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Robust net present value with infinite lifetime

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Abstract:
In this study, Robust Net Present Value (RNPV) has been developed for the evaluation of projects with infinite life time. In this method, the changes of uncertain net incomes in a financial cash flow are postulated in a convex, continuous, and closed region. It has been indicated that RNPV, in the infinite life horizon, is calculable only when the net incomes are uncorrelated. Compared to traditional methods, this study considers the variance matrix of net incomes, takes uncertainty into account during the evaluation of investment projects with infinite life period. One important finding when using this method is that one does not need to calculate the covariance matrix in the evaluation of projects with infinite life. The only requirement is to estimate the value of maximum variance for the given financial cash flow. The proposed method is also easy to both calculate and understand in practice. MATLAB software is used for implementation. Lastly, the features of the developed method have been analyzed using some numerical examples for a project with infinite lifetime.

Keywords: Robust net present value, Robust approach, Project with infinite life, Economic evaluation of investment projects.
JEL Classifications: G31, C60.

1 Introduction

From the perspective of financial theory, NPV is one of the most well-known and popular methods of evaluation when a projects cash flow parameters are certain [6,16]. Remer and Nieto reviewed and explained each project evaluation method in which all parameters were assumed to be certain. They divided these methods into 5 major groups and introduced NPV as one of the most important methods among the 25 existing ones [14]. However, despite the high practicality and popularity of NPV as the main evaluation method, in recent years there have been critics who claim that in uncertain conditions, NPV is not sufficient and does not yield reliable results [12]. This criticism is based on the concept that unpredicted future events

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will naturally affect all financial parameters such as revenue, expense, profit, etc., and furthermore considering that the estimation of cash flows with certainty rarely occurs in practice. Thus, decision-making with the assumption of certainty might seriously damage the future investment.

1.1 Literature Review

Ingersoll [9] stated that since it is not possible to incorporate decision-makers beliefs into point estimation, it is better to use expected NPV estimation instead of NPV point estimation. However, in the presence of variance and correlation among input parameters, this method would be highly sensitive to uncertain financial cash flow parameters, especially when a large variance of changes exists. According to Anderson and Fennell, in order to achieve a better and more realistic result in the evaluation of investment projects, one must enter risk, uncertainty, and investors goals into their investigations [2]. Jovanovic [10] claimed that sensitivity analysis method can be utilized for achieving better information about the effects of uncertain financial cash flows on decision-making indices like NPV. Moreover, Groenendaal & JH [15] argued that, in practice, sensitivity analysis is limited to a certain sensitivity or scenario analysis, which is not sufficient for determining the whole range of NPV variability. Accordingly, they preferred using design of experiments, one of the global sensitivity analysis techniques, to simultaneously investigate input parameters variability in association with a regression method. In addition, Xu & Gertner [17] argued that using design of experiments, one can also study the effect of dependent variables on the model output. However, Nabradi and Szollosi [13], criticizing sensitivity analysis as being considered the best risk analysis method, pointed out that this method does not properly take the relationship among basic parameters of the project into consideration.

Monte Carlo simulation method is employed for taking into account uncertainty in concurrent changes of financial cash flow parameters in NPV calculations. Generally speaking, simulation analysis estimates the probability of different potential outputs, and Monte Carlo simulation is a tool for investigating all possible combinations [13]. Furthermore, it is more reliable than traditional approaches as it takes into account some information as a standard deviation of uncertain parameters. Coates and Kuhl [7], by taking the mean and standard deviation of input parameters into account, attempted to solve engineering economics problems using a simulation method.

As mentioned earlier, several methods have been proposed for evaluating investment projects under uncertain conditions, each of which have their own specific advantages and drawbacks. The main pitfall of such methods under uncertainty is that they tend to overlook the variance and statistical correlation among uncertain parameters in a financial cash flow and the complexity involved in the simultaneous investigation of uncertain parameters. One successful evaluation method under uncertainty, which has been proposed in recent years and is able to overcome some weaknesses found in previous methods, is Robust Net Present Value (RNPV). Hanafizadeh and Latif [8] assumed that the changes of uncertain parameters in a financial cash flow, i.e. net incomes, fall within a convex and closed region. To construct the variability region of uncertain parameters, they used information of the first and second moments, mean vector, and covariance matrix of uncertain net incomes. Considering the worst-case behavior of uncertain parameters in a financial cash flow, the robust net present value was formulated.

Another approach that applied robust optimization in capital budgeting problems was proposed by Kachani and Langella [11]. They formulated capital budgeting problems using robust optimization approach as a linear programming model. In their model, it is assumed that uncertainty in both objective function and constraints exists [11]. However, Bas [3] argued that this approach does not offer any specific solution for solving their own model.

