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Research paper

Designing an updatable long-term health insurance

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Abstract:
In this paper, we consider the long-term health insurance (LTHI) as a sequence of short-term insurance contracts that the contracting parties negotiate to maximize their desired utility. We consider optimal premium and optimal insurance coverage as the optimal contract, and determine it based on the negotiation model. The negotiation parameter of the optimal contract is determined among the set of Pareto-optimal contracts by the Nash solution. We use state-contingent bi-linear approach and time series modeling to estimate health costs. We calculate the bootstrap projection interval for the optimal premium in the coming years due to the uncertainty in estimating the parameters of the predictive model. Thus, the policyholder is aware of the premium projection interval at the time of contract conclusion.

Keywords: Reclassification, Pareto-optimal Contract, Nash solution, Bootstrap method
JEL Classification: I130, C78, C53.

1 Introduction

The population of many countries is aging due to the mortality and fertility rates and downturn in the prevalence of the disease. With incremental life expectancy, medical costs and consequently the demand for health insurance is increasing. Insurance companies usually offer the short-term health insurance where estimation of medical costs is less uncertain. At the time of signing such a contract, the premium is determined as the present value of the expected costs in the next year. Moreover, the insurers are able to underwrite risks and adjust the next year's premiums due to health state of insured in the current year. However, if the insured's health condition deteriorates, the amount of next year premium and insurance coverage will be indeterminate. In short-term health insurance, the insured is exposed to the risk of reclassification and doesn't know whether he/she would have insurance coverage

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for the next year or not! Hence, such short-term health insurances are not very attractive to policyholders. Conversely, in some cases the insured, learning, he/she will have relatively high health costs, buys short-term health insurance. Therefore, the potential of exciting adverse selection is higher than the long-term one.

Claxton et al. (2017) described undesirable features of short-term health insurance such as a raised uninsurance rate of population, unaffordable premiums for high risk individuals and premium fluctuations due to the changes in health conditions. To overcome this issue, using standard regulatory tools such as community-rated premiums and guaranteed issuance, lead to cross-subsidization from the low risk towards the high risk. The main unintended consequence of cross-subsidization is an adverse selection and an alternative is the long-term health insurance contract (Atal et al 2020).

In conventional long-term health insurance, the level premium is set for all periods of contract and there is a unilateral commitment to terminate the contract which means the insured can replace the current contract with better ones but the insurer is not allowed to terminate the contract except in special cases. Under long-term health insurance high risk insureds pay relatively low premium and compensate by paying relatively high premium in healthy times. In theory, a long-term contract can reduce the risk of premium fluctuations due to the reclassification risk by ensuring participation and eliminating adverse selection. However, fluctuation in the health condition of insureds may increase the insurer's risk.

Cochrane (1995) considered a sequence of short-term insurance policies in the form of time consistent insurance policies, combined with a special account and replaced with long-term contract. He determined the optimal premium based on maximizing the insured utility function. In his model the health condition of the insured was monitored at the end of each year. If his health condition improved, the insured should deposit the present value of the premium deduction, the so-called severance payments, to the account otherwise the insurer should deposit the present value of excess premium into the account. Hendel and Lizzeri determined the optimal contract in life insurance with one-side commitment.

Atal (2016) considered the impact of insured lock-in to the contract on the matching between individuals and health care providers in Chile. Fleitas et al (2018) studied limited dynamic pass-through of expected medical costs into premiums in the small group market.

Wiseman (2018) designed a dynamic model for long-term health insurance, in which the premiums and the insurer's coverage were defined based on the insured's income and health state. In his model the optimal coverage was calculated by maximizing only the utility function of insured due to the budget constraint. Ghili et al. (2019) showed that the optimal contract with one-sided commitment only partially insured reclassification risk, because fully omitting reclassification risks needs large front-loaded payments. They also, studied the welfare implications of such contracts relative to alternative insurance market. Atal et al. (2020) provided a systematic welfare analysis of an existing long-term health insurance contract with a distinct advantage of low information requirements for implementation. Their proposed contract, even though theoretically was not optimal, but provided a close approximation in terms of welfare to the optimal contract derived by Ghili et al. (2019).

