Journal of Mathematics and Modeling in Finance (JMMF)
Vol. 3, No. 1, Winter & Spring 2023
Research paper

Mean-standard deviation-conditional value-at-risk portfolio optimization

Maziar Salahi[1], **Tahereh Khodamoradi**[2], **Abdelouahed Hamdi**[3]

[1] University of Guilan
salahim@guilan.ac.ir

[2] University of Guilan
ta_khodamoradi@yahoo.com

[3] Qatar University
abhamdi@qu.edu.qa

Abstract:
The use of variance as a risk measure is limited by its non-coherent nature. On the other hand, standard deviation has been demonstrated as a coherent and effective measure of market volatility. This paper suggests the use of standard deviation in portfolio optimization problems with cardinality constraints and short selling, specifically in the mean-conditional value-at risk framework. It is shown that, subject to certain conditions, this approach leads to lower standard deviation. Empirical results obtained from experiments on the S&P index data set from 2016-2021 using various numbers of stocks and confidence levels indicate that the proposed model outperforms existing models in terms of Sharpe ratios.

Keywords: Portfolio optimization, mean-CVaR, variance, standard deviation.
JEL Classifications: G11, G32, C61.

1 Introduction

Value-at-Risk (VaR) as a risk measure has been criticized for not being subadditive and convex [1]. To resolve this drawbacks, Rockafellar and Uryasev [17] proposed another risk measure as the mean of the left tail distribution called Conditional Value-at-Risk (CVaR). They changed Mean-CVaR portfolio optimization problem into a linear programming problem using scenarios. Konno et al. [14] showed that CVaR is useful to control downside risk in portfolio optimization. Pineda and Conejo [16] presented a method to solve the Mean-CVaR model efficiently for a large number of stocks and scenarios. Also, Kobayashi et al. [13] studied Mean-Variance-CVaR (MV-CVaR) portfolio optimization using cardinality constraint for limiting the number of assets. Solving such a mixed-integer optimization model

[2]Corresponding author
Received: 07/04/2023 Accepted: 05/06/2023
https://doi.org/10.22054/JMMF.2023.73297.1086

is computationally challenging. So to overcome this issue, they proposed a bilevel cutting-plane algorithm. Roman et al. [18] further included variance in the Mean-CVaR model to obtain a range of balanced solutions that are usually discarded by both MV and Mean-CVaR models. Elahi and Abd Aziz [4] proposed a linear weighted sum method to solve the MV-CVaR multi-objective optimization problem. They showed their method achieves better results and helps investors to manage their portfolio better. Shi et al. [19] studied the MV-CVaR model for an insurer with an investment business and showed that the model is able to provide more potential investment strategies for an insurer. Recently Khodamoradi and Salahi [8] applied the conditional scenarios technique and the penalty alternating direction method to solve the extended Mean-CVaR optimization problem for large number of scenarios. They investigated the convergence of the PADM and showed that the proposed approach significantly reduces computational times while keeping an acceptable degree of accuracy. One of the important factor in portfolio is short selling in which it is the sale of a stock that is not owned by the seller. The seller borrows the stock to repay in the future and sells the stock in the market. After some time, the seller buys the stock from the market and pays back to the lender. Investors use short selling when they believe that the stock price will decline [2]. So Khodamoradi et. al [11] also studied Mean-CVaR portfolio optimization with cardinality constraints and short selling under uncertainty. To reduce the level of conservatism, they proposed multi-intervals uncertainty sets instead of the single uncertainty interval and showed that the proposed robust Mean-CVaR model is equivalent to a mixed integer linear programming problem. Also, Hamdi et al. [7] studied MV-CVaR optimization problem under some realistic constraints. To tackle its mixed-integer quadratic optimization model for a large number of scenarios, they used penalty decomposition method.

Although variance as a risk measure has been used in several portfolio optimization models, but it has several drawbacks such as not being a coherent risk measure [1]. However, standard deviation is coherent and more useful for measuring market volatility. Thus, in this paper we propose to combine it with the extended Mean-CVaR model. We show that under certain condition, the proposed model gives better variance compared to the MV-CVaR model. The rest of this paper is as follows. In Section 2, we present the Mean-CVaR and MV-CVaR models. In Section 3, we present the details of the Mean-Standard Deviation-CVaR (Mean-SD-CVaR) model. Finally, in Section 4 numerical results are conducted to show the advantages of using standard deviation instead of variance.

