1}^2}\right\} x_{t-p} + \varepsilon_t, \tag{1}$$

where ε_t is a random variable with identical independent distributions and also independent with x_i and the parameters c_i, π_i, γ should be predicted through observations (Haggan and Ozaki, 1981 [10]).

Model number (1) is the simplest exponential autoregressive model. Ozaki (1981) introduced another model as follows:

$$x_t = \sum_{i=1}^{p} \left\{ c_i + \left(\pi_0^{(i)} + \sum_{j=1}^{k_i} \pi_j^{(i)} x_{t-1}^j \right) e^{-\gamma . x_{t-1}^2} \right\} x_{t-i} + \varepsilon_t \tag{2}$$

where the parameters $\pi_0^{(i)}, c_i, \gamma$ should be predicted. r

Tang (1990) also proposed a model of exponential autoregression that had many applications in the field of financial mathematics as follows:

$$x_t = \left(c_0 + \pi_0 e^{-\gamma (x_{t-d-z})^2}\right) + \sum_{j=1}^{p} \left\{c_j + \pi_j e^{-\gamma (x_{t-d-z})^2}\right\} x_{t-j} + \varepsilon_t. \tag{3}$$

Another appropriate model has been proposed by Teräsvirta (1994) which is as follows.

$$x_t = \left(c_0 + \pi_0 e^{-\gamma (x_{t-d-z})^2}\right) + \sum_{j=1}^{p} \left\{c_j + \pi_j e^{-\gamma (X_{t-d-z})^2}\right\} x_{t-j} + \varepsilon_t, \tag{4}$$

where z and d are scalar parameters and an integer respectively. Proper prediction of the parameters in the exponential autoregressive model is a significant optimization problem. Haggan and Ozaki (1981) [10] addressed these nonlinear problems by transforming them into linear least squares problems through the substitution of the γ parameters in the equations. This approach was effective when there was only one nonlinear parameter in the exponential autoregressive model. However, in the model we are introducing, there will be multiple nonlinear parameters. Careful examination of the equations in the exponential autoregressive model reveals that this model is a linear combination of several nonlinear functions.

2.1 Ergodicity of the model

Considering the exponential autoregressive models, let us first introduce the variables we need:

$$X_t = (x_t, \ldots, x_{t-p+1})^T$$

$$\omega_t = (\varepsilon_t, 0, \ldots, 0)^T$$

$$C_0 = (c_0, 0, 0, \ldots, 0)^T$$

$$A(X) = \begin{bmatrix} \varphi_1(X) & \varphi_2(X) & \ldots & \varphi_{p-1}(X) & \varphi_p(X) \\ 1 & & & 000 & \\ \vdots & & \vdots & \ddots & \vdots & \vdots \\ 0 & & 0 & \ldots & 1 & 0 \end{bmatrix}$$

Exponential autoregressive models can be written as follows:

$$x_t = c_0 + \sum_{i=1}^{p} \varphi_i(X_{t-1}) .x_{t-1} + \varepsilon_t. \tag{5}$$

Or in matrix form we have

$$X_t = C_0 + A(X_{t-1}) .X_{t-1} + \omega_t \tag{6}$$

It is necessary to prove the ergodicity of relation (6) for the exponential autoregressive model to be ergodic, we can rewrite equation (5) as follows:

$$x_t = f(x_{t-1}, \ldots, x_{t-p}) + \varepsilon_t. \tag{7}$$

Or in matrix form we have

$$X_t = g(X_{t-1}) + \omega_t, \tag{8}$$

Where $g(X_t) = (f(X_{t-1}), x_{t-1}, \ldots, x_{t-p+1})^T$.

In this article, we first propose the following exponential autoregressive model, and we discuss the conditions of ergodicity on it. Then we introduce the model with the allocation of coefficients.

$$x_t = \sum_{i=1}^{p} \left(c_i + \pi_i e^{-\gamma x_{t-1}^2} \right) x_{t-i} + \varepsilon_t \tag{9}$$

The following relation is a generalized model (9):

$$x_t = c_0 + \sum_{i=1}^{p} \left(c_i + \pi_i e^{-\gamma (x_{t-d-z})^2} \right) x_{t-i} + \varepsilon_t \tag{10}$$

We will examine the conditions of the model being ergodic.

