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Research paper

A dynamical system model-driven approach to pricing with smart volatility: a case study of catastrophe bonds pricing for Chinas flood

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Abstract:
This study explores the application of dynamic systems for modeling and valuing catastrophe bonds to establish a more intelligent and adaptive approach to determining their volatility parameter. These financial instruments hold significant importance for insurance companies in safeguarding against the risk of insolvency stemming from the escalating frequency and severity of natural disasters worldwide. Employing mathematical principles, this research formulated a pricing partial differential equation and introduced a dynamic system for its resolution. The damage model was assumed to follow a stochastic process, and a radial basis neural network was utilized to estimate the volatility parameter of this stochastic process by leveraging historical data. The study scrutinized the pricing framework of catastrophe bonds related to floods and storms in China, ultimately demonstrating that the proposed methodology proved effective and computationally efficient when contrasted with alternative approaches.

Keywords: Dynamical systems, Catastrophe bonds, Pricing, Volatility, Radial basis function neural networks.
Classification: 37N40, 91-00.

1 Introduction

Dynamic systems have various applications in various fields and can analyze numerous phenomena. By employing the characteristics of a particular phenomenon, a dynamic system can be established to study its behavior. Although not all aspects of a phenomenon can be transformed into dynamic systems, an approximation

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can be made to meet the learning needs partially. However, it is only possible to find an ideal model for some phenomena using dynamic systems or other branches of study. This paper explores one of the practical aspects of dynamic systems, particularly their finance application. Financial crises have been analyzed using multi-factor dynamic systems by Castellacci, and Choi [4], which have been used as a negative shock to economic equilibrium that disrupts the balance. Cheriyan and Kleywegt [5] used dynamic systems to investigate market bubbles and found that investors may predict and decide upon price bubbles in seemingly sensible ways. Choi and Douady [7] studied the effect of increasing leverage on economics using dynamic systems and bifurcation. In addition, Choi [6] examined the stability of an economic structure divided into two unequal subdivisions. This paper discusses another application of dynamic systems: the pricing of catastrophe bonds (CAT bonds).

Sometimes, wind movements can be powerful enough to create fast airwaves, known as storms. These storms can cause trees and structures to collapse, resulting in large-scale destruction and financial losses. Table 1, extracted from https://www.iii.org/fact-statistic/facts-statistics-hurricanes, lists ten expensive storms in the United States.

Table 1: 10 Costly storms in the United States

Rank	Year	Hurricane	Dollars when occurred $\times 10^3$\$	In 2020 dollars $\times 10^3$\$
1	2005	Hurricane Katrina	65,000	85,570
2	2012	Hurricane Sandy	30,000	33,530
3	2017	Hurricane Harvey	30,000	31,590
4	2017	Hurricane Irma	29,900	31,320
5	2017	Hurricane Maria	29,670	31,100
6	1992	Hurricane Andrew	16,000	29,360
7	2008	Hurricane Ike	18,200	21,510
8	2005	Hurricane Wilma	10,670	13,840
9	2018	Hurricane Michael	13,250	13,550
10	2004	Hurricane Ivan	8,720	11,870

There have been numerous studies on natural disaster risk management in recent years. For instance, Scally [8] proposed different suitable locations for shelter by assessing the risk of tropical storms in Rarotonga, and Mai et al. [14] used a novel approach to create a fundamental interaction between the endogenous characteristics of a flood-prone society. However, in a natural disaster, the question of compensating for financial losses arises. Insurance is one way to cover the risk of such losses. As an influential player in a country's economy, examining the factors that can impact the insurance industry is essential. Sometimes the damage caused by natural disasters is so severe that insurance cannot reimburse the entire cost. Therefore, insurance companies use reinsurance to prevent bankruptcy. However, in some cases, the damages caused by natural disasters are so significant that even reinsurance cannot cover them. To address this, the insurance industry has developed CAT bonds to cover the risks arising from such catastrophes. This study focuses on modeling and pricing CAT bonds.

While many studies have been conducted on modeling CAT bonds, this study specifically focuses on China's CAT bond market, which has seen significant growth in recent years due to increased storm damage. The Unger [21] modeling method is used, which considers the damage model as a jump-diffusion stochastic process. However, it may not be appropriate to use the jump-diffusion model in the context of storm and flood damage in China, and thus, this study considers a non-jump model. Previous research, such as Gang Ma and Qun Ma [13], presented a formula for securities based on a non-homogeneous Poisson process, and Burnecki and Giuricich [3] modeled and calculated CAT bonds by considering the approximating tail probabilities in the general compound renewal process framework. The proposed model can provide valuable insights into the design of preventive CAT bonds for China, which is highly vulnerable to storm damage.

The contribution of this research is listed as follows:

- Introducing a novel method for solving the Black-Scholes PDE: The Black-Scholes partial differential equation (PDE) is a widely used model in finance for pricing options. This research introduces a new method for solving this PDE, which can improve the accuracy and efficiency of option pricing.