Employing robust optimization method, Bas [3] proposed laws for investors judgement using only NPV and Internal Rate of Return (IRR) indices for simple projects that have only one rate of return. Though her proposed method was similar to Hanafizadeh's and Latif's [8], she argued against RNPV method, pointing out that it evaluates projects very conservatively, and it does not possess the necessary flexibility for adjusting to different degrees of uncertainty in the evaluation of projects [3]. However, by explaining the uncertainty region and its corresponding RNPV through use of the radius of uncertainty region and the norm degree, the RNPV method provides investors with the flexibility necessary for enlarging or shrinking the uncertainty regions size in a financial cash flow.

When evaluating the projects with infinite time horizon, NPV with infinite life period or capital expense is used [4]. Public regional and national projects such as dams, bridges, or powerplants, with useful life periods of 30 to 40 years, can be categorized in this group [5]. It is noteworthy that with most investment projects with infinite life period, a reinvestment capital should be periodically considered. Moreover, when a business or firm is valued, as opposed to an individual asset, it is often considered to have no finite life period. Since a firm reinvests sufficient amounts into new assets each period, it could theoretically keep generating cash flows forever [1].

Considering the importance of evaluating public projects in a country's infrastructure sector or the valuation of a business or firm, this study intends to develop an RNPV model for the evaluation of such projects under uncertain conditions. It should be mentioned that the kind of information obtained from uncertain parameters in a financial cash flow is assumed to be probabilistic; therefore, in the scope of the present study, other types of information, such as fuzzy, are not considered. To review methods which regard information as fuzzy, you are refered to Table 1 in [3].

The rest of this paper is organized as follows: in the third section, robust net present value will be reviewed briefly and the mathematical formula for RNPV with infinite life horizon will be presented. In Section 4, RNPV with infinite life period will be solved with some numerical examples and its results will be investigated. Lastly, Section 5 is devoted to conclusions.

2 Robust Net Present Value

The RNPV method was first proposed by Hanafizadeh and Latif [8]. They argued that the main pitfall of traditional methods was that they ignored the variance and correlation of data during calculations. These factors play a crucial role in the realization of the results, and furthermore, taking only the nominal mean into account does not contribute much to the estimation of a projects future value. They took into consideration the variance and covariance of uncertain parameters, as the variance-covariance matrix, in order to account for uncertainty during the calculation of NPV [8]. They called this new method Robust Net Present Value. They also stated, RNPV means that even with deviation from the nominal values of uncertain parameters in the uncertainty region, NPV calculation keeps its features (its positivity or negativity).

In that model, it is assumed that changes of uncertain parameters occur in a convex, continuous, non-empty and closed region. In addition, the radius of this region changes proportionally to an investors predicted risk. In the model, the covariance matrix (C) is positive definite and symmetric, and its root square (W) is used in calculations.

It should be mentioned that in the present paper, bold-typed upper case letters signify matrices and bold-typed lower case Italicized letters stand for vectors.

Some important concepts in this method are as follows:

- i: interest rate
- n: project lifetime
- t: is an index refers to year (or period of a cash flow)
- Cash flow discount vector: $x_t = \left(\frac{1}{1+i}\right)^t \qquad \mathbf{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$
- Net income vector:
$$\mathbf{a} = \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix}$$
- $\boldsymbol{\mu}_{\mathbf{a}}$: ů mean of uncertain net income vector
- c_0: primary investment

- $B_q(r)$: is a norm-body which defines by a q-norm and a radius of r
- U: uncertainty region

$$U(q,r) = \{\mathbf{a} : \mathbf{a} = \boldsymbol{\mu}_{\mathbf{a}} + \mathbf{W}\mathbf{u} | \mathbf{u} \in B_q(r)\} \tag{1}$$

$$B_q(r) = \{\mathbf{u} \varepsilon R^n | \|\mathbf{u}\|_q \leq r\} \tag{2}$$

- $\boldsymbol{u}$ can be considered as white noise which falls within norm-body $\mathrm{B}_q(r)$. The root square of the covarinace matrix is:

$$\mathbf{W} = \mathbf{C}^{\frac{1}{2}} \tag{3}$$

Based on the above concepts, we have:

$$NPV = -c_0 + \mathbf{a}^T\mathbf{x} \tag{4}$$

And the formula of robust net present value will be as follows:

$$RNPV = -c_0 + \boldsymbol{\mu}_{\mathbf{a}}^T\mathbf{x} - r\|\mathbf{W}^T\mathbf{x}\|_p$$

$$RNPV(q,r) = NPV^0 - r\|\mathbf{W}^T\mathbf{x}\|_p; \tag{5}$$

$$\frac{1}{p} + \frac{1}{q} = 1 \tag{6}$$

NPV^0 represents the traditional NPV method, including the mean of uncertain income in its caculation. For detailed calculations of robust net present value (Eq. 6), one can refer to [8].

3 Robust net present value with infinite life horizon

In the presence of correlation among net incomes, RNPV method for evaluating projects with infinite life period is divergent (See the Appendix for the proof).

