Journal of Mathematics and Modeling in Finance (JMMF)
Vol. 1, No. 2, Summer & Autumn 2021
Research paper

Network centrality and portfolio optimization using the genetic algorithm

Asghar Abolhasani Hastiany[1], **Alireza Zamanpour**[2]

[1] Economics Department, Payame Noor University, Tehran, Iran
Abolhasani2003@yahoo.com

[2] Financial Engineering Department, Islamic Azad University, Islamshar Branch, Tehran, Iran
Alireza_zamanpour@yahoo.com

Abstract:
This study aims to optimize the portfolio using the genetic operator and network centralization approach. The statistical population of the study is the top 50 companies of Tehran Stock Exchange, in the first quarter of 2021, and to calculate the size of centrality, we used the difference in the overall performance of each company in comparison to all the top companies, based on a standard hybridization indicator. Then based on the companies performance in the capital market, the geometric mean of risk, and return of efficient companies are determined, and given the real-life limitations of the budget, the requirements and expectations of the investors in comparison to the markets performance, and the risk-free investment, the decision-making problem for the composition of the investment is formulated, in the form of a multi-purpose paradigm. We optimized the investments composition by using the modified optimization algorithm and the genetic algorithm with dual operators. Finally, we evaluate the effect of individual and systemic operators on determining the investment strategy by using the compound linear regression with data analysis approach to, and the represented results indicate the positive effect of these two operators.

Keywords: Portfolio optimization, Network centralization, Genetic algorithm, Risk, Return volatility
MSC2010 Classifications: 65Nxx, 65M10.

1 Introduction

Nowadays, various mathematical programs are used to determine the optimal combination of investment or the optimal portfolio. In analyzing the stock market, both statistical and mathematical methods are studied. One of the mathematical methods of analyzing the stock market is using different indicators of the initial multi-objective mathematical scheme of mean-variance portfolio optimization (MVPO), which can be divided into four categories: 1) convergence-based indicators, 2) diversity-based indicators 3) hybridization-based indicators and 4) risk-adjusted indicators.

[2]Corresponding author
Received: 2021-11-26 Accepted: 2021-12-30
http://dx.doi.org/10.22054/jmmf.2021.13844

Convergence-based indicators show the approximate proximity of theoretical Pareto optimality level. In this regard, we can mention indicators such as the mean of Euclidean distance, variance of return error, mean of return error, average error percentage, Epsilon (Liagoras 2018;) and the like.

The second category is the diversity based indicators, which indicate the distribution of the obtained investment combinations along the level or range of the Pareto hybridization principle. In this regard, we can mention indicators such as quantitative distance and scattering measure (Sotiwong and So Daniel 2016) and such.

The third category is hybridization-based indicators, which represent the combination or hybridization of the two categories of convergence and diversity-based indicators. In this regard, we can mention indicators such as Hypervolume indicator and the like. We describe the most important of these indicators in the following:

A) Hypervolume indicator:

The first of the hybridization indicator we talk about is the Hypervolume indicator. This indicator measures the volume of a multi-dimensional region dominated by a set of non-dominant solutions and it is provided by a multi-objective algorithm (Zeitzler and Tille 1999). Higher values of this indicator represent better approximation of the set of answers or the investment combination.

B) $D1_R$

The second case of diversity-based indicators is "$D1_R$". This hybridization based indicator provides information about the average distance between the closest solution and the forward convergence of the optimal solution based on the Pareto principle (Levin et al. 2017).

1.1 Risk-adjusted indicators

Risk-adjusted indicators are the fourth category of indicators that measure the performance of the portfolio optimization algorithm. They represent a combination or a hybridization of return and risk; these indicators simultaneously show the positive effects of gain and the negative effects of loss or risk. In this regard, we can mention the Sharpe index and such. We describe the most important of these indicators in the following:

A) Sharpe ratio:

The first of the risk adjustment indicator we talk about is the Sharpe ratio index. This risk-adjusting indicator is used to measure the adjusted return of a risk-based investment combination Sharpe (1966). In other words, using the Sharpe Ratio Performance Indicator or measure (e.g., in purchasing securities), one can calculate how the return can hedge the investors expected risk.

The higher Sharpe ratio means better performance of the chosen investment combination, in the financial decision-making for the determined decision.

B) Omega ratio:

The second risk-adjusted indicator is the omega ratio indicator. This risk-adjustment-based indicator records all the information of changing to higher return in distribution of the return and is also sensitive to the return level. While Sharpe risk-based performance indicator requires assuming an average structure for variance and the input data that is usually distributed (Ban et al. 2018).

In addition to mathematical methods, there is a group of statistical methods based on autoregressive moving average (ARMA), autoregressive integrated moving average (ARIMA), generalized autoregressive conditional heteroskedasticity (GARCH) (Frances and Chichesles 1999), and smooth transition autoregressive (STAR) (Sarantis 2001), all of which use delay-dependent variable structure. Other types of statistical methods that have been used in recent years include linear detachable analysis (LDA) quadratic discriminant analysis (QDA), linear regression (LR), and support vector machines (SVM). Each of these methods usually consist of multi-input variables. In addition most of the mentioned methods are limited by linearity and the linear independence of explanatory variables in the field of financial predicting. On the contrary, artificial intelligence models such as artificial neural networks (ANNs), Fuzzy systems, and various genetic algorithms are used based on multi-variable data and without any specific assumption. Many of these methods have also been used in the stock market to predict financial variables.

Usually, stock market scheduling systems are used to make a system that supports independent or compatible decision-making, regarding the trading rules. For example, in this field, we can mention researches performed by Barak, Danayi and Techi (2015), Serovolonov, Guyarro and Mishinyuk (2015), Chen and Chen (2016), Chiang, Anke, Wu and Wang (2016), Chormozidis And Chetzoglu (2016). Multivariate analysis using artificial neural networks (ANN), based on nonlinear, data, and generalizable methods, has become a popular and dominant tool in finance and economics. Stock markets are affected by various factors, most of which are used as possible input variables during the development of the stock market prediction system. Therefore, if you expect an efficient and accurate prediction in using ANN, you need to choose the effective and representative inputs from various predicting measures. These sort of choices are the main function of dimension reduction technology.

Dimension reduction can be performed in two different ways: either by selecting relevant variables from the original data set (which is usually referred to as feature selection) or by generating a small group of new variables, each of which is a specific combination of older input variables. Researchers in statistics, computer science, and applied mathematics have worked many years in this field, and have identified and used various linear and nonlinear reduction methods. Sorzano, Vargas, and Pascal-Montano (2014), also classify many dimension reduction methods in the related mathematical insight.

1.2 Optimal portfolio and network centrality

Nowadays there is a wide debate concerning the network. In particular in sociology, the circumstance of measuring the centrality of a particular factor that exists within a network of relations is discussed. The importance of such criteria stems from the implicit assumption of added power or the situation associated with individuals is highly focused. Despite its intuitive meaning, the concept of centrality is somewhat ambiguous and its measuring depends on the specific fundamental process that is used. For instance, in a social network, the time factor that interacts with other factors is considered the central factor. While contrary to the mentioned case, in a bargaining process, the centrality of the i-th factor is derived from its relation to other non-central factors.

Bonasic (1972), in his paper explaining centrality, proposes a centrality criterion that has become a standard for determining centralization in the network literature. Further he discusses this concept in a financial market and determines its relationship with the weights used to determine an optimal portfolio.

1.3 Measuring centrality

Generally, a network is an ordered pair from the set of $G = N, \Omega$ in which $N = \{1, 2, ..., n\}$ is defined as a set of nodes and Ω is a set of relationships between each ordered pair of this set. Now if we assume there is a relation from node i to node j, then $(i, j) \in \tilde{\Omega}$. An appropriate method of sorting the information in $\tilde{\Omega}$ is to use the mean values of the adjacent points' matrix, where $\Omega = [\Omega_{ij}]$. Ω is an $n * n$ matrix in which $\Omega_{ij} \neq 0$ indicates the existence of a relation between i and j nodes.