2 Extended Mean-CVaR model

Let the random vector y has probability distribution function $p(y)$ and $f(x, y) = -x^T y$ be a loss function depending on a decision vector x. By considering $\gamma \in \mathbb{R}$, the cumulative distribution function of the loss associated with x is

$$\Psi(x,\gamma) = \int_{f(x,y)\leq\gamma} p(y)dy.$$

The $\beta-$VaR of the loss associated with portfolio x for a confidence level $\beta \in (0,1)$ is

$$VaR_\beta(x) = \min\{\gamma \in R : \Psi(x,\gamma) \geq \beta\}.$$

Also, the $\beta-$CVaR of the loss associated with portfolio x is defined as follows:

$$CVaR_\beta(x) = \frac{1}{1-\beta}\int_{f(x,y)\geq VaR_\beta(x)} f(x,y)\ p(y)dy.$$

For the sake of simplicity, the auxiliary function is considered as follows [17]:

$$F_\beta(x,\gamma) = \gamma + \frac{1}{1-\beta}\int_{f(x,y)\geq\gamma} (f(x,y)-\gamma)\ p(y)dy,$$

or

$$F_\beta(x,\gamma) = \gamma + \frac{1}{1-\beta}\int (f(x,y)-\gamma)^+\ p(y)dy, \tag{1}$$

where $b^+ = \max\ \{b, 0\}$. Now, the CVaR optimization problem is

$$\min_{x\in X,\gamma} F_\beta(x,\gamma), \tag{2}$$

where X is the feasible set. Since the calculation of the density function $p(y)$ in (1) is often impossible or undesirable, scenarios y_j, $j = 1, ..., m$ are used instead. Then the approximation of F_β is obtained as follows:

$$\bar{F}_\beta(x,\gamma) = \gamma + \frac{1}{(1-\beta)m}\sum_{j=1}^{m}(f(x,y_j)-\gamma)^+$$

which leads to the following model instead of (2):

$$\min_{x\in X,\gamma} \quad \gamma + \frac{1}{(1-\beta)m}\sum_{j=1}^{m}(f(x,y_j)-\gamma)^+. \tag{3}$$

Here we consider $X = \{\sum_{i=1}^{N} x_i = 1,\ \sum_{i=1}^{N} z_i = K,\ l_i z_i \leq x_i \leq u_i z_i,\ r_i x_i \geq 0,\ z_i \in \{0,1\},\ i = 1, ..., N\}$ where N denotes the number of stocks, x_i is the proportion of investment in stock i, m is the number of scenarios, K is the desired number of stocks in the portfolio, so, $z_i = 0$ shows that stock i is not in the portfolio. Also, r_i is the expected return of stock i and the lower and upper bounds of the proportion of investment in stock i, are denoted by l_i and u_i, respectively. In order to consider the negative proportions of investment for stocks that are in the short selling positions, we use constraints $r_i x_i \geq 0$, $(i = 1, 2, ..., N)$ in the model, [9, 10]. The equivalent form of (3) and its combination with portfolio return and risk aversion parameter is as follows:

$$\begin{aligned} \min_{x,t,\gamma} \quad & \lambda(\gamma + \frac{1}{(1-\beta)m} \sum_{j=1}^{m} s_j) + (1-\lambda) \sum_{i=1}^{N} x_i r_i \\ s.t. \quad & s_j \geq f(x, y_j) - \gamma, \quad j = 1, \dots, m, \\ & s_j \geq 0, \quad j = 1, \dots, m, \\ & x \in X. \end{aligned} \tag{4}$$

By incorporating variance in model (4), we get the following extended MV-CVaR portfolio optimization model [12]:

$$\begin{aligned} \min_{x,s,z,\gamma} \quad & \lambda_1 \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \sigma_{i,j} - \lambda_2 \sum_{i=1}^{N} x_i r_i + (1 - (\lambda_1 + \lambda_2)) \left(\gamma + \frac{1}{(1-\beta)m} \sum_{j=1}^{m} s_j \right) \\ s.t. \quad & s_j \geq - \sum_{i=1}^{N} (x_i y_i^j) - \gamma, \quad j = 1, \dots, m, \\ & \sum_{i=1}^{N} x_i = 1, \\ & \sum_{i=1}^{N} z_i = K, \\ & l_i z_i \leq x_i \leq u_i z_i, \quad i = 1, ..., N, \\ & x_i r_i \geq 0, \quad i = 1, ..., N, \\ & z_i \in \{0, 1\}, \quad i = 1, ..., N, \\ & s_j \geq 0, \quad j = 1, \dots, m, \end{aligned} \tag{5}$$

where λ_1 and λ_2 adapt balance between variance, expected return and $CVaR_\beta(x)$.