Theorem 3.1. *If the means of net incomes are equality , and there is a lack of correlations among them in a cash flow, then the lower bound of RNPV with infinite lifetime for different values of the norm degrees equals:*

$RNPV(\infty, r) \geq -c_0 + \frac{\mu_a - r\sigma_{\max}}{i}$

$RNPV(2, r) \geq -c_0 + \frac{\mu_a}{i} - \frac{r\sigma_{\max}}{((1+i)^2-1)^{1/2}}$

$RNPV(1,r) \geq -c_0 + \frac{\mu_a}{i} - \frac{r\sigma_{\max}}{1+i}$

Where σ_{max} is the maximum variance, which is more than the variance of net incomes in a cash flow. μ_a is the mean of equal net revenues in a cash flow. It is common when valuating a company or a firm, particularly in high-growth scenarios, that the growth rate of incomes or dividends will drop to a stable or constant rate, forever, at some time in the future [1]. Therefore, this is an acceptable assumption in the literature.

As correlations equal zero, the square root of covariance matrix will be as follows:

$$\mathbf{W} = \begin{pmatrix} \sigma_1 & 0 & \cdots & 0 & 0 \\ 0 & \sigma_2 & 0 & \ldots & 0 \\ \vdots & 0 & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & \sigma_{n-1} & 0 \\ 0 & 0 & \cdots & 0 & \sigma_n \end{pmatrix}_{n\times n}$$

$$RNPV(q,r) = -c_0 + \boldsymbol{\mu}_{\mathbf{a}}^T\mathbf{x} - r\|\mathbf{W}^T\mathbf{x}\|_p \tag{7}$$

$$RNPV = -c_0 + \lim_{n\to\infty} \sum_{t=1}^{n} \frac{\mu_t}{(1+i)^t} - \lim_{n\to\infty}(r\|\mathbf{W}^T\mathbf{x}\|_p) \tag{8}$$

By considering the mean of net incomes in a cash flow in the first term of Eq. (9), we will have:

$$\lim_{n\to\infty} \sum_{t=1}^{n} \frac{\mu_t}{(1+i)^t} = \boldsymbol{\mu}_a \sum_{t=1}^{\infty} \frac{1}{(1+i)^t} = \frac{\boldsymbol{\mu}_a(\frac{1}{1+i})}{(1-\frac{1}{1+i})} = \frac{\boldsymbol{\mu}_a}{i} \tag{9}$$

As can be seen in Relation (10), cash flows in RNPV with infinite life horizon, just like NPV with infinite life horizon, should resemble annual cash flows. Blank & Tarquin [4] have suggested that when using NPV with infinite life horizon or capital cost method, all cash flows, whether periodic or not, should be changed to annual cash flows. As mentioned, during the valuation of a firm, its cash flow is considered over an infinite life horizon with a stable or constant rate income after sometime in future [1].

In the calculation of the second term of Eq. (9), we have:

$$\mathbf{W} = \begin{pmatrix} \sigma_1 & 0 & \cdots & 0 & 0 \\ 0 & \sigma_2 & 0 & \dots & 0 \\ \vdots & 0 & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & \sigma_{n-1} & 0 \\ 0 & 0 & \cdots & 0 & \sigma_n \end{pmatrix}_{n\times n} \begin{pmatrix} \frac{1}{1+i} \\ \frac{1}{(1+i)^2} \\ \\ \frac{1}{(1+i)^n} \end{pmatrix} = \begin{pmatrix} \frac{\sigma_1}{1+i} \\ \frac{\sigma_2}{(1+i)^2} \\ \\ \frac{\sigma_n}{(1+i)^n} \end{pmatrix} \tag{10}$$

As explained earlier, the uncertainty region is as follows:

$$U(q,r) = \{\mathbf{a} : \mathbf{a} = \boldsymbol{\mu}_\mathbf{a} + \mathbf{W}\mathbf{u} | \mathbf{u} \in B_q(r)\} \tag{11}$$

$$B_q(r) = \{\mathbf{u} \in R^n | \|\mathbf{u}\|_q \leq r\} \tag{12}$$

Where, the following relationship holds for the norms l_1, l_2 and l_∞:

$$\|v\|_\infty \leq \ldots \leq \|v\|_2 \leq \|v\|_1 \tag{13}$$

As stated in Hanafizadeh and Latif [2], on the basis of the definition of dual norm, l_p is the dual norm of l_q, it means the following relationship holds between p and q.

$$\frac{1}{p} + \frac{1}{q} = 1 \tag{14}$$

On the other hand, as mentioned earlier, for RNPV relationships we have:

$$RNPV(q,r) = -c_0 + \boldsymbol{\mu}_\mathbf{a}^T \mathbf{x} - r\|\mathbf{W}^T\mathbf{x}\|_p \tag{15}$$