- Solving the Black-Scholes PDE with minimal running time compared to two prevalent approaches in this area: This new method also shows improved performance compared to two widely used approaches for solving the Black-Scholes PDE, which can potentially save time and resources for financial institutions.

- Facilitates implementation in programming: This method is designed to be easily implemented in programming, making it accessible and practical for financial industry applications.

- Using the method in many financial industries for the first pricing of financial derivatives based on the Black-Scholes PDE before joining the supply-demand market: By applying this method to price financial derivatives based on the Black-Scholes PDE, this research provides a valuable tool for financial industries to assess the risk of these derivatives before they enter the market.

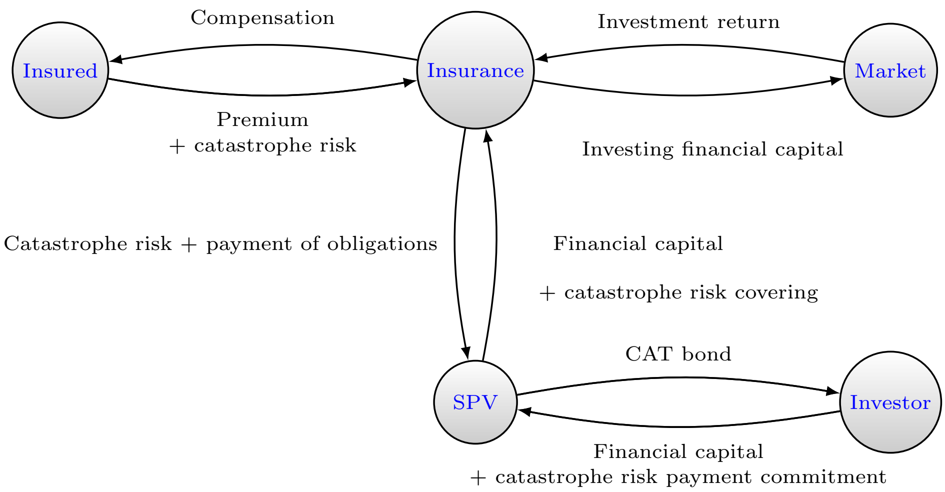
- Damage volatility parameter estimation using neural network: Besides the Black-Scholes PDE, this research also uses neural networks to estimate the volatility parameter for modeling CAT bonds for flood and typhoon damage in China, which can improve the accuracy of these models.

- Using actual data to model CAT bonds for flood and typhoon damage in China: This research also contributes to the understanding and modeling of CAT bonds specifically for flood and typhoon damage in China, which can potentially help the insurance industry in China to manage better the risks associated with these natural disasters.

The rest of this research is described as follows. The next section presents a PDE-based price model using mathematical concepts. In section 3, we solve the PDE using dynamic systems and examine its efficiency by the call and put option. Finally, Section 4 presents the price graph of CAT bonds of China's storm and flood using accurate data using Python software.

2 CAT bonds

CAT bond is one of the most important securities in insurance and has found a special place due to the current weather conditions. As mentioned in the previous section, these bonds are designed to prevent bankruptcy according to compensation for natural disasters. In this section, the components related to these bonds are stated, so to understand their mechanism better, the following graph is presented:



The general structure of these bonds is that insurance companies (or reinsurance companies) first establish a Special Purpose Vehicle (SPV) or cooperate with it, if any, and use it to transfer the risk of natural disasters to one or more investors. Considering the amount of damage, the SPV company issues a particular type of bond and sells it to the investor. Insurers trade as risk sellers, and investors as risk buyers. The SPV states in the contract with the investor that if the damages exceed the expected threshold of an insurance company, the excess damages will be reduced from the principal or coupon of CAT bond interest in the event of a catastrophic event. The SPV transfers the financial resources from the sale of the CAT bond to the insurance company. The insurance company invests the resources in the market and, with a swap contract at a fixed interest rate, guarantees the return on that investment and returns the proceeds to the investor by SPV.

2.1 The model

In this section, we intend to model these bonds. Therefore, the damage model is introduced first, and then the CAT bond is symbolized using it. Finally, a PDE model will be presented using mathematical concepts. To begin modeling, according to the type of data used in the next section, assume that the damage follows a random process as

$$dS = \alpha S dt + \sigma S dw_t, \tag{1}$$

where the S is damage, α is damage rate of data, σ is the volatility around the average and w_t is Wiener process. We also assume that $C(S,t)$ is the price of the CAT bond at the time of t and the amount of the damage S, so using Ito lemma [**fi**, **mas1**], the change of CAT bond is as follow [1,9,21]:

$$dC = \underbrace{(C_t + \alpha S C_S + 0.5\sigma^2 S^2 C_{SS})}_{\varphi_C} dt + \underbrace{\sigma S C_S}_{\Delta_C} dw_t, \tag{2}$$