If $\Omega \neq \Omega^T$, it is called an oriented network, therefore if $(i, j) \in \tilde{\Omega}$, it automatically indicates that $(j, i) \in \tilde{\Omega}$. Note that for undesirable networks, there is no causal relation between the links, and these relations are visually represented as the $(j - i)$ line. On the other hand, if $\Omega \neq \Omega^T$ be an oriented network and Ω_{ij} indicates a causal relation from node j to node i, which may not necessarily exist inversely. In this case, relations between the nodes are shown as arrows i.e. $(j \to i)$. In addition, if $\Omega_{ij} \in \{0, 1\}$ then G is called non-weighted. However, when $\Omega_{ij} \in \mathbb{R}$ he relations between nodes in the network convey information related to the intensity of the interaction between the nodes that lead to a weighted network. For a detailed discussion in this context, we refer the reader to the articles of Newman (2010) and Jackson (2010). As stated in Bonasics research (1972), in expressing real centrality, we assume the centrality of node i be ν_i. It is adequate to the sum of the central weights of the adjacent points as follows:

$$\nu_i \equiv \lambda^{-1} \sum_j \Omega_{ij} \nu_j \tag{1}$$

By substituting relation (1) in the matrix form, the eigen centrality of an evaluated source, ν, is defined based on a specific input Ω concerning the specific value λ, while the largest real value in this field is the preferred one and is defined as follows:

$$\lambda\nu = \Omega\nu \tag{2}$$

Definition 1.1. Suppose a non-oriented network and the weighted network $G = \{N, \Omega\}$, with N as a set of nodes and Ω as a matrix of adjacent points. Then centrality of the eigenvector i along with the i-th component of eigenvector Ω corresponds to the largest eigenvalueλ_1. While the set λ_1^{-1} is the proportional factor.

It should be noted that (1) indicates that each node can be considered centrality of the network, provided it is in association to other nodes (in positive range) or a few central points. This value is also calculated for weighted and non-weighted networks. However, for the mentioned oriented structure, such central measurement has some deficiencies that are not recommended for its implementation.

main result of selecting the optimal portfolio

A review of this researchs literature shows that the theory of portfolio optimization was first introduced by Markowitz (1952), subsequently, this theory has been used as a basis for the outline of the proposed model. We assume that in a portfolio we have risky assets with an expected return vector μ and a covariance matrix Σ. Then portfolio optimization will be defined as the problem of determining the desired weights' vector w, that minimizes this portfolios variance as the sum of portfolio risk, provided the sum of the assigned weights to each asset of the portfolio is equal to one in other words $w^T\mathbf{1} = 1$. This strategy is commonly known as the sum variance minimization strategy (risk sum) or in brief m-var. Therefore, the said strategy is defined in the form of a minimization program:

$$\begin{gathered} \min \sigma_p^2 = w^T \Sigma w \\ s.t. \\ w^T 1 = 1 \end{gathered} \tag{3}$$

And the optimal solution to the defined mathematical model in (6) is expressed as the quantity in (7) and is calculated as follows:

$$w_{mv}^* = \frac{1}{1^T \Sigma^{-1} 1} \Sigma^{-1} 1 \tag{4}$$

We assume that the matrix of return correlation is Ω, the standard deviation return of stock i is σ_i, and $\Delta = diag(\sigma_i)$. Finally, the relation between the correlation and the covariance matrix is obtained from $\Sigma = \Delta\Omega\Delta$. Therefore (7) with respect to Ω can be defined as followers:

$$\widehat{w}^*_{mv} = \varphi_{mv}\Omega^{-1}\epsilon \tag{5}$$

while $\widehat{w}^*_{mv} = w^*_{mv} * \sigma_i$, $\varphi_{mv} = \frac{1}{1^T\Sigma^{-1}1}$, and $\epsilon_i = \frac{1}{\sigma_i}$.

Considering the problem defined in (6), which includes a risk-free asset with the return r_f. Therefore, the defined portfolio is a combination of $n+1$ assets, n represents risky assets and 1 is the risk-free asset. In this case, the excess return of asset $i(r_i - r_f)$ is represented as r^e_i and the excess return vector reaches to μ^e. The problem of minimizing the variance of the portfolio for a certain level of excess return R^e is expressed as follows:

$$\begin{gathered} \min \sigma_p^2 = w^T\Sigma w \\ s.t. \\ w^T\mu^e = R^e \end{gathered} \tag{6}$$

The investment strategy defined in (10) is known as the mean-variance strategy or M-var. Of course, we should note that $w^T\mathbf{1} = 1$ is not a limit in (10), since part of the investor's wealth can be assigned to the risky asset, then $w_f = 1 - w^T\mathbf{1}$. However, when we consider the set of investment portfolio, $w_f = 0$. Anyways, the optimal solution for the M-var strategy will be obtained as follows:

$$w^* = \frac{R^e}{\mu^{eT}\Sigma^{-1}\mu^e}\Sigma^{-1}\mu^e \tag{7}$$

Following the same logic as before, (11) can be transformed to the correlation matrix format as follows:

$$\widehat{w}^* = \varphi\Omega^{-1}\widehat{\mu}^e \tag{8}$$

while $\widehat{w}^*_i = w^*_i * \sigma_i$, $\varphi = \frac{R^e}{\mu^{eT}\Sigma^{-1}\mu^e}$, and $\widehat{\mu}^e_i = \mu^e_i/\sigma_i$.

1.4 The relation between optimal portfolio weights and stocks' centralities

Suppose a network of the financial market is defined as $CM = \{N, \Omega\}$ in which N is a set of stocks and Ω is the adjacent points matrix defined by the return correlation matrix. If the main diagonal of the matrix Ω leads to zero, there will be no need to deal with the absurd structure of the connection circles in this network. Of course, it can be proved that this neither changes the structure of the eigenvector nor its' purpose. Thus, Theorem 1 and conclusion 1 show that the relation between the desired weight of asset i, is obtained from a certain range, and the desired or optimal weights in making this decision, are based on m-var and M-var investment strategies.

Theorem 1.2. *If we assume $CM = \{N, \Omega\}$ be a financial market network and correspondingly $\{\nu_1, ..., \nu_n\}$ and $\{\lambda_1, ..., \lambda_n\}$ depending on the case be a set of special inputs and eigenvalues of matrix Ω. Then, the desired securities' weights or the desired level of the selected assets in the optimal portfolio of (9) and (14) can be defined as follows:*

$$\widehat{w}^*_{mv} = \varphi_{mv}\epsilon + \varphi_{mv}\big(\frac{1}{\lambda_1} - 1\big)\epsilon_m v_1 + \Gamma_{mv} \tag{9}$$

$$w^*_{mv} = \varphi\hat{\mu}^e + \varphi\big(\frac{1}{\lambda_1} - 1\big)\hat{\mu}_M * ev_1 + \Gamma \tag{10}$$

while $\epsilon_M = (v_1^T\epsilon)$ and $\hat{\mu}^e_M = v_1^T\hat{\mu}^e$.

Theorem 15 clearly determines the relation between the optimal weights and the first special input of the correlation matrix, which represents the principal assignor centrality in network theory. In addition, from the viewpoint of principal component analysis, it can be interpreted as the inverse of the standard return deviation and the return to risk ratio in the market. In (17) and (21), the immature idea of investing in an asset that merely the risk or the return to risk ratio associated with it is considered. In this case, by decreasing or increasing the standard return deviation (return to risk) for the i-th asset, the corresponding weight in the optimal portfolio depending on the case should decrease or increase. Here this is referred to as the individual performance of the asset i. In the case of substituting the relation by (17) and (21), the corresponding centrality to each asset can be calculated. Therefore, we will consider the calculated centrality as the systemic performance of asset i. Corollary 25 creates a negative relation between the assigned money to a particular asset and its particular centrality.