According to the mentioned equations, it can be concluded that by increasing q, p decreases. Thus, to consider the most pessimistic situation, q should be equal to infinity, which explains the biggest uncertainty region. In that case, this leads to $p = 1$. That is to say that the most pessimistic situation occurs in the most uncertain state. Hence, according to Eq. (6), (11) and (12), to estimate the most pessimistic state of Eq. (9), p should be considered 1. In that case:

$$((\frac{\sigma_1}{1+i})^p + (\frac{\sigma_2}{(1+i)^2})^p + \cdots + (\frac{\sigma_n}{(1+i)^n})^p)^{\frac{1}{p}} \leq (\frac{\sigma_1}{1+i} + \frac{\sigma_2}{(1+i)^2} + \cdots + \frac{\sigma_n}{(1+i)^n}) \tag{16}$$

Also, since uncertainty increases as time progresses, it can be assumed that in years distant from the present, the standard deviation of net incomes in a cash flow will be higher. Assuming that the variance of each period of financial cash flow is smaller than or equal to the maximum variance, σ_{max}, we have,

$$\max_{i\in N} \sigma_i \leq \sigma_{max} \tag{17}$$

$$\begin{aligned}&\lim_{n\to\infty}\left(\frac{\sigma_1}{1+i}+\frac{\sigma_2}{(1+i)^2}+\cdots+\frac{\sigma_n}{(1+i)^n}\right)\\&\le\lim_{n\to\infty}\left(\frac{\sigma_{\max}}{1+i}+\frac{\sigma_{\max}}{(1+i)^2}+\cdots+\frac{\sigma_{\max}}{(1+i)^n}\right)=\frac{\sigma_{\max}}{i}\end{aligned} \tag{18}$$

As observed in Inequality (15), in the extreme limit condition, i.e. when n is increasing and converges to infinity, the left-hand side of Inequality (15) is smaller than or equal to $\frac{\sigma_{\max}}{i}$.

Also, by considering Inequality (14) for the variances of a cash flow and the extreme limit relation of the sum of geometric progression in the second part of Eq. (9), for $q = 1$ and $q = 2$ we have:

$$\begin{aligned}\lim_{n\to\infty}(r\|\mathbf{W}^T\mathbf{x}\|_p) &\le r\lim_{n\to\infty}\left(\sum_{t=1}^{n}\left|\frac{\sigma_{\max}}{(1+i)^t}\right|^2\right)^{\frac{1}{2}}\\&=r\sigma_{\max}\left(\frac{\frac{1}{(1+i)^2}}{\left(\frac{(1+i)^2-1}{(1+i)^2}\right)}\right)^{1/2}=\frac{r\sigma_{\max}}{((1+i)^2-1)^{1/2}}\end{aligned} \tag{19}$$

q=1 and p= ∞

The norm of matrix $\mathbf{A}$ for p=∞ is:

$$\|\mathbf{A}\|_\infty=\max(a_{ij}) \tag{20}$$

Where a_{ij} is the element of i^{th} row and j^{th} column of matrix $\mathbf{A}$. Based on Eq. (17) and considering the fact that by increasing the value of n, the elements of vector $\mathbf{W}^T\mathbf{x}$ decrease, and the highest value of the elements of the matrix occurs in the first element, the second part of Eq. (9) for p=∞ is:

$$\lim_{n\to\infty}(r\|\mathbf{W}^T\mathbf{x}\|_\infty)\le r\frac{\sigma_{\max}}{(1+i)^1} \tag{21}$$

According to Eq. (9), (15), (16), and (19), the final formulae of the robust net present value for different value of q's are as follows:

$$RNPV(\infty,r)\ge -c_0+\frac{\mu_a-r\sigma_{\max}}{i} \tag{22}$$

$$RNPV(2,r)\ge -c_0+\frac{\mu_a}{i}-\frac{r\sigma_{\max}}{((1+i)^2-1)^{1/2}} \tag{23}$$

$$RNPV(1,r)\ge -c_0+\frac{\mu_a}{i}-\frac{r\sigma_{\max}}{1+i} \tag{24}$$

The right hand side of Inequality (22), (25) and (27) are the lower bound of the robust net present value with infinite life period for differing qs. An investor can use the lower bounds during calculation of the actual RNPV as its estimation. The estimation of RNPV is called $R\hat{N}PV$.

4 Analysis of a project's robust net present value with infinite lifetime

On the basis of Inequality (22), (25), and (27), the estimation of robust net present value of projects with infinite life horizon may be calculated by subtracting the values of $r\frac{\sigma_{\max}}{i}$, $\frac{r\sigma_{\max}}{((1+i)^2-1)^{1/2}}$ and $r\frac{\sigma_{\max}}{(1+i)^1}$ from the nominal net present value of projects with infinite life period, respectively. Such values depend on factors like the radius of uncertainty region, interest rate, and maximum variance of net incomes, σ_{max}, which represents the covariance matrix, or is somehow the maximum of expected disorders. All these values are real and non-negative. Thus, as expected, the RNPV equation with infinite life period is always smaller than NPV. In the numerical examples, the effect of each of these parameters is investigated.