3 Mean-SD-CVaR model

As variance is not a coherent risk measure, while standard deviation is, here we include standard deviation in the extended Mean-CVaR model. The Mean-SD-

CVaR model becomes the following mixed-integer second order cone program:

$$
\begin{aligned}
\min_{x,s,z,\gamma} \quad & \lambda_1 ||\Sigma^{\frac{1}{2}} x|| - \lambda_2 \sum_{i=1}^{N} x_i r_i + (1-(\lambda_1+\lambda_2))\Big(\gamma + \frac{1}{(1-\beta)m}\sum_{j=1}^{m} s_j\Big) \\
s.t. \quad & s_j \geq -\sum_{i=1}^{N} (x_i y_i^j) - \gamma, \quad j = 1,\dots,m, \\
& \sum_{i=1}^{N} x_i = 1, \\
& \sum_{i=1}^{N} z_i = K, \\
& l_i z_i \leq x_i \leq u_i z_i, \quad i = 1,...,N, \\
& x_i r_i \geq 0, \quad i = 1,...,N, \\
& z_i \in \{0,1\}, \quad i = 1,...,N, \\
& s_j \geq 0, \quad j = 1,\dots,m,
\end{aligned}
\tag{6}
$$

where Σ is the covariance matrix. In the following theorem, we give condition under which model (6) has lower standard deviation compared to model (5).

Theorem 3.1. *Let* (x^*, s^*, γ^*) *and* $(\bar{x}, \bar{s}, \bar{\gamma})$ *be the optimal solutions of* (5) *and* (6). *If* $||\Sigma^{\frac{1}{2}} x^*|| + ||\Sigma^{\frac{1}{2}} \bar{x}|| - 1 \leq 0$ *then* $||\Sigma^{\frac{1}{2}} \bar{x}|| \leq ||\Sigma^{\frac{1}{2}} x^*||$.

Proof. Since the feasible regions of both models are the same, we have

$$
\begin{aligned}
\lambda_1 {x^*}^T \Sigma x^* - \lambda_2 {x^*}^T r + (1-(\lambda_1+\lambda_2))(\gamma^* + \frac{s^*}{(1-\beta)m}) \leq \lambda_1 \bar{x}^T \Sigma \bar{x} - \\
\lambda_2 \bar{x}^T r + (1-(\lambda_1+\lambda_2))(\bar{\gamma} + \frac{\bar{s}}{(1-\beta)m})
\end{aligned}
$$

and

$$
\begin{aligned}
\lambda_1 ||\Sigma^{\frac{1}{2}} \bar{x}|| - \lambda_2 \bar{x}^T r + (1-(\lambda_1+\lambda_2))(\bar{\gamma} + \frac{\bar{s}}{(1-\beta)m}) \leq \lambda_1 ||\Sigma^{\frac{1}{2}} x^*|| - \\
\lambda_2 {x^*}^T r + (1-(\lambda_1+\lambda_2))(\gamma^* + \frac{s^*}{(1-\beta)m}).
\end{aligned}
$$

By summing up these two inequalities, we get

$$
{x^*}^T \Sigma x^* + ||\Sigma^{\frac{1}{2}} \bar{x}|| - \bar{x}^T \Sigma \bar{x} - ||\Sigma^{\frac{1}{2}} x^*|| \leq 0.
$$

This is further equivalent to

$$
(||\Sigma^{\frac{1}{2}} x^*|| - ||\Sigma^{\frac{1}{2}} \bar{x}||)(||\Sigma^{\frac{1}{2}} x^*|| + ||\Sigma^{\frac{1}{2}} \bar{x}|| - 1) \leq 0.
$$

Now if $||\Sigma^{\frac{1}{2}} x^*|| + ||\Sigma^{\frac{1}{2}} \bar{x}|| - 1 \leq 0$, then $||\Sigma^{\frac{1}{2}} \bar{x}|| \leq ||\Sigma^{\frac{1}{2}} x^*||$. □

Remark 3.2. Under the condition of Theorem 3.1, it is expected model (5) gives better returns compared to model (6) as it has higher standard deviation.