Corollary 1.3. *Assume $\lambda_1 > 1$, ϵ_M, and $\hat{\mu}^e_M$ are positive quantities, then the exclusive large centrality of the eigenvector of stock i with low weights would be the desired portfolio in either of m-var or M-var strategies.*

Corollary 25 states that under plausible conditions, those stocks that account for a large amount of investors investment in an optimal portfolio will be placed towards the outer part of the network. This corresponds to the results of the study of Poozi et al. (2013). However, in the said study, individual performance, as well as interaction with systematic (central) performance is completely eliminated. Peralta and Zareei (2014), provided evidence that this is the relation of time and market dependency. Therefore, there exist time periods in which most of the core (central) systems are also the best individual assets that lead to dispute in choosing the investment type.

2 Different models in portfolio optimization

In this section, we describe the various models used in portfolio optimization based on the researches conducted in this field.

Single-objective optimization model

The main mean-variance model (MV), which aims to minimize investment risk (variance or dispersion of return) for the desired return level, is considered a single-objective model. In this model, the parameters used in decision-making are the number of available assets (investment fields), N, expected return of i-th asset (investment field), μ_i, the covariance of i-th and j-th asset (investment field),σ_{ij}, and the desired level of expected return, R^*.

In addition to the decision-making parameters that are obtained based on performance data and obtained from the above definitions, w_i is also a decision variable in this optimization scheme, which is defined as the percentage or relative share of investment in the i-th type asset (investment field). Accordingly, considering the stated parameters and variables, the optimization model is defined as follows:

$$\min(c) = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} \tag{11}$$

$$\begin{aligned} & s.t. \\ & \sum_{i=1}^{N} w_i \mu_j = R^* \\ & \sum_{i=1}^{N} w_i = 1 \\ & 0 \leq w_i \leq 1, \quad i = 1, ..., N \end{aligned}$$

In this scheme, the objective function is defined based on risk minimization (of sum of covariances). In this scheme, the first restriction is based on obtaining the return at the desired return level, the second restriction refers to the investment combination, which is equal to 1 or 100 percent overall, and the third restriction states the relative share of investment in each asset or investment field in this portfolio as a relative quantity between 0 and 1 or percent-wise between 0 and 100 percent.

The principal single-objective mean-variance investment model can also be rewritten based on maximizing the expected return in the sum of investment for a given risk level. In either case, the obtained portfolio by solving (27), is called an efficient set by considering the minimum risk for a certain return level or the maximum of return or the investors expected return and for a certain level of risk.

However, to find an efficient portfolio, the risk tolerance level of the investor, or the investors desired return, should be determined. In fact, such conditions may not be a plausible in reality. Therefore, to find an efficient portfolio among different combinations of investments in rational space, instead of considering a single goal, researchers should consider all goals at once. Therefore, to solve the problem of considering the decision making criteria, risk and expected return in financial decision-making at once and even in some cases considering other goals rather than risk and return, researchers have converted the single-objective model into a multi-objective model (Clichy et al. 2019).

Multi-objective optimization model

Reviewing the research literature shows that based on Zeitzlers (1999) approach, the mathematical approach of multi-objective mean-variance portfolio optimization (MVPO) is defined as follows:

$$\begin{gathered} \min f(x) = \{f_1(x), f_2(x), ..., f_p(x)\} \\ s.t. \\ e(x) = \{e_1(x), e_2(x), ..., e_m(x)\} \leq 0 \\ x = (x_1, x_2, ..., x_n) \in X \end{gathered} \tag{12}$$

In this approach, $f_1(x)$ to $f_p(x)$ are different objective functions that can be defined based on risk or other objective criteria (of course minimization criteria). $e_1(x)$ to $e_m(x)$ are different constraints that must be considered in choosing a portfolio or investment combination and can be defined based on investment budget, investment expectations, combination restrictions, and such. Eventually, the decision variables are $x = (x_1, x_2, ..., x_n)$, which are based on the amount of investment in the asset or the evaluated stock as an investment option and are defined relatively. The defined constraints are in fact the justified space or the answer and in other words the possible combinations of investment or different investment portfolios in decision-making.

A) Feasible set:

A feasible set x_f, is defined as a vector of decision variables (here an investment combination or portfolio) in which all the limitations of the investment model are considered and in other words, the obtained answer for the amount of investment in each asset to choose the desired portfolio applies to all restrictions, and at the same time, none of these values are negative as feasible levels of investment in each asset or company shares.

B) Pareto Dominance:

In the multi-objective scheme in (29), based on Debbs opinion (2001), Pareto's dominance principle is considered in finding the optimal solution for selecting the desired investment combination, and a set of answers (of an investment combination) is called the reference combination, provided it contains a smaller amount of objective function than the other combination.

C) Pareto Optimality:

In the multi-objective scheme in (29), according to Zitzlers opinion (1999), Paretos optimality principle is considered in selecting the final investment combination or the desired portfolio, and a set of answers (of an investment combination) is called the optimal combination, provided it contains a smaller amount of the objective function than all other combinations.

Methods for solving multi-objective models

In multi-objective models, optimization of all objective functions may not be possible at once. Therefore, it is necessary to determine the priorities based on management or decision-making preferences, by using methods such as the weighted sum of functions or applying approaches based on the dominance of some goals over others; and to solve the problem based on these priorities.

A) Weighted sum approach:

In the weighted sum of objective functions method, a set of objectives are combined into a single objective function by assigning a related weight as the priority preference of one over another. Due to its simple structure and ease of implementation, in addition to the most use of the classic approach for solving multi-objective optimization problems (Debb 2005), this approach is the most popular solution for multi-objective problems in optimizing portfolio based on the mean-variance paradigm. However, despite the simplicity of this method, there exist major problems for optimizing the multi-objective model, by using this method to achieve Paretos optimal solutions, we have an optimal Pareto non-convex solution space. Thus, the main disadvantage of the weighted sum approach is that the said approach cant produce all Pareto's optimal solutions that coincide with the non-convex solution space levels (Zeitzler 1999).

Regardless of the limitations and complexities of complying with contradictory goals in the objective function on the one hand and considering the limitations related to real-life realities on the other hand, in any case, the general framework of the weighted sum of the objective functions in solving multi-objective portfolio optimization problems with the mean-variance criterion is expressed as follows:

$$\min(C) = \sum_{i=1}^{p} \lambda_i f_i(x) \tag{13}$$
$$s.t.$$
$$x \in X_f$$

While in this relation λ_i is the weight assigned to each of the previous objective functions based on the degree of importance or its preference over other defined objectives. The mentioned weight coefficients or preferences can be determined by methods based on expert surveys and approaches like fuzzy logic. The mean-variance portfolio optimization (MVPO) multi-objective mathematical paradigm can be formulated based on the weighted sum approach of the objectives as follows in (31). In this paradigm, two main goals are considered, minimizing the risk and maximizing the return to optimize the portfolio, which are conflated as a sum function with the management preferences (Chang et al. 2000).

$$\min(C) = \lambda\left[\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij}\right] - (1-\lambda)\left[\sum_{i=1}^{N} w_i \mu_i\right] \tag{14}$$

$$s.t.$$

$$\sigma_{i=1}^{N} w_i = 1$$

$$0 \leq w_i \leq 1, \quad i = 1, ..., N$$

As can be seen in (31), two opposing objectives (minimizing risk and maximizing return) are combined by a parameter λ. The weight parameter λ takes different values between 0 and 1. While the objective function of the model seeks the maximum of the return on one hand and the minimum of the risk on the other, λ obtains an exchange between risk and return.

B) Pareto-based approaches:

Approaches based on the Pareto principle can control large search spaces and exchange between multiple option trades at the same time in a single optimization implementation (Zeitzler 1999). In this approach, unlike the objectives weighted sum approach, which converts a multi-objective structure into a single objective structure, there is no united criterion for evaluating the quality of exchange between objectives. Defining qualitative evaluation criteria in this method is relatively difficult.