4.1 Numerical example

In this study, a public interest project with an initial cost of 10000 dollars and an infinite lifetime (here it is assumed to be 40 years) is considered. The annual net revenue is predicted to be $2000. This type of project in capital investment is called 'conventional' project. This type of investment project is defined as one in which the initial outlay is followed by a stream of positive net incomes in the form: - + + + . . .; or if the outlay takes place over a number of years, the cash flow has the form: - - + + ... Because of relation (8), cash flows are considered to be annual for simplicity. An uncertainty region with a radius of $r = 2$ and $\sigma_{max} = 150$ is considered in order to construct the changes in annual net income, and to investigate whether the project is economical using the proposed method. First, in Fig.1, the results of the proposed method $R\hat{N}PV$ when $q = \infty$ are compared with the results of NPV method using the mean value of net income with infinite life horizon. Then, the results of the proposed method in different norms are analyzed considering the changes of radius and maximum variance. Finally, in Observation 3, the results of the proposed method (the estimation of RNPV with infinite lifetime) are compared with the results of actual RNPV method calculated with the real lifetime of the project, 40 years. The value of the estimation of RNPV for different values of radius and σ_{max} are calculated and its sensitivity is analyzed.

Fig. 1 graphs the changes of $R\hat{N}PV$ and NPV as functions of the discount rate. By increasing the interest rate in the horizental axis, $r\frac{\sigma_{\max}}{i}$ decreases. Therefore, it can be argued that by increasing the interest rate, the value of $R\hat{N}PV$ tends toward NPV in the infinite state.

According to Fig. 1, the rate of return (ROR) of the traditional approach is equal to 0.20, whereas the robust rate of return (RROR) is 0.15. Considering the minimum attractive rate of return (MARR), decision-making in either of these two approaches will be different.

A. If MARR is less than 0.15, both approaches evaluate the project as economi-

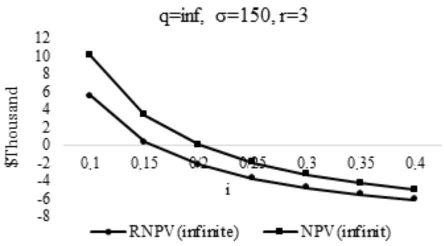

Figure 1: Comparison of $R\hat{N}PV$ and NPV

cal.

B. If MARR is more than 0.20, both approaches reject the project.

C. If MARR is between 0.15 and 0.20, then the traditional approach evaluates the project as economical. However, its acceptance or rejection by the robust approach depends on the investor's risk-taking or risk-aversion profile, which investor's characteristics represent by radius and degree of norm of uncertainty region indicates changes of net incomes concurrency. Of course, as r = 2 is specified for use in this example, it is not recommended to take part in this project.

Observation 1. By an increase in σ_{max}, $R\hat{N}PV$ value with infinite lifetime decreases (see Fig. 2, 3 and 4). In Fig. 2 and 3, all curves of $R\hat{N}PV$ with different σ_{max}s are almost overlapping with NPV. In these figures, $R\hat{N}PV$ with infinite lifetime is calculated for three different σ_{max}s. The radius of uncertainty region is assumed to be a fixed (r=2), and the results of different σ_{max}s are also compared with the results of NPV method with infinite lifetime.

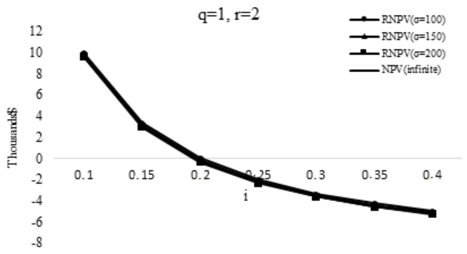

Figure 2: Comparison of different $R\hat{N}PV$s with different σ_{max} when q = 1

Due to an increase in uncertainty or variations of net incomes' volatility, the degree of norms (from 1, 2 and then to infinity) and the variance of uncertain net incomes (from 100, 150 and 200), and consequently their maximum variance, are

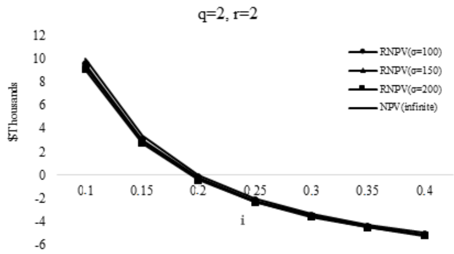

Figure 3: Comparison of different $R\hat{N}PV$s with different σ_{max} when q = 2

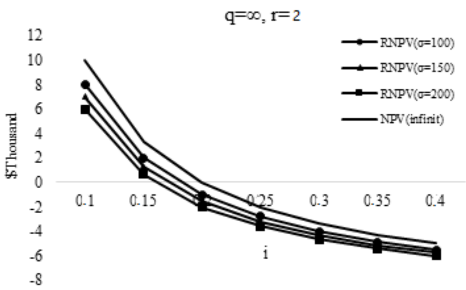

Figure 4: Comparison of different $R\hat{N}PV$s with different σ_{max} when q = ∞

increased. As observed in Fig. 2, 3 and 4, this increase leads to a decrease in the robust rate of return.