2.2 Investigating Model Conditions

Lemma 2.2. *If we assume that the model function (7) is a measurable and bounded function, and the noise function ε_t has an almost everywhere positive density and $E(\varepsilon_t) = 0$, then both models (7) and (8) are ergodic. If*

$$\lim_{\|\theta\| \to \infty} \frac{\left| f(\theta) - \rho^T(\theta) \right|}{\|\theta\|} = 0, \tag{11}$$

where $\|.\|$ is the Euclidean norm and $\rho = (\alpha_1, \ldots, \alpha_p)^T$ has the following conditions:

$$z^p - \alpha_1 z^{p-1} - \ldots - \alpha_p \neq 0, \tag{12}$$

For all $|z| \geq 1$.

If we assume that the model function (7) is bounded, and the density function ω_t is almost everywhere positive, then $\{X_t\}$ in (8) is non-oscillatory, and μ is an irreversible measure on the normal topological space.

Theorem 2.3. *If $\{X_t\}$ is an irreversible measure on the normal topological space and has continuous probability, then there exist sufficient conditions for ergodicity such that a compact set k and a constant $0 < \tau < 1$ exist such that*

$$E(\|X_{t+1}\| \, | X_t = \theta) < \begin{cases} \infty & \theta \in k \\ \tau \|\theta\| & \theta \notin k \end{cases} \tag{13}$$

If $\{X_t\}$ is a non-periodic matrix and if h is a positive integer such that $\{X_{ht}\}$ is ergodic, then the series $\{X_t\}$ is ergodic (Chan and Tong, 1985).

With the stated conditions, ergodicity of exponential autoregressive models can be concluded as follows:

Firstly, it is evident that $|\emptyset_i(X_{t-1})| \leq |c_i| \, |\pi_i|$ for i=1, 2, , p considering that $\emptyset_i = |c_i| + |\pi_i|$. This means that if we assume ε_t has a density function in the exponential autoregressive model on R that is positive everywhere, and if all the roots of the equation below are inside the unit circle, then in this case, the model (10) is an ergodic time series:

$$z^p - \emptyset_1 z^{p-1} - \ldots \emptyset_p = 0$$

Based on this, let's assume that the density function ε_t in the model on the line R is positive everywhere, if all the roots are characteristic function inside the unit circle, then the model (10) is ergodic (Tjostheim, 1990 [29]).

Also, with the proposition that we assume that the density function ε_t in model (10) is positive everywhere on the R line, if all the roots of the characteristic function $z^p - c_1 z^{p-1} - \ldots c_p = 0$ are inside the unit root circle, then the model (9) is ergodic.

2.3 Implementation method on the nonlinear model

In this article, to obtain the coefficients, we need a nonlinear method. Nonlinear least squares, considering the selection of parameters β, minimizes the sum of squared residuals. For this purpose, the following nonlinear model is considered:

$$y = f\left(x; \beta\right) + \varepsilon(\beta) \tag{14}$$

Here, y represents the observed values, x is a vector of predictors, β is the vector of parameters to be estimated, and $\varepsilon(\beta)$ is the error term. The goal is to find the values of β that minimize the sum of squared differences between the observed and predicted values. This is typically done using optimization techniques.

Let us assume T is the number of observations, we can rewrite equation 14 as follows:

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_T \end{bmatrix}, \qquad f\left(x_1, \cdots, x_T; \beta\right) = \begin{bmatrix} f\left(x_1; \beta\right) \\ f\left(x_2; \beta\right) \\ \vdots \\ f\left(x_T; \beta\right) \end{bmatrix}$$

$$y = f\left(x_1, \cdots, x_T; \beta\right) + \varepsilon(\beta) \tag{15}$$

2.4 Prediction and Implementation on the Model

The objective of this section is to find the best k-dimensional level (y_t, x_t) considering t = 1,2,,N. The method involves minimizing a function over the data values. We consider the function (2.13)

$$\begin{aligned} Q_N\left(\beta\right) &= \frac{1}{N}\left(y - f\left(x_1, \cdots, x_N; \beta\right)\right)^T \left[y - f\left(x_1, \cdots, x_N; \beta\right)\right] \\ &= \frac{1}{N}\sum_{N=1}^{k}\left[y_N - f\left(x_t; \beta\right)\right]^2, \end{aligned} \tag{16}$$

Where $\nabla_\beta Q_N(\beta)$ is the gradiant function of $Q_N(\beta)$.