Since in long-term health insurance, premium and insurance coverage (reimbursement part of health costs) are determined at the issuance time of contract, therefore it is reasonable to consider the utility of both parties. In order to find optimal contract, including optimal premium and insurance coverage, the insurance contract can be modeled as a two-person bargaining game in which the two parties negotiate with each other to maximize their profits. Several papers investigated optimal contracts in different contexts. Borch (1960) obtained the optimal set of Pareto contracts and then identified one of them, which was related to the Nash solution. Arrow (1974) proved that the optimal policy for an insurance buyer is one that offers complete coverage, beyond a fixed deductible. Borch (1992) examined the equilibrium as a result of a multi-player bargaining game, taking into account the reinsurance market. Golubin (2002) studied a problem of parametrical optimization of insurance policies under various assumptions on the risk distribution. Golubin (2006) analyzed Pareto's optimal insurance policies when the insurer and the insured were risk averse and solved the problem in a situation where premiums were a function of the insurer's risk. Boonen et al. (2016) assumed that insurance companies shared additive utility functions and optimal reinsurance contract was determined by Nash bargaining solution. Ghossoub (2017) identified the optimal Pareto insurance policies under the heterogeneity of distribution based on the Arrow case (1971). Jiang et al (2019) assumed the two parties of the reinsurance contract may not agree on the loss distribution during negotiations. After determining the Pareto's optimal contracts, they specify the optimal insurance contract commensurate with Nash(1950) and KalaiSmorodinsky(1975) bargaining solution. In this paper, we consider long-term health insurance as a sequence of annual insurance policies in which optimal contract in the given period, including optimal premium and optimal coverage, determined based on negotiation model where the negotiation parameter is obtained by Nash solution. Health costs are determined using the model of Christiansen et al. (2018) in relation to insureds age, initial health state and calendar year of the contract. Health states are classified according to the health costs so that the parameters of projective model depend on the health state. Due to the uncertainty in estimating the parameters of health costs predictive model, the projection interval for future optimal premiums is determined at the beginning of the contract using bootstrap methods. Assuming that the insured and the insurer are risk averse, they enter into a long-term contract that ensure the benefit of both parties in this way, the insured is covered against reclassification but the coverage is state-contingent and the projection interval of the optimal premiums is available for all periods at the issuing time of contract.

The remainder of this paper is structured as follows. Section 2 2 analyzes the methodology of our proposed model for long-term health insurance. Section 3 3 presents the application of our model to real data. Finally, we concluded this paper in Section 4 4.

2 Methodology

We propose a T-period health insurance contract in which insured's health costs at age x in the calendar year t denoted by $Y_x(t,)$. We assume that the insurer and the insured are risk averse. Following [9], $Y_x(t)$ drawn from normal distribution with parameters $\left(\alpha_x^t + \beta_x^t k^t, \sigma^2\right)$ Where α_x^t is the age index and exhibit the typical shape of the average annual medical costs. k^t accounts for the time trend due to improvements in longevity, medical inflation, etc. which is projected by $ARIMA(0,1,0)$ model while β_x^t is an age response that modulate k^t and $\beta_x^t k^t$ is the interaction of age and time.

Based on the health expenses, we assume health states as the 3 bounded classes. If $a_i \leq Y_x(t) \leq b_i$ then $Y_x(t)$ belongs to class i, and $Y_x(t,i)$ fallows normal distribution with parameters $a_x^{t,i} + \beta_x^{t,i} K^{t,i}, \sigma^2$. Such that the parameters of Normal distribution related to class i. Let, $C_x(t,i)$ and $\pi_{x,s}^t$ denote the insurance coverage and premium for a person with initial health state s and current state i and health expenses $Y_x(t,i)$ respectively. $C_x(t,i)$ is continues and based on indemnity principle in insurance can not exceed $Y_x(t,i)$. The average health expenses for an x years old insured with an initial health state s at the calendar year t of contract and discounted average health costs across all T years are as follows:

$$\mu_{x,s}^t = \sum_{i=1}^{n} E\left(Y_x(t,i)\right) P\left(S_x(t) = i | S_x(0) = s\right) \tag{1}$$

$$\mu_{x,s} = \sum_{t=1}^{T} \sum_{i=1}^{n} \nu^{t-1} E\left(Y_x(t,i)\right) P\left(S_x(t) = i | S_x(0) = s\right) \tag{2}$$

If the initial state is unknown the unconditional average health costs for an x years old insured at the calendar year t of contract and discounted unconditional average health costs across all T years will be shown by $\overline{\mu}^{x+t}$ and μ_x respectively and calculated as follows:

$$\mu_x^t = \sum_{s=1}^{n} \mu_{x,s}^{x+t} P\left(S_x(0) = s\right) \tag{3}$$

$$\mu_x = \sum_{t=1}^{T} \sum_{s=1}^{n} \nu^{t-1} \mu_{x,s}^{x+t} P\left(S_x(0) = s\right) \tag{4}$$