4 Computational experiments

In this section, we compare the performance of models (4), (5) and (6) from different perspective by the data of S&P index for 2016-2020 with 110 stocks when $\lambda = \frac{1}{2}$, $\lambda_1 = \lambda_2 = \frac{1}{3}$, $u_i = -l_i = 0.2$ for different K and β values. All computations are performed in MATLAB R2017a on a 2.50 GHz laptop with 4 GB of RAM, and CVX 2.2 is used to solve all optimization models [6]. To solve the mixed-integer models, we used Mosek in CVX. Tables 1 and 2 show the returns, risks(standard deviation), Sharpe ratios and CVaR values of models (4), (5) and (6) for different m and β values with monthly and annual returns. The results show that using standard deviation in the model significantly reduces the risk and increases the Sharpe ratio of model (6) though reduces the returns. These results are depicted in Figures 1-4 for returns, risks, CVaRs, and Sharpe ratios differently. As we see in Figure 1, in terms of returns, model (4) is the best, model (5) is the second best and model (6) is the worst, while in terms of risks in Figure 2 the results are reverse and model (6) is the best and model (4) is the worst. In terms of CVaR, as can be seen in Figure 3, no model is the best always and in terms of Sharpe ratios as can be seen in Figure 4, model (6) significantly outperforms the other two models.

The results show that the returns of model (5) have increased in the range of 33 to 51 percents compared to model (6), while the standard deviation of model (6) have reduced in the range of 36 to 75 percents compared to model (5). Also, the Sharpe ratios of model (6) have increased in the range of 53 to 70 percents compared to model (5). As Sharpe ratio indicates the efficiency of a portfolio, thus model (6) outperforms the other two models. Similarly, with annual return, the returns of the model (5) have increased in the range of 7 to 15 percents compared to model (6), while the standard deviation of model (6) have reduced in the range of 84 to 99 percents compared to model (5) and their Sharpe ratios have increased in the range of 67 to 89 percents compared to model (5). To see all results together,we have depicted all of them in Figures 5.

4.1 Out-of-sample experiments

Sine Sharpe ratio is a widely used performance indicator of the portfolio relative to its risk [15], we compare the performance of models (4), (5) and (6) in terms of out-of-sample Sharpe ratio, return and standard deviation with S&P index data for 2016-2021. It represents the additional amount of return that an investor receives per unit of increase in risk. In comparing two portfolios, the one with higher Sharpe ratio is better [5].

We use the rolling-horizon procedure for out-of-sample performance [3]. The out-of-sample Sharpe ratio is an indicator that balances the return and risk, given

Table 1: Return, risk, CVaR and Sharpe ratio values of models (4), (5) and (6) for different m values and different confidence levels β for S&P index data when $K = 30$ with monthly returns for 2016-2020.

		Model (4)				Model (5)				Model (6)			
β	m	Return	Standard deviation	CVaR	Sharpe ratio	Return	Standard deviation	CVaR	Sharpe ratio	Return	Standard deviation	CVaR	Sharpe ratio
$\beta = 0.9$	m=100	0.0853	0.1190	0.0058	0.7169	0.0796	0.0908	0.0030	0.8763	0.0518	0.0411	0.0021	1.2590
	m=500	0.0856	0.1230	0.0097	0.6958	0.0826	0.0911	0.0094	0.9070	0.0475	0.0342	0.0085	1.3908
	m=1000	0.0856	0.1183	0.0098	0.7236	0.0832	0.0918	0.0097	0.9060	0.0498	0.0362	0.0079	1.3755
	m=2000	0.0861	0.1205	0.0112	0.7148	0.0835	0.0932	0.0111	0.8969	0.0480	0.0341	0.0088	1.4099
	m=3000	0.0862	0.1231	0.0108	0.7000	0.0832	0.0928	0.0105	0.8969	0.0471	0.0335	0.0085	1.4036
	m=4000	0.0860	0.1222	0.0105	0.7036	0.0839	0.0951	0.0106	0.8820	0.0489	0.0345	0.0093	1.4185
	m=5000	0.0862	0.1245	0.0111	0.6926	0.0839	0.0945	0.0109	0.8875	0.0478	0.0335	0.0090	1.4278
$\beta = 0.95$	m=100	0.0850	0.1111	0.0050	0.7647	0.0822	0.0997	0.0038	0.8242	0.0484	0.0375	0.0041	1.2908
	m=500	0.0852	0.1191	0.0138	0.7152	0.0831	0.0955	0.0135	0.8699	0.0467	0.0336	0.0084	1.3899
	m=1000	0.0855	0.1231	0.0135	0.6946	0.0831	0.0928	0.0134	0.8956	0.0444	0.0319	0.0090	1.3925
	m=2000	0.0856	0.1210	0.0144	0.7071	0.0833	0.0929	0.0145	0.8968	0.0420	0.0296	0.0097	1.4172
	m=3000	0.0857	0.1238	0.0139	0.6924	0.0835	0.0951	0.0140	0.8776	0.0426	0.0307	0.0098	1.3900
	m=4000	0.0862	0.1222	0.0148	0.7053	0.0835	0.0935	0.0143	0.8933	0.0445	0.0320	0.0098	1.3891
	m=5000	0.0864	0.1201	0.0157	0.7197	0.0829	0.0907	0.0145	0.9131	0.0439	0.0314	0.0102	1.3988