Assuming that D is another CAT bond similar to C as $dD = \varphi_D dt + \Delta_D dw_t$, we have the following portfolio:

$$\Pi = x_1 C + x_2 D, \tag{3}$$

where x_1 and x_2 are the volume C and D in the portfolio, respectively. Assume the portfolio is risk-free, so

$$\begin{aligned} d\Pi &= r\Pi dt, \\ E[x_1 dC + x_2 dD] &= r x_1 C + r x_2 D. \end{aligned} \tag{4}$$

Assuming $x_1 = \Delta_D$ and $x_2 = -\Delta_C$, from the previous expression, the following ratio is obtained:

$$\frac{\varphi_C - rC}{\Delta_C} = \frac{\varphi_D - rD}{\Delta_D}. \tag{5}$$

This ratio is called the market price of risk and is denoted by q. By inserting the values associated with the symbols, the following PDE is obtained:

$$C_t + (\alpha S - q\sigma S) C_S + 0.5\sigma^2 S^2 C_{SS} - rC = 0. \tag{6}$$

According to the mechanism of these bonds, the terminal condition or pay off is as follows:

$$C(S,T) = C^* \mathrm{I}_{S \le S^*} + \max(C^* - (S - S^*), 0) \mathrm{I}_{S > S^*}, \tag{7}$$

where I is indicator function, T is maturity time and S^* is the maximum indemnity that insurance can pay. If the amount of damage is less than S^*, this bond will act as a usual bond with a fixed-rate; otherwise, the amount of excess damage will be deducted from the amount of fixed-rate bond C^*.

Also, the boundary conditions are as follows:

$$\begin{aligned} & C(0,t) = C^* e^{-r(T-t)}, \\ & \lim_{S\to\infty} C(S,t) = 0. \end{aligned} \tag{8}$$

The financial interpretation of the above phrase means that if the amount of damage is zero, it acts as a usual bond with a fixed-rate, and vice versa, if it tends to infinity, no one will show interest in these bonds and their price will be zero.

2.2 Smart Volatility

Volatility is an essential quantity in stochastic modeling, especially financial modeling. It shows the stochastic variable's degree of fluctuation or unpredictability and is crucial for forecasting the investment risk. The importance of analyzing the volatility parameter is based on the following factors [10, 11, 15–17]:

- Investors use volatility to control risk in their portfolios. Investors change their investing approach to account for the increased risk of greater volatility. By modeling volatility, investors may determine risk tolerance and make educated investing choices.

- Volatility is significant in pricing financial assets, especially derivatives such as options and futures. The anticipated volatility of the underlying asset determines the value of these assets. Consequently, it is vital to comprehend the volatility characteristic to price these securities appropriately.

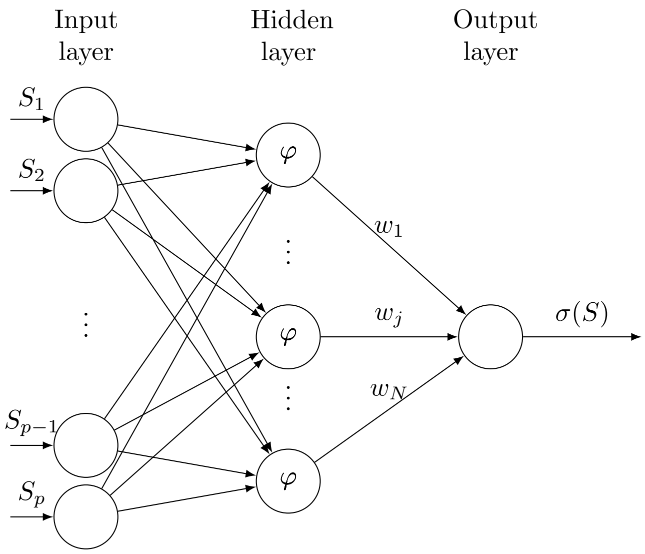
- Hedging is a practice used to lessen the risk associated with financial investments. By modeling volatility, investors may determine the most effective hedging techniques to protect themselves from unfavorable market swings.

- Volatility models may be used to predict future market fluctuations. This is crucial for investors who want to make educated choices on whether to acquire or sell assets.

The study of the volatility parameter is crucial for comprehending the risk associated with financial investments and making educated investment choices. Modeling volatility offers a potent instrument for risk management, asset pricing, hedging, portfolio optimization, and forecasting.

In this research, we assume that the loss model's volatility is proportional to the amount of reported loss. To estimate the volatility function, we use a radial basis neural network (RBFNN) as described in the follow.

This network will be trained using the Mean Square Error loss function and Adams optimization in Tensorflow.



To create a data set, we examine the research methodology of Azizi and Neissy [19, 20]. They determine the volatility value at each time step using the primary data. Thus, we may view the dataset as follows:

$$\{(S_1, \sigma_{1(S_0,S_1)}), ..., (S_p, \sigma_{p(S_0,...,S_p)})\} \tag{9}$$

After obtaining the $\sigma(S)$ value we can rewrite the PDE 6 as:

$$C_t + (\alpha S - q\sigma(S)S)C_S + 0.5\sigma(S)^2 S^2 C_{SS} - rC = 0. \tag{10}$$

3 The dynamic system solution of CAT bonds PDE

In this section, we intend to provide a solution method for PDE 6. Depending on the type of PDE, it isn't easy to get a close form, and researchers usually use other methods, such as numerical methods. We want to find a solution to this matter using the dynamic system. In detail, with a converting in PDE 6, we achieve a homogeneous linear dynamic system and solve it by the usual method.