To optimize the problem, we have a relationship with k number of unknown parameters and k nonlinear equation (14).

$$\nabla_\beta Q_T(\beta) = \frac{-2}{T}\nabla_\beta f(x_1,\cdots,x_T;\beta)\left[y - f(x_1,\cdots,x_T;\beta)\right]\triangleq 0, \tag{17}$$

where

$$\nabla_\beta f(x_1,\cdots,x_T;\beta) = \begin{bmatrix}\nabla_\beta f(x_1;\beta) & \nabla_\beta f(x_2;\beta)\cdots & \nabla_\beta f(x_T;\beta)\end{bmatrix}. \tag{18}$$

These parameters have associated errors, and the objective is to minimize this value. In this regard, we proceed with the iterative solution of this problem, considering the first-order solution method. To continue this process, we define the second order as $\nabla^2_\beta Q_T\left(\overline{\beta}\right)$ and by repeating this procedure, we will have an optimal solution for the parameters for nonlinear data fitting. It is noteworthy that the optimal solution may not be unique, and local optimal solutions for error minimization are possible, which is of importance.

3 Mathematical modeling

In this section, a method for parameter optimization and increasing prediction accuracy on the nonlinear exponential autoregressive model is introduced. It is worth noting that this method is effective for time series models and is a general approach for optimization in the time series domain, applicable to various time series models. In this article, we specifically focus on the examination of the model (10).

Consider the following model:

$$x_t = c_0 + \sum_{i=1}^{p}\left(c_i + \pi_i e^{-\gamma_i(x_{t-i}-z_i)^2}\right)x_{t-i} + \varepsilon_t. \tag{19}$$

At first, we prove its ergodicity, then we compare its results with previous models.

3.1 Examining the ergodicity of the introduced model

Theorem 3.1. *Assume that the density function ε_t in model (19) is positive everywhere on the line R. If all the roots are characteristic functions $z^p - c_1 z^{p-1} - \ldots c_p = 0$ inside the unit circle, then this model is ergodic.*

Proof: Consider $f_i(x_{t-i}) = \emptyset_i$ and $\rho = (\alpha_1,\ldots,\alpha_p)^T$. According to the statements of ergodicity of models (9) and (10), we have:

$$
\begin{aligned}
&\lim_{\|X\|\to\infty} \frac{\left|f(X)-\rho^{T}(X)\right|}{\|X\|} \\
&\quad = \lim_{\|X\|\to\infty} \frac{\left|c_0+\sum_{i=1}^{p}\left(c_i+\pi_i e^{-\gamma_i(x_i-c_i)^2}\right)x_i-\rho^{T}(X)\right|}{\|X\|} \\
&\quad \le \lim_{\|X\|\to\infty} \frac{|C_0|}{\|X\|} + \lim_{\|X\|\to\infty} \frac{\left|\sum_{i=1}^{p}\left(c_i+\pi_i e^{-\gamma_i(x_i-c_i)^2}\right)x_i\right|}{\|X\|}
\end{aligned} \tag{20}
$$

Since γ_i is greater than zero we conclude that:

$$
\lim_{\|X\|\to\infty} \frac{\left|\sum_{i=1}^{p}\left(c_i+\pi_i e^{-\gamma_i(x_i-c_i)^2}\right)x_i\right|}{\|X\|} = 0
$$

Therefore, according to the definition of α, we can easily conclude the condition (12), so the ergodicity of the model (19) is proved by knowing the ergodicity of the model (10) and the relation (20).

3.2 Model Order Determination

There are various methods available for determining the order of the model, such as Akaike's Information Criterion (AIC), Bayesian Criterion, and many more. For numerical examples and simulations, we choose the AIC method.

$$
AIC\,(p) = (N-m)\log\left(\widehat{\delta}_p^2\right) + 2\,(2p+1)\,, \tag{21}
$$

where p is the optimal order, and N is the total number of observations. There are 2p+1 estimated parameters, and $\widehat{\delta}_p^2$ is the residual variance with the model used by predicting the least squares estimation.

3.3 Step by step explanation of the novel coefficient Allocation method

Below is a detailed step-by-step explanation of this method.

(i) Problem Definition: Clearly define the problem that requires coefficient allocation. Identify the variables, constraints, and objectives involved in the allocation process.

(ii) Data Collection and Preprocessing: Gather all necessary data that will influence the allocation of coefficients. This data might include historical data, statistical measures, or domain-specific metrics.