by (SR) is calculated as

$$SR = \frac{\mu}{\sigma},$$

where μ is expected portfolio return and σ is the standard deviation of portfolio where

$$\mu = \frac{1}{T-L} \sum_{t=L}^{T-1} (x_t' \; r_{t+1}),$$

and

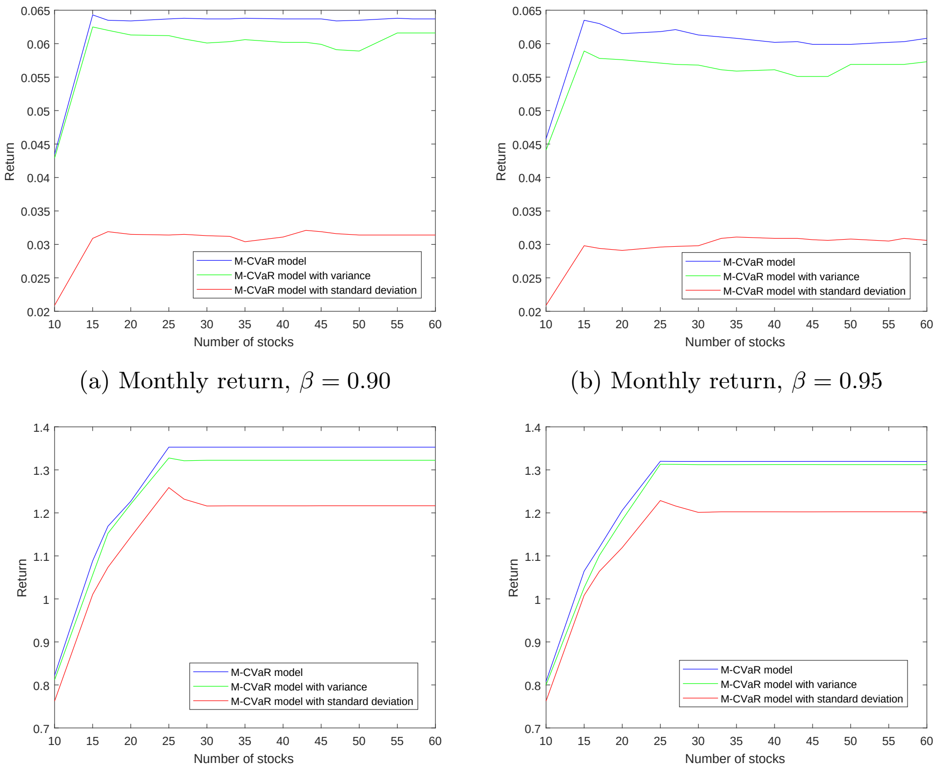
$$\sigma = \sqrt{\frac{1}{T-L-1} \sum_{t=L}^{T-1} (x_t' \; r_{t+1} - \mu)^2},$$

here x_t is the optimal weight at time t, $t = L, L+1, ..., T-1$, L is the length of the estimation time window T is the total number of returns in the data set, and r_{t+1} is the stock return. We use monthly stock return data, corresponding to 5 years for an estimation window of $L = 60$ data from 2016 to 2020 and using 2021 data for the out-of-sample. Results of solving models (4), (5) and (6) are reported in Table 3 when $N = 110$, $K = 30$, $u_i = -l_i = 0.2$ and for different number of scenarios. As we see, the Sharpe ratios of model (6) are greater than the other models.