Another influential factor creating discrepancy between NPV and $R\hat{N}PV$ is the radius of uncertainty region. When uncertainty is high or the investor is risk-averse, then the radius of uncertainty region is assumed to be larger. The following observation explains the impact of the uncertainty region's radius on $R\hat{N}PV$.

Observation 2. By an increase in the radius of uncertainty region, the value of RNPV with infinite lifetime decreases. Fig. 5, 6 and 7 confirm this observation. In these figures, the effect of the uncertainty region's radius is investigated using a standard deviation equal to 150. In both Fig.5 and 6 ,$R\hat{N}PV$ and NPV curves are almost overlapping.

As mentioned earlier, given the level of uncertainty and an investor's risk aversion profile, different radiuses are used for calculating the robust approach. When the uncertainty level is high or the investor is risk averse, a larger r can be assumed. For instance, as seen in Fig. 7, when $r = 3$, RROR is 0.15, while for decision-making with less uncertainty, or when the investor is at a higher risk prone, r and RROR would be 1 and 0.18, respectively. Furthermore, ROR would be 0.20 using the traditional approach.

Observation 3. By increasing the uncertainty regions' norm, $R\hat{N}PV$ value decreases with infinite lifetime.

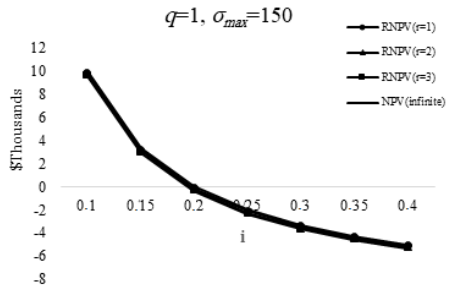

Figure 5: Comparison of different $R\hat{N}PV$s with different σ_{max} when q = 1

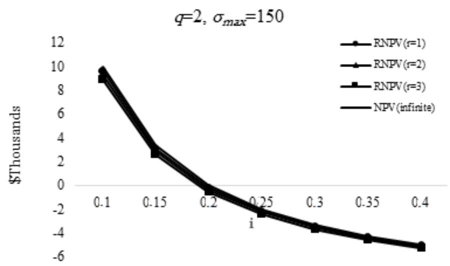

Figure 6: Comparison of different $R\hat{N}PV$s with different σ_{max} when q = 2

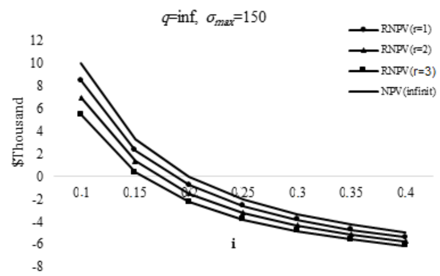

Figure 7: Comparison of different $R\hat{N}PV$s with different σ_{max} when q = ∞

As indicated in Fig. 8, by increasing the uncertainty regions' norm (from 1, 2, and then *inf*), the value of $R\hat{N}PV$ with infinite lifetime decreases (it was indicated in Inequality (22), (25) and (27) that $R\hat{N}PV$ with infinite lifetime differs for different norms). The findings illustrate that by increasing the degree of norms value, the value of $R\hat{N}PV$ with infinite lifetime decreases further. Hence, the discrepancy of values when $q = 1$ and $q = 2$ is lower, whereas it reaches the most pessimistic state, i.e. the maximum value, when $q= \infty$.

Observation 4. $R\hat{N}PV$ with infinite lifetime is always smaller than or equal to its actual robust net present value.

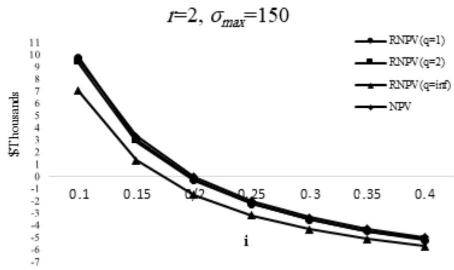

Figure 8: Comparison of different $R\hat{N}PV$s for different norms

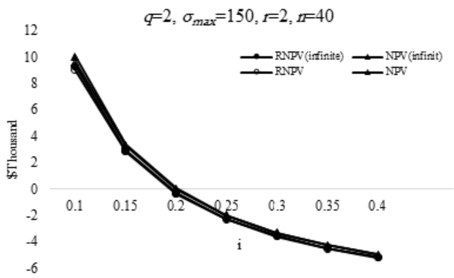

Figure 9: Comparison of the results of actual RNPV with $R\hat{N}PV$ infinite lifetime

It must be noted in these comparisons that according to relations (22), (25) and (27), the project lifetime is not taken into consideration when calculating an estimation of robust net present value with infinite lifetime, while it is taken into account when calculating the actual robust net present value. In this example, the lifetime is deemed to be 40 years. As is evident from Figures 9 and 10, considering q =2, and relation (25), the values of actual robust net present value and the estimation of robust net present value with infinite lifetime are very close to each other.