Equations (2) and (4) agree with the second and first best outcome in Wiseman (2018). We aim to find the optimal values of $C_x(t,i)$ and $\pi_{x,s}^t$ which maximize the utility function of both parties according to the Nash solution The utility function of the insured and insurer are denoted by U and V respectively. Assuming that the insured and the insurer are risk averse, for a contract to be Pareto-optimal, we give the necessary and sufficient set of conditions, O, where the insurance must lead to the improvement of the utility for the insured and the insurer. Otherwise the existence of the insurance will be irrational. Let ν be the discount rate and s be the insured initial health state, we consider the following conditions as rational constrains:

$$O := \begin{cases} \sum_{t=1}^{n} \nu^{t-1} E\left[U\left(C_x(t,i) - Y_x(t,i) - \pi_{x,s}^t\right)\right] \geq \sum_{t=1}^{n} \nu^{t-1} E\left[U\left(-Y_x(t,i)\right)\right] \\ \sum_{t=1}^{n} \nu^{t-1} E\left[V\left(\pi_{x,s}^t - C_x(t,i)\right)\right] \geq 0 \\ 0 \leq C_x(t,i) \leq Y_x(t,i) \end{cases} \tag{5}$$

2.1 Baseline

By assuming that the insured's initial health state is known, we present the following main problem to find the set of Pareto-optimal long term health insurance contracts according to Borch (1960), Gerber(1998) and Jiang et al.(2019).

$$max_{C_x(t,i)\in[0,Y_x(t,i)],\pi_{x,s}^t\in\left[0,M_{x,s}^t\right]} \tag{6}$$

$$H(C,\pi) = \rho \sum_{t=1}^{T} \nu^{t-1} E\left[U\left(C_x(t,i) - Y_x(t,i) - \pi_{x,s}^t\right)\right] +$$

$$(1-\rho) \sum_{t=1}^{T} \nu^{t-1} E\left[V\left(\pi_{x,s}^t - C_x(t,i)\right)\right]$$

$$s.t. \sum_{t=1}^{T} \nu^{t-1} E\left(C_x(t,i)\right) \leq \sum_{t=1}^{T} \nu^{t-1} \mu_{x,s}^t \tag{7}$$

$M_{x,s}^t$ is the maximum premium payable due to the risk aversion of the insured which is determined as follows:

$$M_{x,s}^t = (1+\theta)\,\mu_{x,s}^t \tag{8}$$

For solving the main problem , we consider two following steps:

Step1) For a fix amount of $\pi_{x,s}^t$ and $\rho \in [0,1]$ we modify the problem by applaying point-wise maximization Raviv(1992) and Ghossoub(2007)

$$\rho \sum_{t=1}^{T} \nu^{t-1} E\left[U\left(C_x(t,i) - Y_x(t,i) - \pi_{x,s}^t\right)\right] + \tag{9}$$

$$(1-\rho) \sum_{t=1}^{T} \nu^{t-1} E\left[V\left(\pi_{x,s}^t - C_x(t,i)\right)\right]$$

$$s.t. \quad \sum_{t=1}^{T} \nu^{t-1} E\left(C_x(t,i)\right) \leq \sum_{t=1}^{T} \nu^{t-1} \mu_{x,s}^t \tag{10}$$

We form the Lagrangian with respect to term under expectation operator:

$$L\left(Y_x(t,i), C_x(t,i), \pi_{x,s}^t, \lambda\right) = \rho \sum_{t=1}^{T} \nu^{t-1} U\left(C_x(t,i) - Y_x(t,i) - \pi_{x,s}^t\right)$$
$$+ (1-\rho) \sum_{t=1}^{T} \nu^{t-1} V\left(\pi_{x,s}^t - C_x(t,i)\right) + \lambda \left[\sum_{t=1}^{T} \nu^{t-1}\left(C_x(t,i) - \mu_{x,s}^t\right)\right] \qquad (11)$$

Now we solve the following problem for the fixed $\pi_{x,s}^t$ and arbitrary λ to find optimal insurance coverage

$$max_{C_x(t,i)\in[0,Y_x(t,i)]} L\left(C_x(t,i), Y_x(t,i), \pi_{x,s}^t\right) \qquad (12)$$

$L_1(.)$, $L_2(.)$ denote the first and second partial derivative of L with respect to $C_x(t,i)$. According to risk aversion of the insured, the insurer and concavity of utility functions we have

$$L\left(Y_x(t,i), C_x(t,i), \pi_{x,s}^t, \lambda\right) = \qquad (13)$$
$$\rho \sum_{t=1}^{T} \nu^{t-1} U''\left(C_x(t,i) - Y_x(t,i) - \pi_{x,s}^t\right) +$$
$$(1-\rho) \sum_{t=1}^{T} \nu^{t-1} V''\left(\pi_{x,s}^t - C_x(t,i)\right) \leq 0$$

Therefore, L is strongly concave, and the equation 13 has a unique solution, which we show by $C^*_{(Y_x(t,i),\lambda^*)}$. Using Theorem 1, Lemmas 1 , we acquire the answer to main problem.