(iii) Initial Coefficient Assignment: Assign initial coefficients to the variables based on predefined rules. These initial coefficients serve as a starting point for further optimization.

(iv) Objective Function Formulation: Develop an objective function that quantifies the goal of the coefficient allocation. This function should reflect the criteria for an optimal allocation, such as minimizing error, maximizing efficiency, or balancing trade-offs.

(v) Constraint Definition: Define the constraints that must be adhered to during the coefficient allocation. Constraints can be equality constraints, inequality constraints, or boundary conditions that the coefficients must satisfy.

(vi) Optimization Algorithm Selection: Choose an appropriate optimization algorithm to solve the coefficient allocation problem.

(vii) Iterative Optimization: Run the optimization algorithm iteratively to adjust the coefficients. During each iteration, evaluate the objective function and constraints to guide the adjustments. Monitor convergence criteria to determine when the optimal or satisfactory coefficients have been reached.

(viii) Validation and Testing: Validate the allocated coefficients by applying them to a test dataset or real-world scenario. Assess the performance against predefined metrics or benchmarks. Make necessary adjustments if the validation results indicate suboptimal performance.

(ix) Final Allocation: Once validated, finalize the coefficients and document the allocation method, including the rationale for each step and the results obtained. Implement the coefficients in the relevant application or system.

(x) Review and Improvement: Periodically review the coefficient allocation method and its performance. Gather feedback and conduct further analysis to refine the method over time. Incorporate new data and insights to continuously improve the allocation process.

4 Finding results

The company Kayson was established in 1975 and is listed on the Tehran Stock Exchange with the symbol **K**SON. The company is engaged in engineering, procurement, construction, and project management activities. Kayson has consistently reported actual profits, achieving 53% in 2018 and 63% in 2017. In 2016, the company was recognized among the top 100 global contractors. In this section, a simulation of the model is considered for Kayson's stock prices. The closing prices of Kayson from September 18, 2018, to April 25, 2019, were obtained from the Tehran Stock Exchange Technology Management Company. The company had 135 trading days during this period.

The prices of the company are plotted in Figure 1. With knowledge of the data and conducting tests on the data, a nonlinear model is applied. Initially, the data is estimated with models (9) and (10), then data analysis is performed with the proposed model, i.e., model (19), and error values are compared. Figure 2 represents the estimation of model (9), Figure 3 shows the estimation of model (10), and Figure 4 presents the simulation of Kayson's data with the introduced model (19). It's worth noting that all simulations were performed using EViews software.

Considering the trend of the autocorrelation function and the delay of one in the partial autocorrelation function, it is possible to choose a suitable model for these data using the first-order Box Jenkins autoregressive method. Figure 2 shows the forecast of the company's stock using the first-order autoregressive model. Using Dickie Fuller's developed tests, he noticed the existence of instability in the data, so the logarithm of the data price difference is investigated. In model selection, ARMA (3,1) model is selected through information criteria using Akaike's criterion.

By knowing the data and performing tests on the data, the implementation of the model is used in non-linear mode. First, by using the data, the parameters of models (9) and (10) and the new model (19) are estimated, then the simulation of the models with the estimated parameters is done. Figures 3 and 4 show the simulated graphs of data using models (9) and (10). The simulation of the data of Caisson company with the introduced model (19) is presented in Figure 5. It should be noted that all the simulations were done in EViews software.

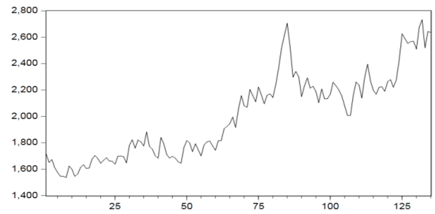

Figure 1: Chart of the stock price of Kayson Company

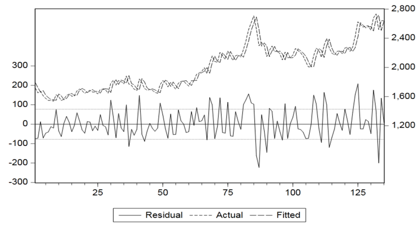

Figure 2: Simulation of the stock price of Kayson company using the first order autoregressive model