By changing the variable $\tau = T - t$, the PDE 6 is rewritten as:

$$C_\tau = (\alpha S - q\sigma(S)S)C_S + 0.5\sigma(S)^2 S^2 C_{SS} - rC. \tag{11}$$

Depending on the structure of the problem, we limit the range of changes S and show it as $[0, S_{max}]$. In the next step, this interval is divided into $n+2$ sections as follows:

$$
\begin{aligned}
&S_0 = 0, S_1, ..., S_n, S_{n+1} = S_{\max}, \\
&dS = S_{i+1} - S_i, \\
&i \in \{0, ..., n\}.
\end{aligned} \tag{12}
$$

Given the above discretization and notation $C(S_i, \tau) = C_i(\tau)$, the derivatives respect to S are considered as:

$$
\begin{aligned}
\tfrac{\partial C(S_i,\tau)}{\partial S} &= \tfrac{\partial C_i(\tau)}{\partial S} \simeq \tfrac{C_{i+1}(\tau) - C_{i-1}(\tau)}{2dS}, \\
\tfrac{\partial^2 C(S_i,\tau)}{\partial S^2} &= \tfrac{\partial^2 C_i(\tau)}{\partial S^2} \simeq \tfrac{C_{i+1}(\tau) - C_i(\tau) + C_{i-1}(\tau)}{dS^2}.
\end{aligned} \tag{13}
$$

By placing these derivatives in PDE 11 and considering all indexes, we have

$$
\underbrace{\begin{bmatrix} \frac{d}{dt}C_0(\tau) \\ \frac{d}{dt}C_1(\tau) \\ \vdots \\ \frac{d}{dt}C_n(\tau) \\ \frac{d}{dt}C_{n+1}(\tau) \end{bmatrix}}_{\dot{C}} = \underbrace{\begin{bmatrix} \gamma & 0 & 0 & ... & 0 & 0 & 0 \\ a_1 & b_1 & c_1 & & & & 0 \\ 0 & a_2 & b_2 & c_2 & & & 0 \\ \vdots & & \ddots & \ddots & \ddots & & \vdots \\ 0 & & & a_{n-1} & b_{n-1} & c_{n-1} & 0 \\ 0 & & & & a_n & b_n & c_n \\ 0 & 0 & 0 & \cdots & 0 & 0 & \nu \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} B_1(\tau) \\ C_1(\tau) \\ \vdots \\ C_n(\tau) \\ B_2(\tau) \end{bmatrix}}_{C},
$$

$$
\begin{aligned}
\gamma &= \begin{cases} \frac{dB_1/d\tau}{B_1} & B_1 \neq 0 \\ -1 & \text{Otherwise} \end{cases}, \\
\nu &= \begin{cases} \frac{dB_2/d\tau}{B_2} & B_2 \neq 0 \\ -1 & \text{Otherwise} \end{cases}, \\
a_i &= \tfrac{0.5\sigma(S_i)^2 S_i^2}{dS^2} - (\alpha \tfrac{S_i}{2dS} - q\sigma(S_i)S_i), \\
b_i &= -\tfrac{\sigma(S_i)^2 S_i^2}{dS^2} - r, \\
c_i &= \tfrac{0.5\sigma(S_i)^2 S_i^2}{dS^2} + (\alpha \tfrac{S_i}{2dS} - q\sigma(S_i)S_i).
\end{aligned} \tag{14}
$$

where B_1 and B_2 are boundary conditions as follows:

$$
\begin{aligned}
B_1(\tau) &= C(0, \tau), \\
B_2(\tau) &= \lim_{S \to \infty} C(S, \tau).
\end{aligned} \tag{15}
$$

Now using the usual methods in dynamic systems, the solution to the linear dynamic system $\dot{C} = AC$ is

$$
C = e^{A\tau} IC \tag{16}
$$

where IC is the initial condition.

3.1 Stability

To study stability, Lyapunov's definition of stability is first stated, and using the properties of the problem, the establishment of this definition for our problem is examined.

Definition 3.1. (Lyapunov stability) [22]: The solution $X(t)$ is a Lyapunov stable if for each ε, there exists a δ that for other solution $Y(t)$, satisfying $|X(t_0) - Y(t_0)| < \delta$, we can draw conclusions $|X(t) - Y(t)|_{t>t_0} < \varepsilon$.