In approaches based on the Pareto principle, usually, a solution ranking strategy based on the concept of Pareto optimal principle is used (Horn et al. 1994). Many multi-objective algorithms are based on Pareto ranking, however many alterations such as 1) Dominance depth algorithm (Deb et al. 2002) and 2) Dominance dimensions algorithm in optimization, are used more than other algorithms (Zeitler et al. 2001). Based on the proposed paradigm by Levin et al. (2014), mean-variance portfolio optimization (MVPO) multi-objective mathematical paradigm based on the Pareto principle can be presented as follows:

$$\min \left[\sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_{ij} \right] \quad \text{and} \quad \max \left[\sum_{i=1}^{N} w_i \mu_i \right] \tag{15}$$
$$s.t.$$
$$\sum_{i=1}^{N} w_i = 1$$
$$0 \leq w_i \leq 1, \quad i = 1, ..., N$$

As can be seen in the objective function (32), two opposing objectives: 1) minimizing the risk and 2) maximizing the return, are evaluated independently to achieve the optimal investment combination in a paradigm based on the Pareto dominance principle.

Case studies based on performance data in different capital markets show that modeling the optimal investment combination to "determine the preference of single-objective or multi-objective paradigms" has a basic assumption. The said basic assumption is that investors are aware of the desired risk or return level of single-objective models. Therefore, multi-objective models seem more realistic than single-objective models.

2.1 Research methodology

This research is based on a theoretical inference method to find a new and indigenous model suitable for Irans capital market conditions to calculate network centrality and portfolio optimization using genetic algorithm, therefore the research can be considered as a theoretical research in this regard. On the other hand, designing the model and employing it to help the investors and the capital market to make better investment decisions, and therefore this research can be considered an applied research in terms of purpose. The statistical population of this study as discussed later (50 top listed companies in the first quarter of 2021) is compatible with the statistical sample and using random methods in studying a section of the statistical population as a random sample is not objectified and mathematical optimization methods have been used to select the optimal portfolio. Accordingly, during the inference, the aim was not to generalize and disseminate the results, and the used tools were descriptive, in other words, descriptive inference method has been used. In the following, the general framework, measuring, and determining relations between variables and in a way the proposed research model is discussed.

2.2 Calculating the centrality

In this section, using the "portfolio optimization" approach, the centrality criterion for each company is calculated and based on the regular and logical algorithm that is used, the obtained findings are described. To determine the centrality, first, we normalize and integrate the data, then based on the difference in the overall performance of each company in comparison to all the top companies, including the performance of the company that is under evaluation, and relying on the standardized integrated criterion of performance, we calculate the centrality size of each company and then rank it relatively. In other words, the difference in the option's return that is under survey compared to all other justified options, in investment decision-making has been used.

In this regard, first: the difference of the hybridization based indicators for each company, and including the company itself, is calculated and is determined as a positive number (absolute value). Second: the sum of performance differences is calculated by summing up the calculated values per 50 companies. Third: the obtained sum for each company is divided by the obtained sum for all companies (rounded by 4 decimals) and controlled (the obtained sum of numbers for all companies is equal to 1). Fourth: Considering that the most optimal performing status is for the minimum value calculated in the previous step, complement calculated, which means the obtained numbers in calculating the previous division are subtracted from the maximum value of the previous column and stated as a positive number. Finally: Standardized, which means the obtained complement is divided and controlled on the sum of complements of 50 companies and is divided as the centrality measure of the evaluated company. The obtained numbers are positive with 5 decimals and their sum is 1 and they are somehow standardized.

It should be noted that the centrality measure per company as a relative quantity is between 0 to 1 and in sum or for all companies equals 1.

2.3 Determining the optimal strategy based on the mean-variance paradigm

In this section, research findings are analyzed based on Markovitzs portfolio theory (1952), mean-variance mathematical optimization paradigm, and applying real limitations in the top companies are studied according to performance data in time period. In this regard, first objective criteria in modeling including risk and return are defined and measured, then decision variables are defined, the objective function and limitations are stated, and eventually, by mathematical optimization, the final model is solved and optimal strategies of investing are determined.

Therefore, by using the research background, the return criterion is selected as one of the most important indicators affecting investment decision-making, and in calculating the average return, Barak and Modares (2015) model has been used as follows, in which R is the average stock return for the studied time periods;

$$R = \sqrt[n]{(1 + \frac{r_1}{100})(1 + \frac{r_2}{100})...(1 + \frac{r_n}{100})}$$

In other words, to calculate the average return the geometric mean method has been used and here, $r_1, ..., r_n$ are the stocks real return from the first to the n-th time period. In this regard, the one-year performance up to 19/3/2021 for the studied companies has been considered and in these 12 months the monthly periods have been the criterion of action and to calculate the return, stock price changes compared to past have been used. This means that the change percentage of the stock price (stock price at the end of the month subtracted from the stock price at the beginning of the month) is divided by the stock price at the beginning of the month and multiplied by 100 is expressed as a percentage, and finally, to be converted to the annual return it is multiplied by 12. The second criterion that has attracted investors in decision-making along with the return, has been the investment risk; according to Markowitz (1952), Nikzad and Zaranezhad (2012), Chang et al. (2012), Burke et al. (2014), Sadaf And Ghodrati (2015), Senol and Oztoran (2016), from which different interpretations have been expressed. In this study, based on Barak and Modaresis model (2015), the risk criterion is based on price changes and is calculated by the following formula:

$$\sigma = \sqrt{\frac{1}{n-1} \sum_{i=0}^{1} \left(r_i - E(r)\right)^2}$$

3 Modeling investment composition

At this stage of decision-making to determine the optimal investment composition based on the initial feasible space, and in other words, the decision made in evaluating the financial efficiency of selected companies and introducing the efficient companies as feasible investment options, modeling the composition of investment is defined as the final model by using these steps: 1) defining the decision variable, 2) defining the objective function, 3) identifying decision-making real constraints and 4) summing up the said steps.

3.1 The first step, defining the decision variable:

Following similar researches, including Chen et al. (2019), in this study, the decision variable is the relative investment between 0 to 1 of the total investment in the i-th efficient company and for each of the 50 companies that are ultimately selected as top stock companies and feasible investment option in determining the initial space (or decision-making) which is defined as follows:

X_i relative investment in the desired efficient company: $i : 1, 2, ..., 50$.

3.2 The second step, defining the objective functions:

Based on the two criteria, risk and return, the objective functions are defined based on Markowitz (1952) mean-variance paradigm as follows:

$$\max(R) = R_1X_1 + R_2X_2 + ... + R_nX_n$$
$$\min(\delta) = \delta_1X_1 + \delta_2X_2 + ... + \delta_nX_n$$

Here, R_i is the average monthly return of the i-th efficient company, X_i is the relative investment in the i-th efficient company, δ_i is the average monthly risk of the i-th efficient company, R is the average monthly return of the investment composition in the efficient companies and δ is the average risk of investment composition in the efficient companies. The investor wants to choose a combination of investments that has the most return and the least risk at same time.