Table 2: Return, risk, CVaR and Sharpe ratio values of models (4), (5) and (6) for different m values and different confidence levels β for S&P index data when $K = 30$ with annual returns for 2016-2020.

β	m	Model (4)				Model (5)				Model (6)			
		Return	Standard deviation	CVaR	Sharpe ratio	Return	Standard deviation	CVaR	Sharpe ratio	Return	Standard deviation	CVaR	Sharpe ratio
$\beta = 0.9$	m=100	1.3415	0.2640	0.0831	0.0462	1.2865	0.1487	0.0530	0.0786	1.2182	0.0272	0.0636	0.4071
	m=500	1.3613	0.2337	0.1230	0.0530	1.3526	0.1836	0.1249	0.0670	1.2704	0.0353	0.1460	0.3270.
	m=1000	1.3661	0.2269	0.1340	0.0547	1.3525	0.1740	0.1322	0.0707	1.2573	0.0230	0.1298	0.4980
	m=2000	1.3682	0.2367	0.1333	0.0526	1.3577	0.1881	0.1326	0.0656	1.2602	0.0257	0.1505	0.4462
	m=3000	1.3679	0.2379	0.1255	0.0523	1.3552	0.1764	0.1249	0.0699	1.2580	0.0265	0.1262	0.4323
	m=4000	1.3684	0.2374	0.1375	0.0524	1.3538	0.1777	0.1353	0.0693	1.2625	0.0277	0.1473	0.4150
	m=5000	1.3671	0.2401	0.1271	0.0518	1.3562	0.1817	0.1273	0.0679	1.2617	0.0268	0.1408	0.4282
$\beta = 0.95$	m=100	1.3308	0.2131	0.1155	0.0568	1.3228	0.1885	0.1134	0.0638	1.2238	0.0401	0.1176	0.2773
	m=500	1.3509	0.2301	0.1312	0.0534	1.3399	0.1543	0.1363	0.0789	1.2446	0.0190	0.1311	0.5956
	m=1000	1.3578	0.2476	0.1738	0.0499	1.3430	0.1795	0.1719	0.0680	1.2452	0.0249	0.1842	0.4541
	m=2000	1.3670	0.2350	0.1873	0.0529	1.3526	0.1640	0.1864	0.0750	1.2596	0.0243	0.1887	0.4715
	m=3000	1.3664	0.2330	0.1838	0.0533	1.3484	0.1743	0.1775	0.0703	1.2538	0.0220	0.1868	0.5187
	m=4000	1.3660	0.2261	0.1838	0.0549	1.3509	0.1660	0.1780	0.0740	1.2613	0.0256	0.1844	0.4471
	m=5000	1.3652	0.2172	0.1881	0.0571	1.3523	0.1668	0.1842	0.0737	1.2567	0.0244	0.1843	0.4689