As observed in Fig. 9 and 10, by increasing the radius of uncertainty regions, the discrepancy between the results of actual RNPV and NPV increases. However, the values of actual robust net present value and the estimation of robust net present value with infinite lifetime are still very close to each other.

Fig. 11 confirms previous findings and by decreasing σ_{max},almost all results are similar to each other. However, by increasing σ_{max}, as observed in Fig. 12, the value of $R\hat{N}PV$ with infinite lifetime is a little less than the actual RNPV value. It can be concluded that σ_{max} has more impact on the value of $R\hat{N}PV$ with infinite lifetime relative to the actual RNPV value.

As shown clearly in Fig. 13 and 14, when increasing the norm of uncertainty regions, the distance between $R\hat{N}PV$ and NPV increases. Meanwhile, when $q = \infty$, $R\hat{N}PV$ with infinite life horizon is a bit smaller than actual RNPV.

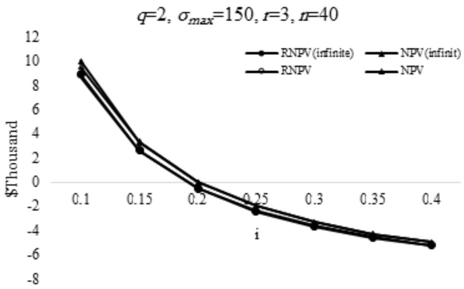

Figure 10: Comparison of the result of actual RNPV with $R\hat{N}PV$ infinite lifetime by the increase of the radius of uncertainty region

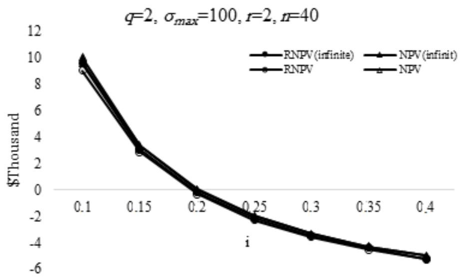

Figure 11: Comparison of the results of actual RNPV and $R\hat{N}PV$ with infinite life horizon by a decrease in σ_{max}

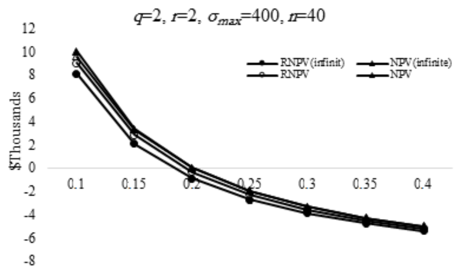

Figure 12: Comparison of the results of actual RNPV and $R\hat{N}PV$ with infinite life horizon by an increase in σ_{max}

5 Conclusion

Evaluation of public interest projects in the infrastructure sector of countries is of paramount importance. Moreover, during valuation of a firm, particularly during high-growth phase, it is common to assume that the growth rate of incomes or dividends shall drop to a stable or constant rate, forever, sometime in the future. On

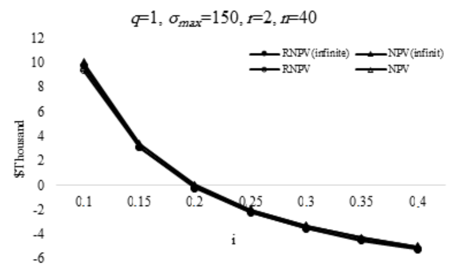

Figure 13: Comparison of the tesults of actual RNPV and$R\hat{N}PV$ with infinite life horizon by a decrease in uncertainty regions using norm of q=1