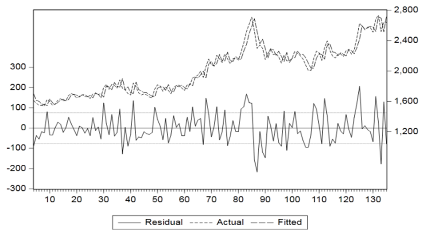

Figure 3: Simulation of Kayson company shares with model 9 (exponential autoregression of the 3rd order

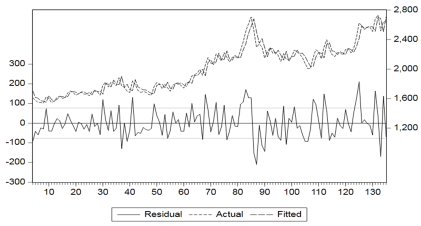

Figure 4: Simulation of Kayson company shares with model (10) (exponential autoregression of the 3rd order)

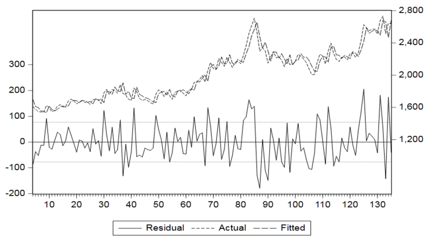

Figure 5: Simulation of the shares of Kayson company with the new model (exponential autoregression of the 3rd order)

5 Model Comparison

Figures 6 and 7 show the prediction of data using models (9) and (10) in the 3rd order exponential autoregressive form. Figure 8 shows the prediction of data using the newly introduced model with the help of assigning coefficients. By comparing the error values of all three models in the guide tables of forecasting figures, we find that the lowest error value is for the model with the allocation of coefficients, i.e. model (19). In comparing the error values of the estimated models, the presented model is more accurate. As can be seen, the model presented in this article is closer to the real values than other models. Then, model (10) and model (9) have less

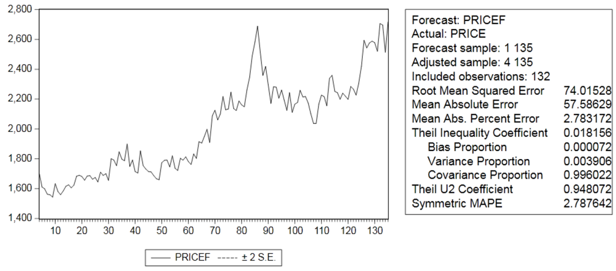

Figure 6: Data prediction with model (9)

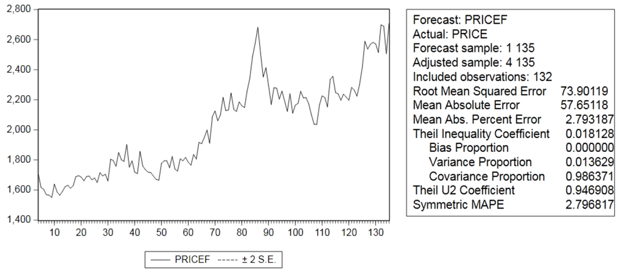

Figure 7: Data prediction with model (10)

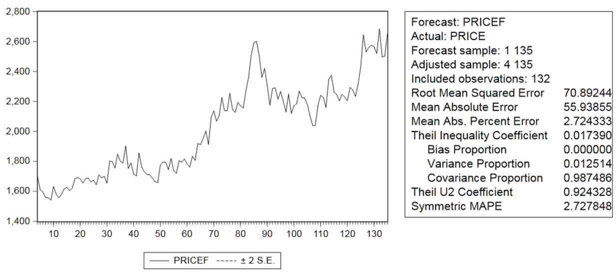

Figure 8: Data prediction with the new model

errors in prediction

6 Conclusion and Recommendations

Time series models represent a robust research domain for estimation and prediction in financial markets, encompassing various linear and nonlinear models. Autoregressive and exponential autoregressive models are among the types, and their performance can be enhanced by adapting them to changes. In this article, by allocating coefficients to significant variables, we were able to increase prediction accuracy. Furthermore, through applying the model to Kayson company data, we conducted simulations and observed an improvement in prediction accuracy and a reduction in errors compared to previous models. Future work in this article could involve exploring well-known nonlinear functions as coefficients for other significant variables in different models. Additionally, predicting data using the new method introduced in this article, combining artificial intelligence networks, is among the future objectives of this research.

Conflicts of interest

The authors declare no potential conflict of interests.

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