3.3 The third step, applying real limitations:

By applying real limitations when choosing the portfolio or optimal investment composition, maximizing returns and minimizing risk in selecting the investment composition, the final decision space in selecting the optimal composition will be determined based on investment limitations. Obviously, these limitations are based on the circumstances of the person who decides and vary from person to person. Therefore, following the Chen et al. (2019), a number of limitations are mentioned here as examples:

A) Investment composition limitation: This limitation is affected by the definition of variables as relative quantity and in fact, is the relative investment in the investment portfolio or relative share of each top company from one investment unit, which assuming relative investment in other companies be 0, the maximum share of the i-th efficient company is equal to 1 and considering that the real decision variables are non-negative the minimum is defined as an unknown relative quantity as follows:

$$0 \leq X_i \leq 1, \quad i : 1, 2, ..., 50$$

B) Investment budget limitation: This limit is based on the existing or available amount of money as a ceiling or maximum of investment by a natural or legal person and varies from a person to the other. Here, according to the general assumption in the budget constraints of investment following Maghwani et al. (2018), we assume that the investor wants to buy a share that is relatively divided between different shares and practically with the individuality of the Rial budget considering dividing the Rial budget by the average price of a share we can calculate the number of final shares with the optimal relative combination. This number is multiplied by the optimal relative combination and the amount of the shares of each company from this specific composition will be determined, and by multiplying the amount by the daily price of the relevant share, the Rials amount of buying shares from each company in the optimal composition will be determined. Accordingly, the budget constraint is generally defined per share as follows:

$$X_1 + X_2 + ... + X_{50} = 1$$

C) Minimum return relative to the market limitation: This limit is based on the minimum risk-free return, for instance, the return on investment in a one-year bank deposit, which is 15% per annum and 1.25% per month according to the Central Bank, and based on this limit, the investment portfolio or composition should be determined in a way that the return on investment will not be less than the risk-free return. In other words:

$$R_1X_1 + R_2X_2 + ... + R_{50}X_{50} \geq 1.0125$$

D) Minimum return relative to the market limitation: This limit is based on the average performance in the capital market and is based on the assumption that in general the investment composition should be determined in a way that the minimum return on investment will not be less than the average return in the capital market. In other words:

$$R_1X_1 + R_2X_2 + ... + R_{50}X_{50} \geq \overline{R}$$

E) Maximum risk limit relative to the capital market: This limit is based on the average performance in the capital market and is based on the assumption that in general the investment composition should be determined in a way that the maximum risk on investment will not exceed the average risk in the capital market. In other words:

$$\delta_1X_1 + \delta_2X_2 + ... + \delta_nX_n \geq \overline{\delta}$$

3.4 The fourth step, final model:

According to variables definition, objective functions and investment limits, the final model of the optimal investment composition will be as follows:

X_i: The relative investment amount in the desired efficient company, $i : 1, 2, ..., 50$.

$$\begin{aligned}
&\max(R) = R_1X_1 + R_2X_2 + ... + R_{50}X_{50}\\
&\min(\delta) = \delta_1X_1 + \delta_2X_2 + ... + \delta_{50}X_{50}\\
&s.t.\\
&X_1 + X_2 + ... + X_{50} \leq 1\\
&R_1X_1 + R_2X_2 + ... + R_{50}X_{50} \geq 15.00\\
&R_1X_1 + R_2X_2 + ... + R_{50}X_{50} \geq \overline{R}\\
&\delta_1X_1 + \delta_2X_2 + ... + \delta_{50}X_{50} \leq \overline{\delta}\\
&0 \leq X_1 \leq 1, \quad i : 1, 2, ..., 50
\end{aligned}$$

4 Portfolio optimization

To select the desired option in investment decision-making, at this stage of the analysis, relying on the genetic algorithm and the following ultra-innovative algorithm, we optimize the portfolio in the form of determining the optimal investment composition of efficient companies, aiming to achieve maximum average returns and the minimum risk and observance of real limitations in decision-making.

4.1 Step 1, Optimization algorithm:

In this regard, the correction mechanism that Stretcher et al. (2004), Levin, Kuo and Kendall (2014), and Sculpadonekt et al. (2007) have used in their studies, is invented to manage budget, floor, and ceiling limits. It should be noted that budget limits are usually avoided unless $_i = 0$. Another modifying mechanism has been developed in Chang et al.'s (2000) study to comply with investment budget, capability, floor and ceiling limits. According to the algorithm used in this study, producing solutions in this study should be based on consistency and quantity. Then, the used algorithm is shown in table 35.

4.2 Step 2, Determining optimization parameters:

Relying on the ultra-innovative mathematical algorithm and using the data mining process of the genetic algorithm, the optimization parameters including the number of generations, the number of iterations, base population, etc. are defined as follows. In this study, the binary genetic algorithm is used. In other words, the genetic operation was not directly performed on the variables, but its coded in the base-2. Also, the production of the initial generation is performed randomly. The initial population size used in this study is 100. The termination condition in the algorithm is reaching a constant objective function or to reach the maximum number of generations, which is 200 in this method.

Table 1: Stock Composition Optimization Algorithm

Portfolio optimization modified algorithm
Proposed repair mechanism
rocedure Repair (ω, ϑ)
$\delta \leftarrow 0$
$I_{nz} = \{i \vert \omega_i > 0\}$
$r_i = (\omega \bmod \vartheta) \qquad \forall i \in I_{nz}$
$I_{LBV} = \{i \vert \omega_i - r < I_i\}$
if $\quad \vert I_{LBV}\vert = 0 \qquad$ then
$\omega_i \leftarrow \omega_i - r_i$
else
$\omega_i \leftarrow \omega_i + (\vartheta_i - r_i) \qquad \forall i \in I_{LBV}$
$\omega_i \leftarrow \omega_i - (\omega_i \bmod \vartheta_i) \qquad \forall i \in I_{nz}$
end if
$\beta = \sum_{i \in I_{nz}} \omega_i$
if $\beta > 1$ then
$a_i \leftarrow I_i + \vartheta_i - (I_i \bmod \vartheta_i) \qquad \forall i \in I_{nz}$
$\omega_i \leftarrow a_i + \vartheta_i + \frac{\omega_i - a_i}{\sum_{i \in I_{nz}}(\omega_i - a_i)}(1 - \sum_{i \in I_{nz}} a_i) \qquad \forall i \in I_{nz}$
else
$a_i \leftarrow u_i - (u_i \bmod \vartheta_i) \qquad \forall i \in I_{nz}$
$\omega_i \leftarrow a_i - \frac{a_i - \omega_i}{\sum_{i \in I_{nz}}(a_i - \omega_i)}(\sum_{i \in I_{nz}} a_i - 1) \qquad \forall i \in I_{nz}$
end if
$r_i = (\omega_i \bmod \vartheta_i) \qquad \forall i \in I_{nz}$
$\delta \leftarrow \sum_{i \in I_{nz}} r_i$
$I = \{i \vert \delta > \vartheta_i\}$
$\vartheta_{min} \leftarrow \min\{\vartheta_i \vert i \in I\}$
Choose an index k from $\{i \vert \vartheta_i = \vartheta_{min} : i \in I\}$
$I_D \leftarrow 0$
while $\delta \geq \vartheta_{min}$ do
$I \leftarrow I \backslash I_D$
if $\omega_k + \vartheta_{min} \leq u_k$ then
$\delta \leftarrow \delta - \vartheta_{min}$
$\omega_k \leftarrow \omega_k + \vartheta_{min}$
else
$I \leftarrow I \backslash \{k\}$
$I_D \leftarrow I_D \cup \{k\}$
end if
$I \leftarrow \{i \vert \delta > \vartheta_i\}$
$\vartheta_{min} \leftarrow \min\{\vartheta_i \vert i \in I\}$
Choose an index k from $\{i \vert \vartheta_i = \vartheta min : i \in I\}$
end while
end procedure

The number of elite chromosomes that will enter the next generation is 3.5% of the population. To scale the value of the fitness function, a rank scale has been used. To determine how to select chromosomes, the Selection Tournament method has been used. The intersection rate, which represents the percentage of the population affected by the intersection operator, is considered 0.8 in the best case of portfolio selection.

Mutations in chromosomes are performed by the Arithmetic Function method, the length of change in the gene depends on the limitations of the problem. The mutation rate, which represents the percentage of the population affected by the mutation operator, is considered 0.1. Then, using the mentioned parameters and Table 4-18 as the justified starting point and applying them in *MATLAB*, $z_1, z_2, z_3, z_4, z_5, z_6$, which have the main role in the fitness function, are calculated. Assuming that the relative investment amounts are the same in all financially efficient companies, the starting point is obtained from dividing 1, i.e. budget limitation, by 50.