Lemma 2.1. *if there exists a $\lambda^* \in \mathbb{R}$ that satisfies these two following condition, the optimal coverage of the insurer $C^*_{(Y_x(t,i),\lambda^*)} \in O$ solves problem in step 1*

(i) $\forall C_x^{t,i} \in O$ *and satisfies in (5),* $L\left(Y_x(t,i), C^*_{(Y_x(t,i),\lambda^*)}, \pi_{x,s}^t, \lambda^*\right) \geq L\left(Y_x(t,i), C_x(t,i), \pi_{x,s}^t, \lambda\right)$

(ii) $\sum_{t=1}^{T} \nu^{t-1} E\left(C^*_{(Y_x(t,i),\lambda^*)}\right) \leq \sum_{t=1}^{T} \nu^{t-1} \mu_{x,s}^t$

Proof. For every $C_x(t,i) \in O$ that satisfy (9) and (10), if we have

$$\rho \sum_{t=1}^{T} \nu^{t-1} E\left[U\left(C^*_{(Y_x(t,i),\lambda^*)} - Y_x(t,i) - \pi_{x,s}^t\right)\right] + \qquad (14)$$
$$(1-\rho) \sum_{t=1}^{T} \nu^{t-1} E\left[V\left(\pi_{x,s}^t - C^*_{(Y_x(t,i),\lambda^*)}\right)\right] + \lambda\left(\sum_{t=1}^{T} \nu^{t-1}\left(\mu_{x,s}^t - C^*_{(Y_x(t,i),\lambda^*)}\right)\right) \geq$$
$$\rho \sum_{t=1}^{T} \nu^{t-1} E\left[U\left(C_x(t,i) - Y_x(t,i) - \pi_{x,s}^t\right)\right] +$$
$$(1-\rho) \sum_{t=1}^{T} \nu^{t-1} E\left[V\left(\pi_{x,s}^t - C_x(t,i)\right)\right] + \lambda\left(\sum_{t=1}^{T} \nu^{t-1}\left(\mu_{x,s}^t - C_x(t,i)\right)\right)$$

If we consider (7) as an equation, assuming γ as loading factor, we have

$$(1+\gamma)\sum_{t=1}^{T}\nu^{t-1}E\left(C^{*}_{(Y_x(t,i),\lambda^*)}\right)=\sum_{t=1}^{T}\nu^{t-1}\mu^{t}_{x,s} \tag{15}$$

If we take expected value of both side due to (7) we have

$$\begin{aligned}&H\left(C^{*}_{(Y_x(t,i),\lambda^*)},\pi^{t}_{x,s}\right)-H\left(cy_i^{x+t},\pi^{t}_{x,s}\right)\geq \\ &\lambda\left(1+\gamma\right)E\left(C^{*}_{(Y_x(t,i),\lambda^*)}-C_x\left(t,i\right)\right)=0\end{aligned} \tag{16}$$

so

$$H\left(C^{*}_{(Y_x(t,i),\lambda^*)}-\pi^{t}_{x,s}\right)\geq H\left(C_x\left(t,i\right)-\pi^{t}_{x,s}\right) \tag{17}$$

And the proof is completed. □

Theorem1. show that the optimal solution of main problem (4) is

$$c^{*}_{(Y_x(t,i),\lambda^*)}=min\left\{Y_x\left(t,i\right),max\left(0,c\left(Y_x\left(t,i\right),\lambda^*\right)\right)\right\} \tag{18}$$

In which $c(Y_x\left(t,i\right),\lambda^*)$ satisfy in

$$L_1\left(Y_x\left(t,i\right),C_x\left(t,i\right),\pi^{t}_{x,s},\lambda\right)=0 \tag{19}$$

Proof. Due to risk aversion of insured, insurer and concavity of utility functions, we have

$$\begin{cases}U'\left(x\right)>0 & U''\left(x\right)<0\\ V'\left(x\right)>0 & V''\left(x\right)<0\end{cases} \tag{20}$$

If ξ is the maximum value of the utility function domain, due to the risk aversion of the insured and the insurer

$$\lim_{x\mapsto\xi}U\left(x\right)=\lim_{x\mapsto\xi}V\left(x\right)=0 \tag{21}$$