Now suppose C is a solution to linear dynamics system 14 with the initial condition C_0. Also suppose F_0 is obtained by adding a perturbation to C_0 and the corresponding solution is F. Hence we have

$$C - F = e^{A\tau} C_0 - e^{A\tau} F_0 = e^{A\tau}(C_0 - F_0). \tag{17}$$

Using the norm property, we have

$$\begin{aligned} &\|C - F\| \le L \,\|(C_0 - F_0)\|\,, \\ &L = \left\|e^{A\tau}\right\|. \end{aligned} \tag{18}$$

Now assuming $\|C - F\| \le L\,\|(C_0 - F_0)\| < \varepsilon$ and $\|(C_0 - F_0)\| < \delta$, for each ε, $\delta = \varepsilon/L$ can be selected to meet the definition of Lyapunov stability.

Example 3.2. To evaluate the effectiveness of this method, we intend to compare the results of the call option and put option with their exact solution. Suppose the price range of an underlying asset is $[0, 100]$, strike price 25 for the call option, strike price 70 for the put option, interest rate 0.05, volatility 0.25, and maturity time 1. Suppose the close-form of the call option is $Call(S,t)$ and the put option is $Put(S,t)$, which are as follows:

$$\begin{aligned} &Call(S_t, t) = N(d_1)S_t - N(d_2)Ke^{-r(T-t)}, \\ &Put(S_t, t) = N(-d_2)Ke^{-r(T-t)} - N(-d_1)S_t, \\ &d_1 = \tfrac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\tfrac{S_t}{K}\right) + \left(r + \tfrac{\sigma^2}{2}\right)(T-t)\right], \\ &d_2 = d_1 - \sigma\sqrt{T-t}. \end{aligned} \tag{19}$$

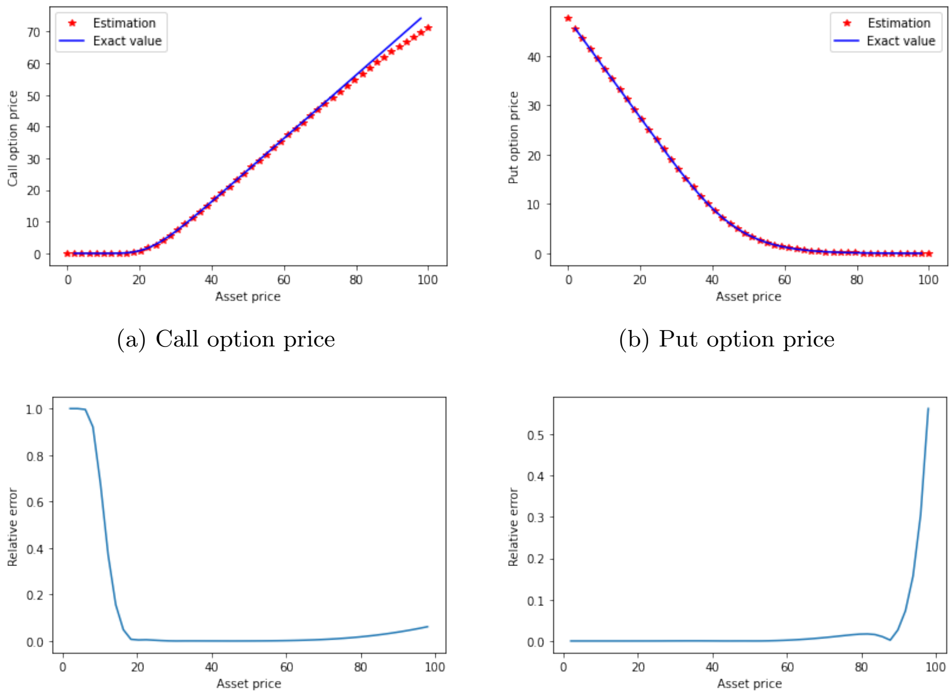
The results of this comparison can be seen in figure 1.

To evaluate the efficiency of this method, a relative error has been used as follows:

$$\frac{|\Omega - \omega|}{|\Omega|}, \tag{20}$$

where Ω is the actual value and ω is the estimated value. Since the actual amount of options in the range between zero and twenty for the call option and ninety to one hundred for the put option is close to zero and is under the ratio, the relative error does not show the correct value, but the relative error in the general case expresses the efficiency of the rolling method used.

To better understand the algorithm of this method, put option Python code has been put in the Appendix (section 6) as an example.



(a) Call option price (b) Put option price

(c) Relative error of Call option (d) Relative error of Put option

Figure 1: Call and Put option price

4 Numerical results

In this section, we intend to run the obtained results, and Python software will be used to display the results. Real data from China's natural disasters have been used for the practical study. Data extracted from https://public.emdat.be.

As stated in Section 2.1, insurance companies estimate the damages that may result from natural disasters, according to the insured, before any natural disasters occur. Then, the amount of damages that the insurance company can pay is deducted from the total estimated amount. In the next step, without losing the whole issue, the SPV issues only one CAT bond with the estimated damage price (this one CAT bond can be divided into any number, and the price is divided by the same number). According to the mechanism of this financial instrument, the amount of Payoff at maturity is calculated by deducting the amount of loss over the specified threshold from the price of the CAT bond (for example, in the chart 3 and 5 the red line of the chart is Payoff). With this payoff, we will price the CAT bond using the solution method provided at time zero or conclude the contract.