4.3 Step 3, Simulation:

Using *MATLAB*, the formulated model, and the modified algorithm, the simulation process was performed with each of the three operators, and after 250 generations of simulation for the selected operators, the simulation operation was terminated. A summary of each simulation is presented in the following, and in the next section, the best answer based on the calculations is represented.

Selecting the optimal portfolio using Tournament genetic operator

After following the steps and determining the assumptions and parameters that were previously explained in the process of implementing the genetic algorithm simulation in the third chapter, its possible to simulate the algorithm using *MATLAB*, with a repeat rate of 250 generations and an initial population of 150 by default. At this stage, using *MATLAB*, the proposed ultra-innovative algorithm with the Tournament operator has been used. Simulation to build an optimal portfolio was performed. Figure (36) depicts the change rate of the fitness function in 250 generations.

Accordingly, figure (37) depicts the distance between each generation of the proposed genetic algorithm compared to the previous generation of answers in 250 generations:

In addition, figure (2) depicts the best, worst, and the average value of the fitness function in each generation of using the genetic algorithm by Tournament operator function:

Eventually, figure (3) depicts, the selected chromosome (of the optimal portfolio) using the genetic algorithm after 250 generations:

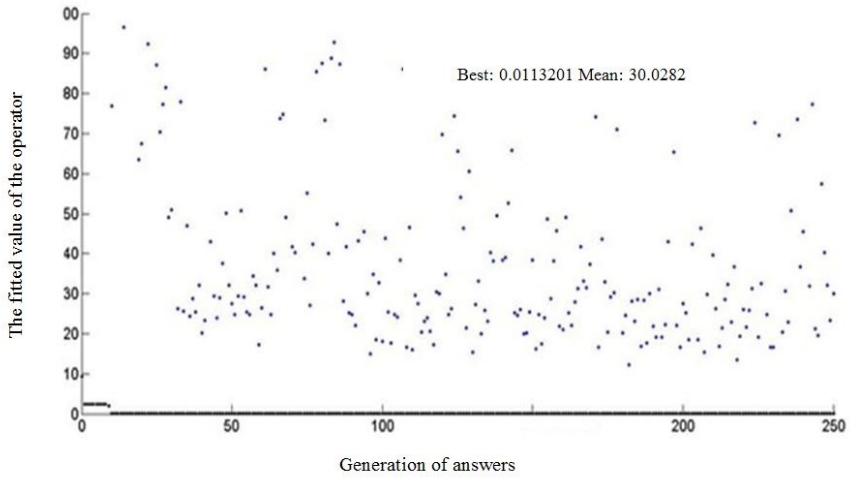

Figure 1: The change trend of Tournament operator in each generation of the simulation (researcher's findings)

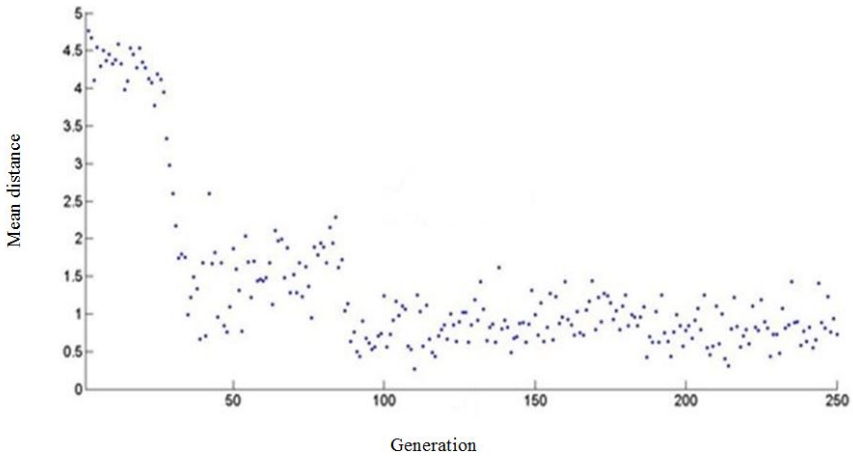

Figure 2: The distance between each generation of Tournament's operator (researcher's findings)

Selecting the optimal portfolio using Roulette Wheel genetic operator:

Following the steps, and determining the assumptions and parameters that were explained in the implementation process of the genetic algorithm simulation in the third chapter, simulation of the algorithm by MATLAB with a repetition rate of 250 generations and an initial population of 150 can be done by default. At this stage, by using MATLAB, the proposed ultra-innovative algorithm with the Roulette

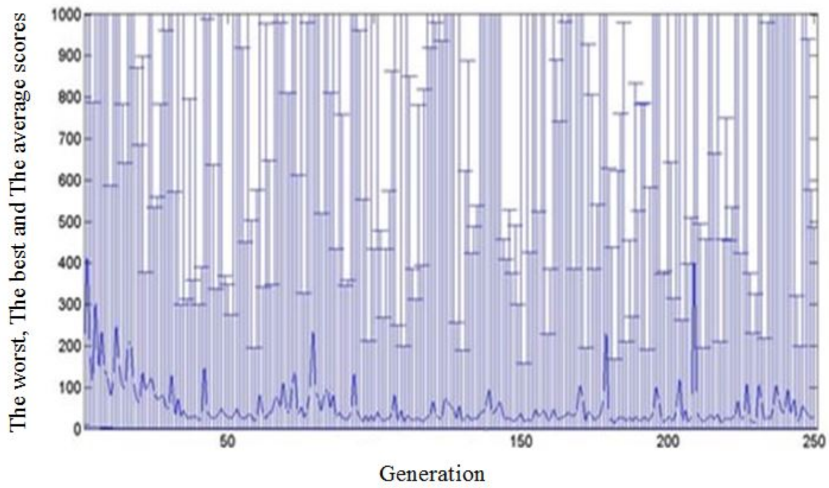

Figure 3: The best, and worst, and the average value of Tournament operator (researcher's findings)

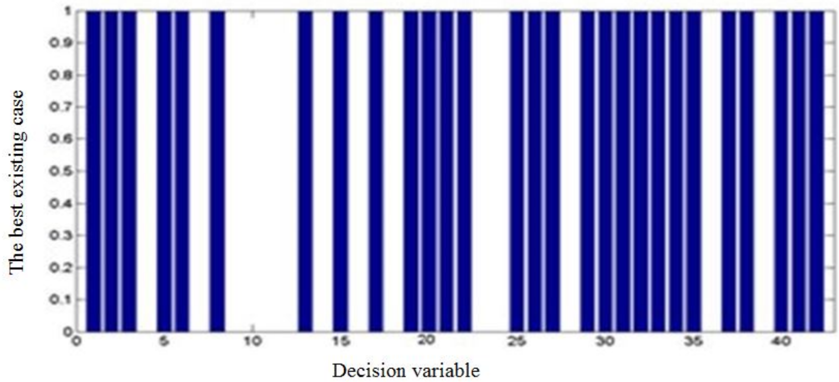

Figure 4: The final chromosome in Tournament operator (researcher's findings)

Wheel operator is used. Simulations were performed to build an optimal portfolio. Figure (4) depicts the rate of change in the fitness function in 250 generations.