Since L_1 is continues in $C_x\left(t,i\right)$

$$\begin{aligned}&L_1\left(Y_x\left(t,i\right),C_x\left(t,i\right),\pi^{t}_{x,s},\lambda\right)= \\ &\rho\sum_{t=1}^{T}\nu^{t-1}E\left[U'\left(c^{*}_{\left(y_i^{x+t},\lambda^*\right)}-y_i^{x+t}-\pi^{t}_{x,s}\right)\right] \\ &+\left(1-\rho\right)\sum_{t=1}^{T}\nu^{t-1}E\left[V'\left(\pi^{t}_{x,s}-c^{*}_{\left(y_i^{x+t},\lambda^*\right)}\right)\right]+\left[\sum_{t=1}^{T}\nu^{t-1}\right]\end{aligned} \tag{22}$$

And satisfies

$$\lim_{c\to\infty}L_1\left(Y_x\left(t,i\right),C_x\left(t,i\right),\pi^{t}_{x,s},\lambda\right)=U'\left(\infty\right)+V'\left(-\infty\right)\leq 0 \tag{23}$$

$$\lim_{c\to-\infty}L_1\left(Y_x\left(t,i\right),C_x\left(t,i\right),\pi^{t}x,s,\lambda\right)=U'\left(-\infty\right)+V'\left(\infty\right)\geq 0 \tag{24}$$

So the solution to (10),$c\left(Y_x\left(t,i\right),\lambda^*\right)$, always exists in $(-\infty,\infty)$. If $C_x\left(t,i\right) \overset{\text{a.s}}{=} 0$, then
$L_1\left(Y_x\left(t,i\right),C_x\left(t,i\right),\pi^t_{x,s},\lambda\right) \leq 0$, so $c\left(Y_x\left(t,i\right),\lambda^*\right) < 0$.
And solution of main problem is $max\left(0,c\left(Y_x\left(t,i\right),\lambda^*\right)\right)$. Otherwise, if $C_x\left(t,i\right) \overset{\text{a.s}}{=} Y_x\left(t,i\right)$, then
$L_1\left(Y_x\left(t,i\right),C_x\left(t,i\right),\pi^t_{x,s},\lambda\right) \geq 0$ consequently $c\left(Y_x\left(t,i\right),\lambda^*\right) \geq Y_x\left(t,i\right)$ almost surely. And solution is
$min\left\{Y_x\left(t,i\right),max\left(0,c\left(Y_x\left(t,i\right),\lambda^*\right)\right)\right\}$. And the optimal coverage will be $c\left(Y_x\left(t,i\right),\lambda^*\right)$.

If $c^*_{\left(Y_x(t,i),\lambda^*\right)}$ is solution in step 1, we determine π^* so that it solves problem in step 2. □

Step2) By replacing $c^*_{\left(Y_x(t,i),\lambda^*\right)}$ in the main problem, we will get the optimal premium, $\pi^{t*}_{x,s}$, which is a function of $E\left(Y_x\left(t,i\right)\right)$ and ρ. By placing π^* in $c^*_{\left(Y_x(t,i),\lambda^*\right)}$ the optimal coverage is determined as a function of $Y_x\left(t,i\right)$ and $E\left(Y_x\left(t,i\right)\right)$ and ρ. After we drive all of Pareto-optimal contracts and detect Pareto-efficient frontier, we obtain a unique solution based on specific ρ to the negotiation problem by maximizing the product of the utility gains of two parties, according to [23].

$$\max_{(c,\pi)} \sum_{t=1}^{T} \nu^{t-1}\left[E\left[U\left(C_x\left(t,i\right)-Y_x\left(t,i\right)-\pi^t_{x,s}\right)\right]-E\left[U\left(Y_x\left(t,i\right)\right)\right]\right] \tag{25}$$
$$*\sum_{t=1}^{T} \nu^{t-1} E\left(\pi^t_{x,s}-C_x\left(t,i\right)\right)$$

2.2 Assuming the insurer's initial state is known

According to Theorem 1, optimal coverage using Equation 19 becomes

$$c^*_{\left(Y_x(t,i),\lambda^*\right)} \overset{\text{a.s}}{=} min\left(Y_x\left(t,i\right),\mu^t_{x,s}+\frac{\rho\beta_1}{\rho\beta_1+\left(1-\rho\right)\beta_2}\left(Y_x\left(t,i\right)-E\left(Y_x\left(t,i\right)\right)\right)\right) \tag{26}$$

Now the optimal insurance premium is obtained by placing $c^*_{\left(y^{x+t,s},\lambda^*\right)}$ in the main problem and maximizing it with respect to $\pi^t_{x,s}$. The optimal premium denoted by equation (27)

$$\pi^{t*}_{x,s} = min\left(M^t_{x,s},max\left(0,\frac{\left(1-\rho\right)\beta_2\mu^t_{x,s}-\left(1-2\rho\right)}{\rho\beta_1+\left(1-\rho\right)\beta_2}\right)\right) \tag{27}$$