First, we want to price the CAT bond of the China storm. Consider the historical data of figure 2.

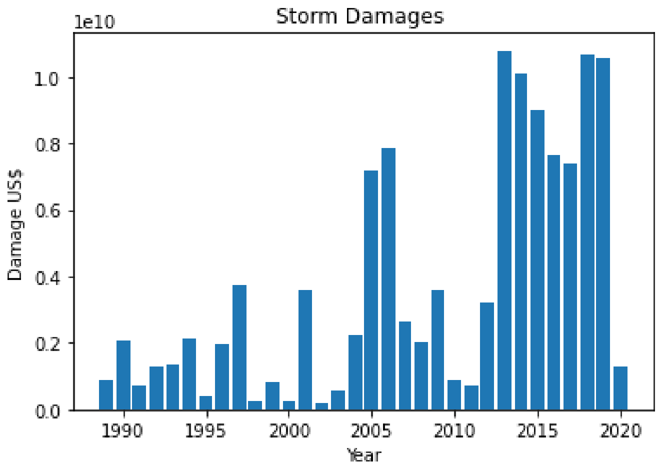


Figure 2: Storm damages

The average loss over the years is 3.7×10^9\$. Suppose the ability of insurance companies to pay compensation is 6×10^9\$; on the other hand, the losses of recent years have reached 10^{10}\$. Therefore, insurance companies request SPV to issue CAT bonds to cover this risk for 4×10^9\$. Using the parameter estimation method by historical data in Bjork's book [2], the drift value is 0.0248. Assume an interest rate is 0.05 and the market price of risk is 0.4. We also consider 500 discretization points to obtain a smooth diagram. The price chart of this bond can be seen in figure 3.

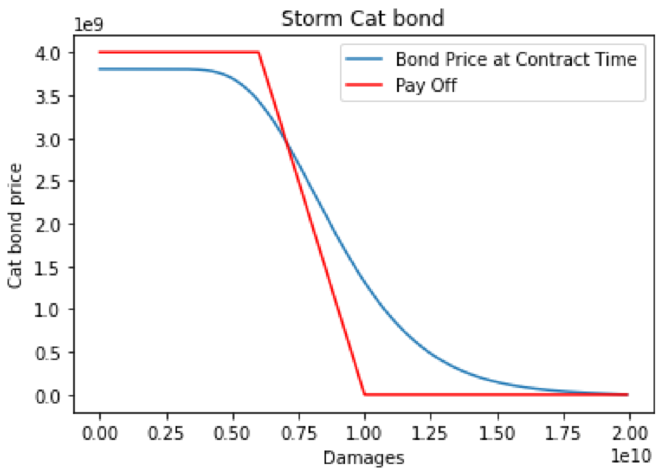


Figure 3: Storm CAT bond

Figure 4 is a graph of flood damage in China. The average loss over the years

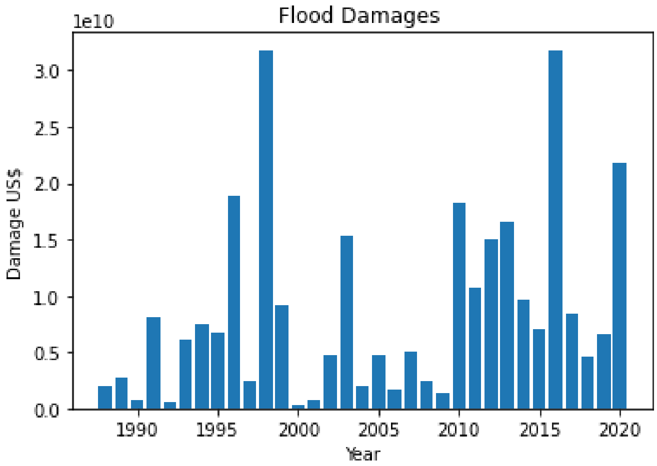


Figure 4: Flood damages

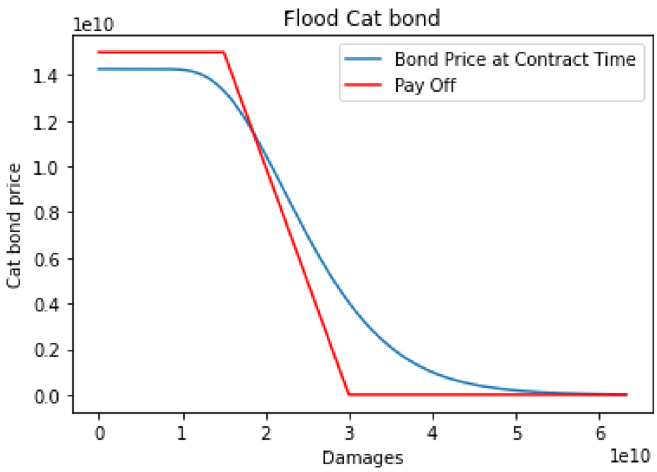


Figure 5: Flood CAT bond

has been approximately 8.7×10^9\$, and suppose the insurance companies can pay only 1.5×10^{10}\$. Since the amount of damages has reached 3×10^{10}\$, using the SPV, the insurance companies issue the 1.5×10^{10}\$ CAT bond, and the price chart can be seen in the figure 5.