Accordingly, figure (1) depicts, the distance between each generation of the proposed genetic algorithm compared to the previous generation of answers by the Roulette Wheel operator in 250 generations:

In addition, figure (7) depicts the best, worst and average value of the fitness function in each generation of using the genetic algorithm by Roulette Wheel operator function:

Eventually, figure (8) depicts, the selected chromosome (of the optimal portfolio) using genetic algorithm and Roulette Wheel operator after 250 generations:

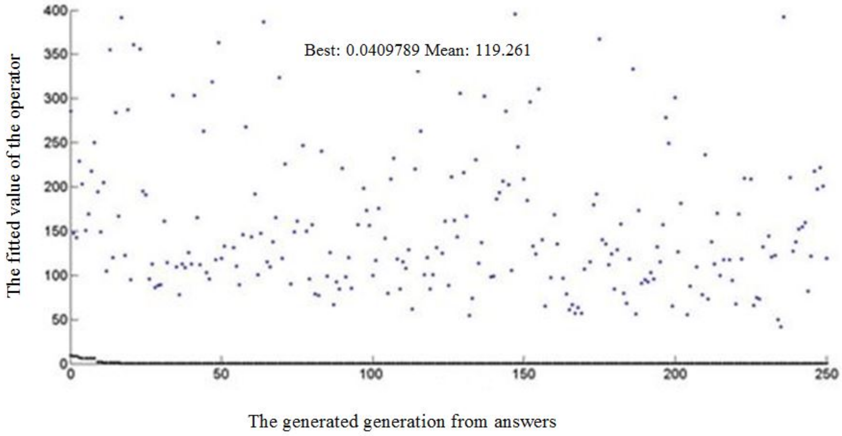

Figure 5: The change trend of Roulette Wheel operator for each generation of the simulation (researcher's findings)

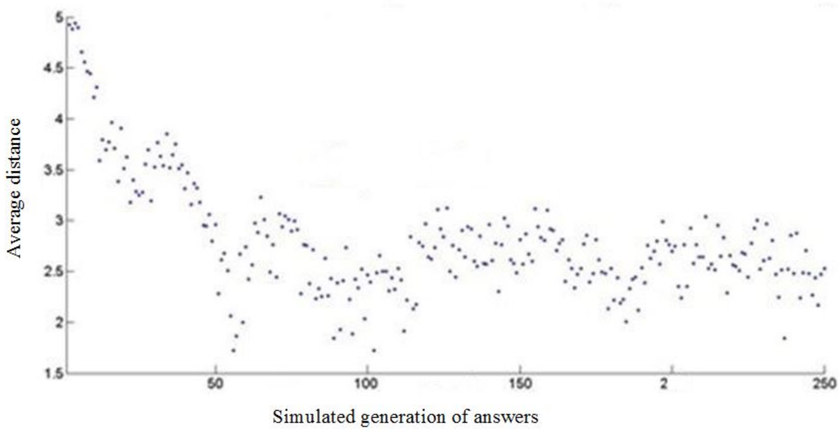

Figure 6: The distance between each generation in Roulette Wheel operator (researcher's findings)

Performance comparison of TR genetic operators:

According to return and risk values obtained in the simulation using the two selection operators Tournament and Roulette Wheel, it is determined that the return of the Tournament operator is somewhat better but it has a higher risk level. In any case, based on the performance efficiency the first operator is better and the performance of the two algorithms is compared in figure (9):

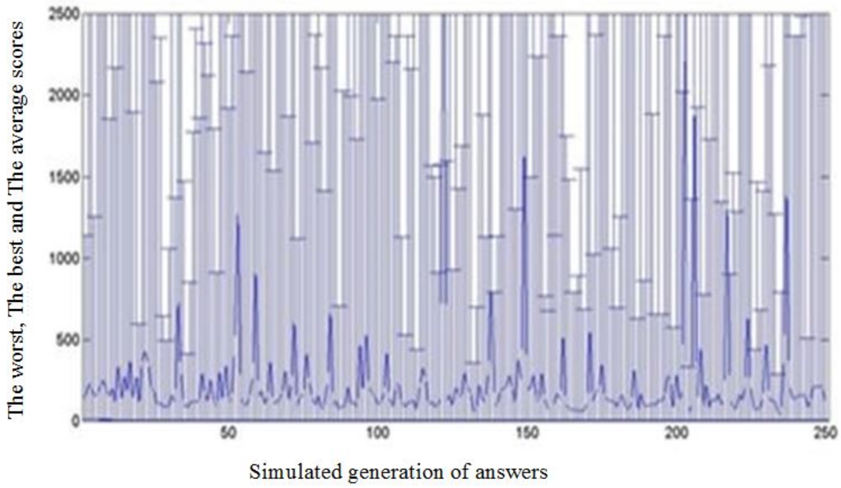

Figure 7: The best, and worst, and the average value of Roulette Wheel operator (researcher's findings)

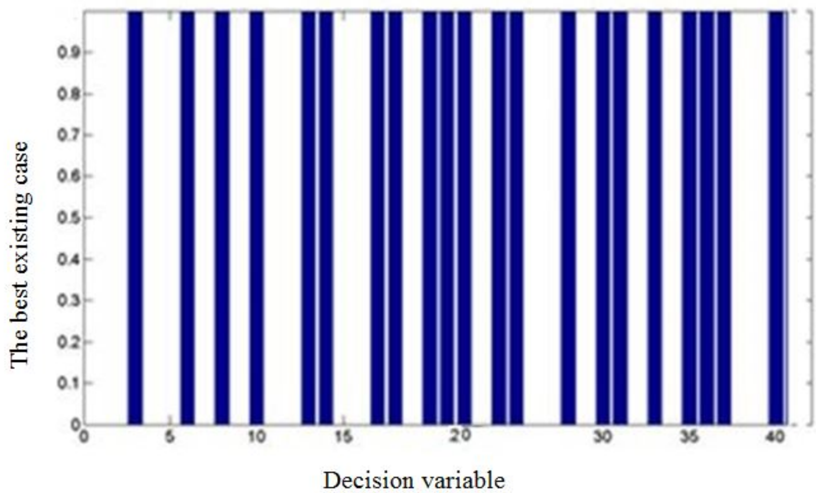

Figure 8: The final chromosome in Roulette Wheel operator (researcher's findings)

4.4 Step 4, Decision-making:

Finally, according to the more efficient operator, i.e. the Tournament operator, the optimal decision, and in other words, the optimal combination of investments or optimal portfolio was determined with a maximum return of 1.169 and investment risk of 0.231.

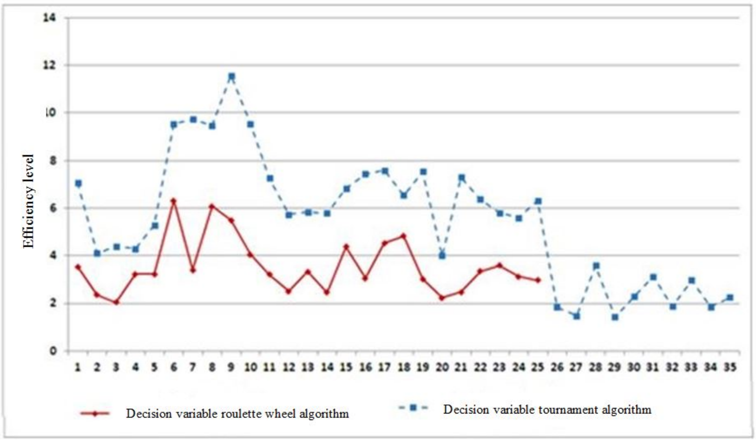

Figure 9: Comparison of the efficiency of the two operators (researcher's findings)

4.5 The effect of Sharpe individual operator, return volatility, and centrality on investment strategy

At this stage, based on the performance data of the top companies in the studied time period, Sharpe (1964) index or individual operator is calculated in investment decision-making and its effect on the optimal investment strategy which is obtained through the Tournament operator is evaluated. Therefore, to evaluate the effect of Sharpe individual operator, return volatility and a systems operation based on the centrality, optimal decision strategy or portfolio, combined linear regression based on cross-section data analysis model and the model proposed by Gastow et al. (2014) has been utilized as follows:

$$w_i^* = \beta_0 + \beta_1 \text{Centrality}_i + \beta_2 \text{Sharpe Ratio}_i + \beta_3 \text{VOL}_i + \epsilon_i \tag{16}$$