Where $M^t_{x,s}$ is the maximum amount of insurance premium. Therefore, the amount of insurance premiums, is independent of insureds the initial state and depends on the age and calendar year of the contract. In other words, the policy holder is insured against the risk of reclassification. However, the insurer's coverage is state-contingent and keeps insurer from detriment

3 Application to Real Data

3.1 Data description

For numerical example, we use the data set issued by the national health insurance of Iran the Salamat Insurance. It covers the period 2016-2020. The response $Y_x(t)$ is indexed by attained age x and calendar year t. It describes the average yearly Laboratory costs for year $t = 2016, \cdots, 2020$ at age $x = 1, 2, \cdots, 69$. The observed $Y_x(t)$ are displayed in Figure 1. The collected data are crude. We smooth the

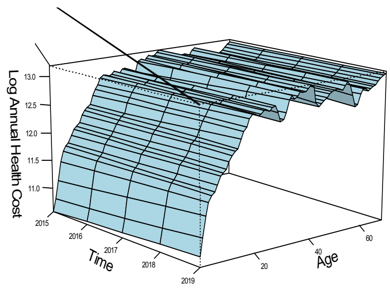

Figure 1: Observed health costs y^{x+t}

collected crude data with Whittaker-Henderson. The shape of the data is somehow similar to a mortality surface. Except in some ages, the effect of inflation is visible, due to an increase in yearly costs as time passes.

According to Christiansen et al. (2018), we predict expected health costs of upcoming years by normal distribution with parameters $\left(m_x^{t,i}, \sigma^2\right)$ in which $m_x^{t,i} = \alpha_x^{t,i} + \beta_x^{t,i}\kappa^{t,i}$ is state-contingent mean and σ^2 is constant variance of normal distribution. Since the health states are impressed directly by medical costs, we assume three states.we assume state (1) consists common expenses such as charge a fee for family doctor and periodic checkups and etc., state (2) consists hospitalization expenses for surgery etc., and state (3) specific diseases such as cancer and organ transplantation. If $a_i \leq Y_x(t) \leq b_i$ then $Y_x(t)$ belongs to class i for $i \in 1, 2, 3$. That is because state (3) shows the costs of specific diseases that don't follow the specific age pattern. Since the number of polices in state 1 is significantly larger than the others, the shape of data in state (1) is close to shape of whole data. In this model entry(i, j) of the transition matrix $P_x(t)$ is transition probability between state j at time t with respect to initial state i at time 0 and is calculated as follows:

$$p_{ij}(t) = p\left(S_x(t) = j \mid S_x(0) = i\right) = p\left(a_j \leq Y_x(t) \leq b_j \mid a_i \leq Y_x(0) \leq b_i\right) \quad (28)$$

which is in the form of the truncated conditional bivariate normal distribution with mean vector $m_x = \left(m_x^{0,i}, m_x^{t,j}\right)$ and variance-covariance matrix $\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_1\varrho\sigma_2 \\ \sigma_1\varrho\sigma_2 & \sigma_2^2 \end{bmatrix}$

Fore example $p_{12}(5)$ is the probability of being in state (2) at time 5 with initial

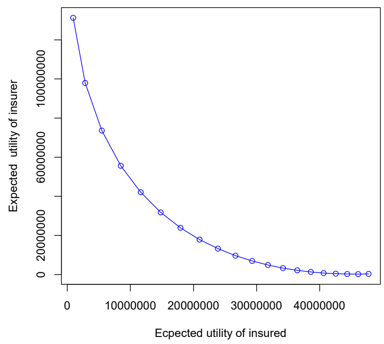

Figure 2: Pareto efficient frontier

state (1) and is computed as follows:

$$p_{12}(5) = \frac{\int_{a_1}^{b_1}\int_{a_2}^{b_2} f_{(Y_x(5,2),Y_x(0,1))}\left(y_x\left(5,2\right),y_x\left(0,1\right)\right) d_{y_x(5,2)}d_{y_x(0,1)}}{\int_{a_1}^{b_1} f_{Y_x(0,1)}\left(y_x\left(0,1\right)\right) d_{y_x(0,1)}} \tag{29}$$

3.2 Examples

We consider $T = 10$ years' health insurance contract and calculate the optimal contract for the 30-year-old insured in 2020. Assume that the utility function of the insurer V and the insured U are quadratic.

$$U\left(x\right) = -\frac{1}{2}\beta_1 x^2 + x, \qquad V\left(x\right) = -\frac{1}{2}\beta_2 x^2 + x \tag{30}$$