As can be seen from the figures 3 and 5, the price of the CAT bond decreases as the damage increases. In the figure related to the storm, before the point 4×10^9\$ and in the figure related to the flood, before the point 1×10^{10}\$, the price graph is like a bond with a fixed-rate. As the results show, the price of the CAT bond is initially below the payoff graph, but from somewhere, it is above the graph. The specifications of the device running this program are Pentium (R) CPU 2117U @ 1.80GHz. Although the system is not robust and the number of discretization points is 500, the mean execution time of the program was less than 1.5 seconds, which indicates the high efficiency of this method and is much faster than other methods, such as the numerical method. As discussed, the error value is meager.

4.1 Run time comparing

In this section, we analyze the time complexity of this method, and for this purpose, two numerical methods are considered. We examine the finite difference and radial basis function approximation run time and compare it with our approach using flood and storm computation.

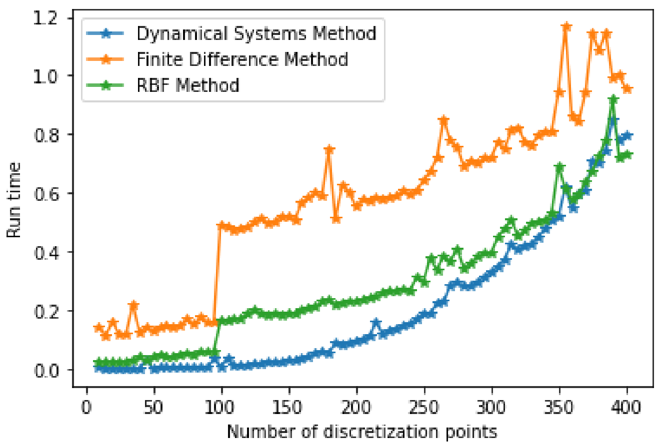


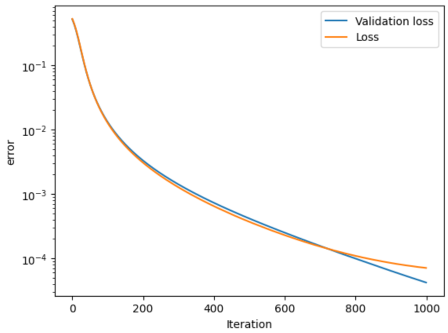
Figure 6: Run time

As shown in figure 6, the dynamic system approach is faster than other numerical methods. In general, for each amount of discretization, the runtime of our method is shorter than the finite difference and RBF method. Although by growing the number of discretized points, the dynamic system method run times converge to the RBF methods spent time, it has an actual distance to the finite-difference approach.

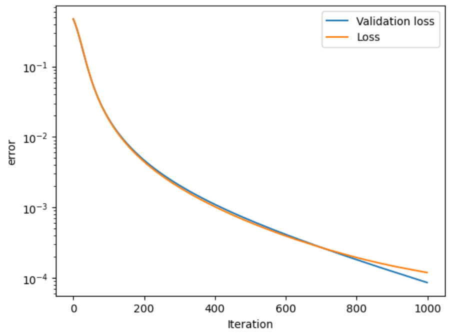
4.2 Cross-validation of volatility estimation

Cross-validation is a statistical method used to test a prediction model's performance. It separates the available data into two sets: the training set, which is used to train the model, and the validation set, which is used to assess the model's performance. The procedure is performed several times, with each iteration including a distinct data division into training and validation sets. The findings are then averaged to assess the model's performance on new, unknown data. Cross-validation is an effective method for evaluating the generalization capabilities of a model, and it is commonly used in machine learning and data science applications.

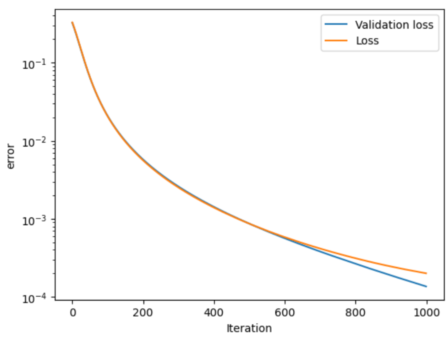
To evaluate the performance of RBFNN, we analyze the cross-validation of the network in figure 7:



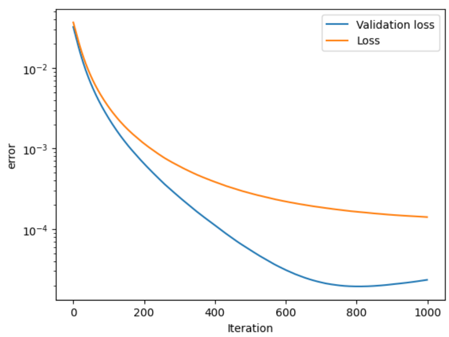
(a) Trained by 80% of data and validated by 20%



(b) Trained by 60% of data and validated by 40%



(c) Trained by 40% of data and validated by 60%



(d) Trained by 20% of data and validated by 80%

Figure 7: Loss function and Cross-validation

As seen in the above figure, the network has very high efficiency for different amounts of training data. It should be noted that even 20% of the data can make a reasonable estimate, indicating this network's proper performance. The results in the figures 7a, 7b, and 7c almost have the same behavior. The noteworthy point is that for 20% training data, the cross-validation value is much lower than the cost function. Very seldom, the cross-validation score will be lower than the loss function value, particularly when the model is not overfitting the training data. Overfitting happens when the model is too complicated and captures noise and random changes in the training data, which may result in a high level of accuracy on the training set but poor generalization performance on new data. In such circumstances, the loss function value may be small, suggesting a good fit to the training data, yet the cross-validation score may be significant, indicating poor generalization performance. Consequently, the loss function and cross-validation are essential to evaluate the model's performance and minimize overfitting. Therefore, in general, we can conclude that this network is reliable.

5 Conclusion

This research investigates the modeling and value of catastrophe bonds using dynamic systems. Since the frequency of natural disasters around the globe increases each year, and they inflict significant harm to insurance firms, it is vital to employ financial instruments to avoid insolvency due to these occurrences. Catastrophe bonds are instruments that shift insurance damage risk to the market. In this study, we first use mathematical principles to derive a pricing PDE and then present a dynamic system to solve it. In this research, the damage model is supposed to follow a random process. Using historical data and a radial basis neural network, we achieved an appropriate estimate of the volatility parameter to improve the accuracy of the investigated damage model. Finally, in the numerical results, considering the historical data of floods and storms in China, the price structure for these bonds was discussed. Furthermore, we discovered that our method is quite effective regarding time complexity compared to other methods.

This article provides a revolutionary pricing technique applicable to several financial goods. This methodology is not straightforward for PDEs with large dimensions, but it may be combined with another numerical method to address the issue.

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6 Appendix

We can see put option Python code in the following:

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import norm
from scipy.linalg import expm


def d1(S, K, r, v, t):
    return (np.log(S/K) + (r +
0.5*v*v)*t)/(v*(t**0.5))

def d2(S, K, r, v, t):
    return d1( S, K, r, v,t)-
(v*(t**0.5))

def exactPutOption(S, K, r, v, t):
    return K*np.exp(-r*(1-t))
*norm.cdf(-d2( S, K, r, v, 1-t)) -
norm.cdf(-d1( S, K, r, v, 1-t))*S


def BC1(t, k, s, r):
    return np.exp(-r*t)*k

def dBC1(t, k, s, r):
    return -r*np.exp(-r*t)*k

def BC2(t, k, s, r):
    return 0

def dBC2(t, k, s, r):
    return 0

def expA(s, n, r, sigma, t):

    ds = s[1]-s[0]
    a =
0.5*sigma*sigma*s*s/(ds*ds)-
r*s/(2*ds)
    b = -sigma*sigma*s*s/(ds*ds)-r
    c =
0.5*sigma*sigma*s*s/(ds*ds)+r*s/(2*
ds)
    a[-1] = 0
    a = a[1:]
    b[0] =
dBC1(t,k,s[0],r)/BC1(t,k,s[0],r)
    b[-1] = 1
    c[0] = 0
    c = c[0:-1]
    A = np.diag(b)+np.diag(a,k=-
1)+np.diag(c,k=1)

    return expm(A*t)

def putOption(s, n, r, k, sigma,
t):

    Z=expA(s, n, r, sigma,t)

    IC=k-s

IC=np.max(np.hstack((IC.reshape((n,
1)),np.zeros((n,1)))),1)

    return Z@IC

smax = 100
# smax ishe maximum value of the
asset.
k, r, sigma, n, t = 50, 0.05, 0.25,
50, 1     # k is Strike price, r is
interest rate,

# sigma is volatility, n is the
asset price

# discretization, t is maturity
time.
s = np.linspace(0,smax,n)
PU = putOption(s, n, r, k, sigma,t)
plt.plot(s,PU,'r*',label='Estimatio
n')

Pu = exactPutOption(s[1:-1], k, r,
sigma, t-1)
plt.plot(s[1:-
1],Pu,'b',label='Exact value')
plt.ylabel('Put option price')
plt.xlabel('Asset price')
plt.legend()
plt.show()

error = np.abs(PU[1:-1]-Pu)
plt.plot(s[1:-1],error/Pu)
plt.ylabel('Relative error')
plt.xlabel('Asset price')
plt.show()
print(np.mean(error))
```

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