Here, $w_{i,gmv}^*$ is the dependent variable of the optimal weights in choosing the optimal investment combination or investment strategies in the mean-variance model. Centrality_i the first independent variable is the systemic operator of stock, based on centrality. Sharpe Ratio_i is the individual operator of stock, based on the Sharpe index, VOL_i represents the return volatility based on the stock return domain and eventually, ϵ_i is the unforeseen part in regression estimation or estimation error. It is expected that in both models of investment strategy, obtained the values are higher than the centrality values, which is affected by the systematic performance of stocks in interaction with each other. The results of using combined linear regression with a cross-sectional data analysis model based on the output of statistical software are briefly stated in Table 2 as follows:

The results of the above table show that due to the negativity of the coefficient of the individual operator of return fluctuations on the investment strategy in the regression estimation relation which is equal to -0.0069, it can be concluded that

Table 2: The impact of Sharpe individual operator on the optimal investment strategy (Researcher's findings)

Description	Impact symbol	Coefficient	T Statistics	Significance level	Relation type
y-intercept	β_0	0.1121	1.617	0.107	*****
Systemic operator	β_1	0.0086	1.935	0.054	Straightforward and meaningful
Sharpe operator	β_2	0.0092	2.221	0.027	Straightforward and meaningful
Return volatility	β_2	-0.069	-3.226	0.001	Inverse and meaningful
Explanation power		Determination coefficient: 0.7825		Modified determination coefficient: 0.7512	
Generalizability of relation		Fisher statistics: 12.181		Significant level: 0.0000	

the individual operator of the stocks return fluctuations had a negative impact on determining the optimal investment strategy and Student's t-statistic is equal to -3.222 and its significance level is 0.001 and less than 5% and therefore the obtained results are significant at the 5% level. The relation justifies an estimation between 75.12 to 78.25% of changes in investment strategy based on the individual Sharpe operator, the centrality system operator, and has high explanatory power and finally, Fisher's significance level is equal to 0.0000 and indicates the significance of the estimated relationship. As a result, the individual operator of companies' stocks in the research field based on the measure of the return fluctuations affects the optimal composition of investment.

Considering the individual Sharpe operator is positive in the estimated regression relation which is equal to 0.0092, it can be concluded that the individual Sherpe operator had a positive effect on determining the optimal investment strategy and Students t-statistic is equal to 2.221 and its significance level is 0.027 and is less than 5% and therefore the obtained results are significant at the 5% level. The relation justifies an estimation between 75.12 to 78.25% of changes in investment strategy based on the individual Sharpe operator, the centrality system operator, and has high explanatory power and finally, Fisher's significance level is equal to 0.0000 and indicates the significance of the estimated relationship. As a result, the individual operator of companies' stocks in the research field (Sharpe ratio) affects the optimal composition of investment

Considering the coefficient of centrality size is positive in the estimated regression relation which is equal to 0.0086, it can be concluded that the size of centrality had a positive effect on determining the optimal investment strategy and Students t-statistic is equal to 1.935 and its significance level is 0.054 and is less than 10% and therefore the obtained results are significant at the 10% level. The relation justifies an estimation between 75.12 to 78.25% of changes in investment strategy based on the individual Sharpe operator, the centrality system operator, and has high explanatory power and finally, Fisher's significance level is equal to 0.0000 and indicates the significance of the estimated relationship. As a result, the investments diversity based on the coefficient of variation of the stocks centrality size affects the optimal composition of investment.

5 Conclusions

The present study has been conducted in order to optimize the portfolio using genetic and network centralization operators in the framework of a case study among companies listed on the Tehran Stock Exchange. The results of the study showed that, in investigating the effect of a centralization based systemic operator, Sharp individual operator, and individual performance of stock based on the return volatility measure on the optimal investment strategy obtained from optimization of the portfolio based on the Mean-variance model and the genetic algorithm (Tournament operator), considering that the coefficient of the individual operator of return volatility of the investment strategy is negative, regarding regression estimation, we can conclude that the individual operator of stock return volatility has a negative effect on determining the optimal strategy of investment. And as stock return volatility and actually investment risk based on scattered return are increased the stock becomes less preferable to be selected as an optimal investment option and owns a lower percentage of the overall investment or portfolio. Also, due to the positive effect of the individual Sharpe operator on determining the optimal investment strategy, as the risk premium of the market of investing in the company is higher and the risk is less, it has become more preferable to be chosen as a desirable investment option and dedicates a higher percentage of total investment or portfolio to itself. Also, due to the positive effect of centrality size on determining the optimal investment strategy, as the sum of differences in the hybridization indicator of the companys performance have become less than other companies, it has become more preferable to be chosen as a desirable investment option and dedicates a higher percentage of total investment or portfolio to itself.

Bibliography

[1] Y. AMIHUD, *Illiquidity and stock returns: cross-section and timeseries effects*, Journal of Financial Markets, 5 (2020), 31-56.

[2] Z. DAI, H. ZHU, F. WEN, *Two nonparametric approaches to mean absolute deviation portfolio selection model*, Journal of Industrial Management Optimization, 16(5) (2020), 2283.

[3] V. DEMIGUEL, Y. PLYAKHA, R. UPPAL, G. VILKOV, *Improving portfolio selection using option-implied volatility and skewness*, Journal of Financial and Quantitative Analysis, (2013), 1813-1845.

[4] E. F. Fama, *A review of theory and empirical work*, The journal of Finance, 25(2) (1970), 383-417.

[5] E. F. Fama, K. R. French, *Multifactor explanations of asset pricing anomalies*, The journal of finance, 51(1) (1996), 55-84.

[6] R. Gopalan, O. Kadan, M. Pevzner, *Managerial decisions, asset liquidity, and stock liquidity*, Asset Liquidity, and Stock Liquidity, (2009).

[7] B. Han, H. Y. Wong, *MeanVariance Portfolio Selection Under Volterra Heston Model*, Applied Mathematics Optimization, (2020), 1-28.

[8] C. Iorio, G. Frasso, A. D'Ambrosio, R. Siciliano, *A P-spline based clustering approach for portfolio selection*, Expert Systems with Applications, 95 (2018), 88-103.

[9] B. Li, Y. Zhu, Y. Sun, G. Aw, K. L. Teo, *Multi-period portfolio selection problem under uncertain environment with bankruptcy constraint*, Applied Mathematical Modelling, 56 (2018), 539-550.

[10] J. Lintner, *Distribution of incomes of corporations among dividends, retained earnings, and taxes*, The American economic review, 46(2) (1956), 97-113.

[11] J. Ma, J. D. Harstvedt, R. Jaradat, B. Smith, *Sustainability driven multi-criteria project portfolio selection under uncertain decision-making environment*, Computers Industrial Engineering, 140 (2020), 106236.

[12] N. Mansour, M. S. Cherif, W. Abdelfattah, *Multi-objective imprecise programming for financial portfolio selection with fuzzy returns*, Expert Systems with Applications, 138 (2019), 112810.

[13] H. Markowitz, *The utility of wealth*, Journal of political Economy, 60(2) (1952), 151-158.

[14] J. A. Ohlson, *Earnings, book values, and dividends in equity valuation*, Contemporary accounting research, 11(2) (1995), 661-687.

[15] G. Peralta, A. Zareei, *A network approach to portfolio selection*, Journal of Empirical Finance, 38 (2016), 157-180.

[16] J. R. Ross, *Constraints on variables in syntax*, (1967).

[17] W. F. Sharpe, *A theory of market equilibrium under conditions of risk*, The journal of finance, 19(3) (1964), 425-442.

[18] Y. Zhang, P. Zhao, B. Li, Q. Wu, J. Huang, M. Tan, *Cost-sensitive portfolio selection via deep reinforcement learning*, IEEE Transactions on Knowledge and Data Engineering, (2020).

[19] X. Zhong, D. Enke, *Forecasting daily stock market return using dimensionality reduction*, Expert Systems with Applications, 67 (2017), 126-139.

How to Cite: Asghar Abolhasani Hastiany[1], Alireza Zamanpour[2], *Network centrality and portfolio optimization using the genetic algorithm*, Journal of Mathematics and Modeling in Finance (JMMF), Vol. 1, No. 2, Pages:113–139, (2021).