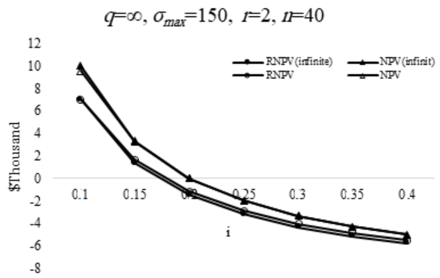

Figure 14: Comparison of the results of actual RNPV and $R\hat{N}PV$ with infinite life horizon by an increase in uncertainty regions using norm of $q= \infty$

the one hand, NPV, as the most important evaluation method, is usually used for evaluating such projects with infinite lifetime. However, despite the high applicability and popularity of NPV as the major evaluation method, critics have recently considered it imperfect and unsuitable for evaluation of projects in uncertain environments. Hence, several methods have been proposed for taking uncertainty into account when evaluating projects with infinite life horizon. One such method, which can be simply applied and is able to rectify most drawbacks of other methods, is RNPV. We have developed this method for evaluating projects with infinite life horizon. In this method, the changes of net incomes are considered in a convex, continuous, and closed uncertainty region. Because it considers the variance of net incomes of a cash flow, this method uses more information for evaluating investment projects, and thereby offers more reliable results compared to traditional NPV. MATLAB software was employed for mathematical programming. The developed method was then analyzed using some numerical examples for a conventional investment project with infinite life horizon. The findings revealed that estimation of RNPV, in evaluation of projects with infinite lifetime, is always smaller than or equal to the actual RNPV of such projects, and can therefore be considered as a efficient estimation of its actual robust net present value. Furthermore, calculating the estimation of RNPV is much simpler than calculating actual RNPV, due to the fact that there is no need to calculate the multiplication of covariance matrix root square by the vector of discount rate. Moreover, when a decision-making situation is very uncertain or the investor is conservative, increasing the radius and σ_{max} of the uncertainty .regions can be considered. The results of numerical calculations indicated that when comparing the estimated RNPV and actual RNPV methods, despite estimated RNPV having a lower bound than actual robust net present value with infinite lifetime, this lower bound is very close to the actual value of RNPV. In addition, one of the important findings from this study is that in the evaluation of projects with infinite life horizon, there is no need to calculate the covariance matrix, and it is sufficient to estimate σ_{max}. It is recommended that future studies consider uncertainty of annual costs and incomes separately.

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6 Appendix

Robust Net Present Value in Infinite Life State

Theorem 6.1. *: In general state, i.e. presence of variance and covariance among net incomes in a cash flow, the RNPV method for evaluation of projects with infinite lifetime is divergent.*

Proof. In this state, the square root of covariance matrix is as follows:

$$\mathbf{W} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n-1} & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2n-1} & \sigma_{2n} \\ \vdots & & \ddots & & \vdots \\ \sigma_{(n-1)1} & \cdots & \sigma_{(n-1)(n-1)} & & \sigma_{(n-1)n} \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_{n(n-1)} & \sigma_{nn} \end{pmatrix}_{n\times n} \quad {}_{n\to\infty}$$

$$RNPV = -c_0 + \boldsymbol{\mu}_{\mathbf{a}}^T\mathbf{x} - r\|\mathbf{W}^T\mathbf{x}\|_p \tag{25}$$

$$RNPV = -c_0 + \sum_{t=1}^{n} \frac{\boldsymbol{\mu}_{\mathbf{t}}}{(1+i)^t} - r((\sum_{j=1}^{n} |\sum_{t=1}^{n} \frac{\sigma_{jt}}{(1+i)^t}|^p)^{\frac{1}{p}}) \tag{26}$$

With the assumption of equality between the means of net incomes in a cash flow, we have:

$$RNPV = -c_0 + \boldsymbol{\mu_a} \sum_{t=1}^{\infty} \frac{1}{(1+i)^t} - r(\lim_{n\to\infty} (\sum_{j=1}^{n} |\sum_{t=1}^{n} \frac{\sigma_{jt}}{(1+i)^t}|^p)^{\frac{1}{p}} \tag{27}$$

Assuming that all variances and correlations are equal to each other,

$$\begin{aligned} RNPV = -c_0 + \lim_{n\to\infty} & \frac{\boldsymbol{\mu_a}(\frac{1}{1+i}(1-(\frac{1}{1+i})^n)}{(1-\frac{1}{1+i})} \\ & - r(\lim_{n\to\infty} \sum_{j=1}^{n} |\sigma_{\max} \frac{\frac{1}{1+i}(1-(\frac{1}{1+i})^n}{(1-\frac{1}{1+i})}|^p)^{\frac{1}{p}} \end{aligned} \tag{28}$$

$$RNPV = -c_0 + \frac{\boldsymbol{\mu_a}}{i} - r(\lim_{n\to\infty} (\sum_{j=1}^{n} |\frac{\sigma_{\max}}{i}|^p)^{\frac{1}{p}} \tag{29}$$

$$RNPV = -c_0 + \frac{\boldsymbol{\mu}_{\mathbf{a}}}{i} - r(\lim_{n\to\infty} |\frac{\sigma_{\max}}{i}| \sum_{j=1}^{n} 1) \tag{30}$$

As was observed, RNPV method for evaluation of projects with infinite lifetime is divergent. Therefore, when having a complete covariance matrix, the lower bound formula of RNPV with infinite lifetime cannot be calculated. As such, in the presence of covariance among net incomes of a cash flow, the real value of a project's life must be substituted in RNPV formula in order to be calculated.

□

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