According to risk aversion of insured, insurer and concavity of their utility functions we let

$$\beta_1 = 0.000005, \qquad \beta_2 = 0.00001, \qquad \theta = 0.1$$

3.3 Assuming the insurer's initial state is known

For $T = 10$ years' health insurance contract, we calculate the optimal contract for the 30-year-old insured in 2020, and we set the projection interval for optimal premium in coming years. According to the quiddity of states and the distribution function of medical costs, Figure 3 shows transition map of this model. First, we calculate all of the Pareto-optimal contracts according to the O condition set by equations (26) and (27) for all 3 initial health state. Figure 2 shows Pareto efficient frontier for appropriate values of ρ with the corresponding contracts. In this case, the insured is protected from the risk of reclassification and only faces the risk of initial health status. However, the insurance coverage depends on the health state and health costs in the coming years of the contract. For the 30-year-old person, considering the negotiation parameter $\rho = 0.96$ and estimating future health costs

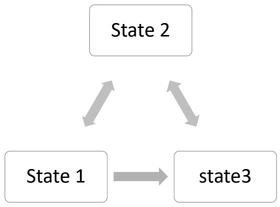

Figure 3: Multi state model

for different initial health states, the insurer's coverage during the contract are indicated in Table 1.

Table 1 : Optimal insurance coverage for estimated health costs

	2020	2021	2022	2023	2024	2025	2026
health cost, s=1	101192	299818	389548	344666	349982	359325	428750
coverage, s=1	101192	299818	389548	344666	349982	359325	350116
health cost, s=2	668072	1123680	1234221	844861	1544702	863938	1321077
coverage, s=2	263554	443291	402414	505765	363309	521165	340823
health cost, s=3	8252195	8372017	8858438	10035483	9126208	9333422	8720331
coverage, s=3	933323	933323	955359	1036539	1196357	1076984	1105341

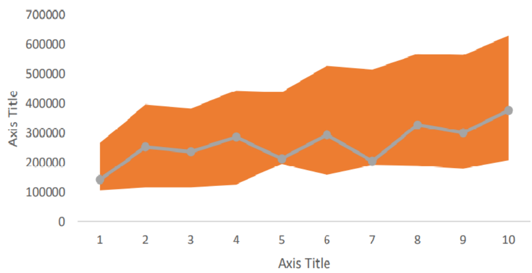

Figure 4: Premium projection interval with initial health state 1

It is important to provide projection interval for optimal premiums according the error affecting the quantities of interest. The optimal premium is calculated in terms of expected health costs and there are two different source of uncertainty that combined. One in estimating the parameters which are non-linear and other forecast errors in the projected time trends as well as uncertainty in the short-term forecast of the time factor, κ_t by the $ARIMA$ model $(0, 1, 0)$. In the current application, it is impossible to derive the relevant prediction intervals analytically because theoretical calculation with the fitted model is too complex. That is why bootstrap procedures are used. Therefore, the insured is aware of the premium projection interval during the contract. The bootstrap procedure for each year of the contract yield, $N = 1000$ sample of normal distribution with the parameters $\left(\alpha_x^{t,i} + \beta_x^{t,i}\kappa^{t,i}, \sigma^2\right)$. To set the projection interval Each time we evaluate the optimal premium. The 0.95th and 0.05th empirical percentiles are, respectively, the 950th and 50th numbers in the increasing ordered list of 1,000 replications of the

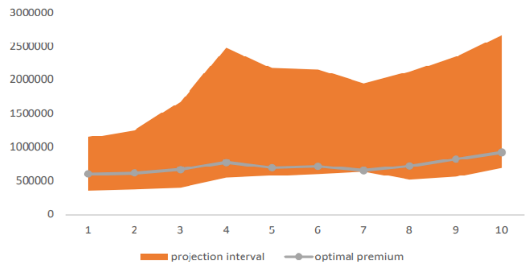

Figure 5: Premium projection interval with initial health state 2

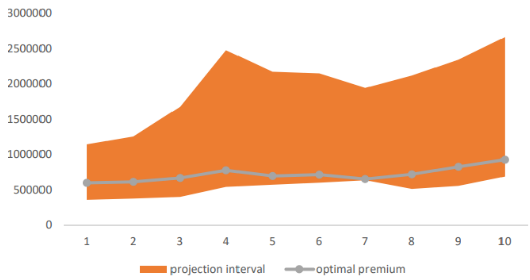

Figure 6: Premium projection interval with initial health state 3

optimal premium. The Figures 4, 5 and 6 shows the $0.95th$ percentiles projection interval for premiums during the contract period for a 30-year-old insured with each initial health state.

4 Conclusions

In this paper, we considered the long-term health insurance based on the Wiseman (2018) model as a sequence of annual health insurance policies. To improve the disadvantages of long-term health insurance, we specify the optimal contract including optimal insurance premiums and optimal insurance coverage for the healthcare costs using a negotiation model. Since the health state changes over time, the insured tends not only to be insured against risk according to his/her health state within the insurance period, but also to be insured against changing health state and reclassification of risk. The insurer also seeks a fair premium appropriate to the insured's risk. To achieve this, we determined the optimal contract based on the negotiation model, in which the negotiation parameter is calculated based on the Nash solution. The optimal premium is independent of health state so that the insured is safe against reclassification. However, the insurance coverage is state-contingent and protects the insurer from detriment. Moreover, due to the uncertainty in estimating the parameters of the prediction model, we specified the projection interval by using the bootstrap method for the optimal insurance premiums in the coming years.

Bibliography

[1] Arrow, K. J. (1974). Optimal insurance and generalized deductibles. *Scandinavian Actuarial Journal*, **1974**(1), 1–42.

[2] Arrow, K. J. (1971). The theory of risk aversion. *Essays in the theory of risk-bearing*, 90–120.

[3] Atal, J. P. (2016). *Essays in Health Economics*, Doctoral dissertation, UC Berkeley.

[4] Atal, J. P., Fang, H., Karlsson, M., and Ziebarth, N. R. (2020). *Long-term health insurance: Theory meets evidence*. National Bureau of Economic Research.

[5] Baton, B., and Lemaire, J. (1981). The core of a reinsurance market. *ASTIN Bulletin: The Journal of the IAA*, **12**(1), 57–71.

[6] Boonen, T. J. (2016). Optimal reinsurance with heterogeneous reference probabilities. *Risks*, **4**(3), 26.

[7] Borch, K. (1960). Reciprocal reinsurance treaties. *ASTIN Bulletin: The Journal of the IAA*, **1**(4), 170–191.

[8] Borch, K. (1992). *Equilibrium in a reinsurance market. In Foundations of Insurance Economics*. Springer, Dordrecht.

[9] Christiansen, M., Denuit, M., Lucas, N., and Schmidt, J. P. (2018). Projection models for health expenses. *Annals of Actuarial Science*, **12**(1), 185–203.

[10] Claxton, G., Levitt, L., and Pollitz, K. (2017). *Pre-ACA Market Practices Provide Lessons for ACA Replacement Approaches*.

[11] Cochrane, J. H. (1995). Time-consistent health insurance. *Journal of Political Economy*, **103**(3), 445–473.

[12] Gerber, H. U., and Pafum, G. (1998). Utility functions: from risk theory to finance. *North American Actuarial Journal*, **2**(3), 74–91.

[13] Ghili, S., Handel, B., Hendel, I., and Whinston, M. D. (2019). *The Welfare Effects of Long-Term Health Insurance Contracts*. Cowles Foundation for Research in Economics, Yale University.

[14] Ghossoub, M. (2017). Arrow's Theorem of the deductible with heterogeneous beliefs. *North American Actuarial Journal*, **21**(1), 15–35.

[15] Golubin, A. Y. (2002). Franchise optimization in the static insurance model. *Journal of Mathematical Sciences*, **112**(2), 4126–4140.

[16] Golubin, A. Y. (2006). Pareto-optimal insurance policies in the models with a premium based on the actuarial value. *Journal of Risk and Insurance*, **73**(3), 469–487.

[17] Handel, B. R., Hendel, I., and Whinston, M. D. (2017). *The welfare effects of long-term health insurance contracts*. Technical Report: No. w23624. National Bureau of Economic Research.

[18] Hendel, I., and Lizzeri, A. (2003). The role of commitment in dynamic contracts: Evidence from life insurance. *The Quarterly Journal of Economics*, **118**(1), 299–328.

[19] Jiang, W., Ren, J., Yang, C., and Hong, H. (2019). On optimal reinsurance treaties in cooperative game under heterogeneous beliefs. *Insurance: Mathematics and Economics*, **85**, 173–184.

[20] Kalai, E., and Smorodinsky, M. (1975). Other solutions to Nash's bargaining problem. *Econometrica: Journal of the Econometric Society*, **43**(3), 513–518.

[21] Kihlstrom, R. E., and Roth, A. E. (1982). Risk aversion and the negotiation of insurance contracts. *Journal of Risk and Insurance*, **49**(2), 372–387.

[22] Lemaire, J. (1991). Cooperative game theory and its insurance applications. *ASTIN Bulletin: The Journal of the IAA*, **21**(1), 17–40.

[23] Nash, J. (1953). Two-person cooperative games. *Econometrica: Journal of the Econometric Society*, **21**(1), 128–140.

[24] Wiseman, T. (2018). Competitive long-term health insurance. *Journal of health economics*, **58**, 144–150.

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