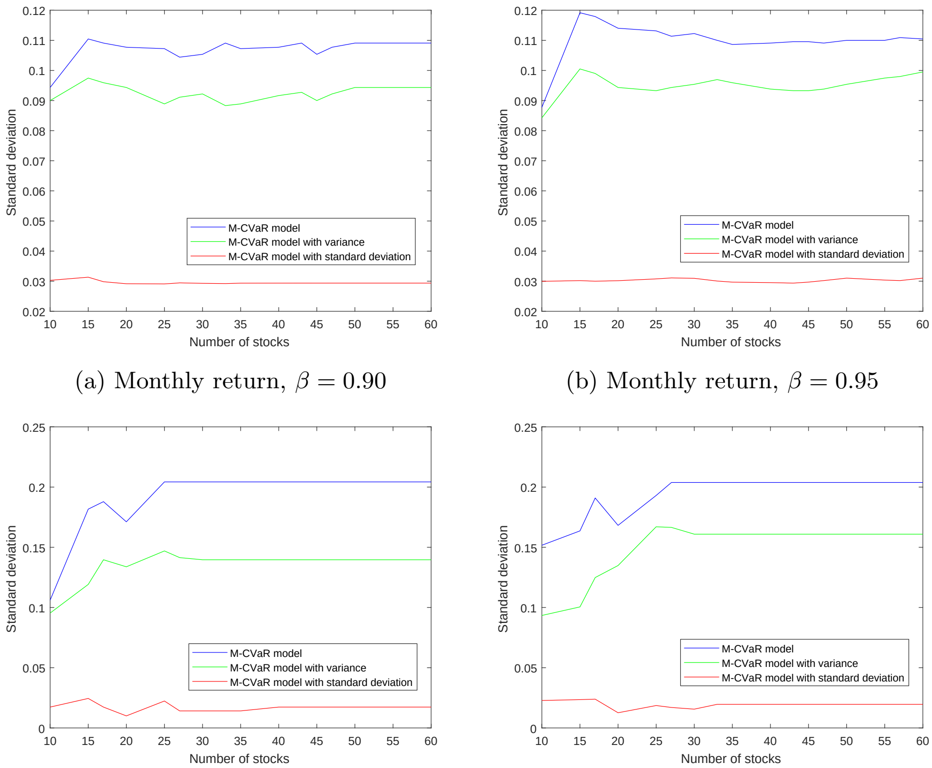
As before, these results are depicted in Figures 6-8 for different number of stocks and two different confidence levels when $m = 1000$. The results follow same trend as in the in-sample case in terms of returns, CVaRs, Sharpe ratios and standard deviations.



(a) Monthly return, $\beta = 0.90$ (b) Monthly return, $\beta = 0.95$

(c) Annual return, $\beta = 0.90$ (d) Annual return, $\beta = 0.95$

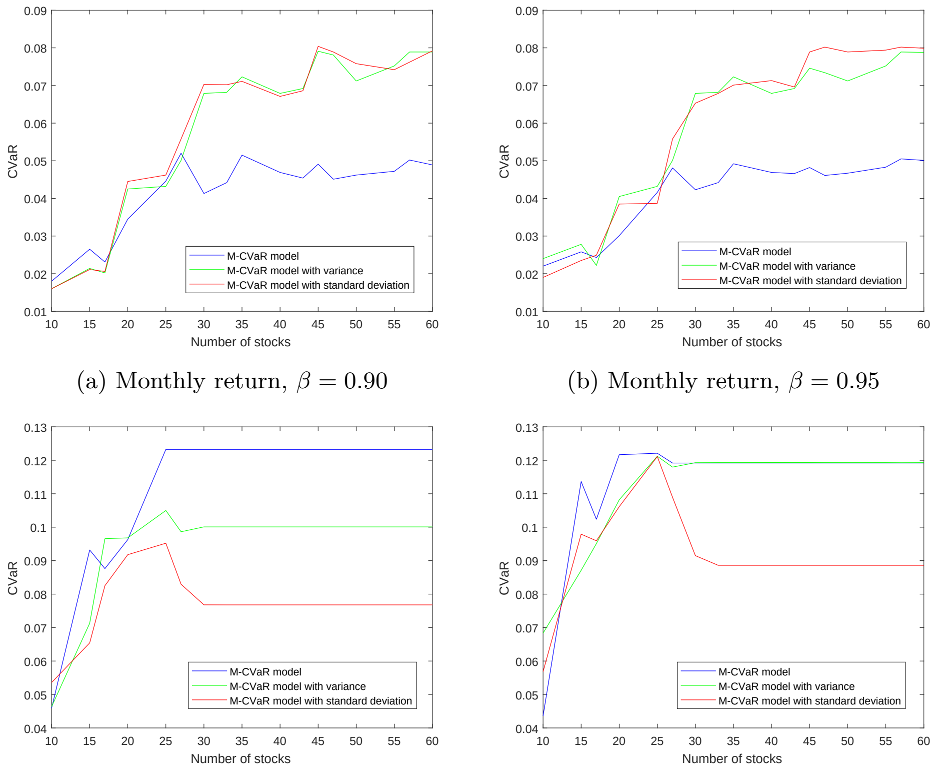
Figure 1: *Comparison of returns of models* (4), (5) *and* (6) *for different number of stocks and two different confidence levels for 2016-2020.*



(a) Monthly return, $\beta = 0.90$ (b) Monthly return, $\beta = 0.95$

(c) Annual return, $\beta = 0.90$ (d) Annual return, $\beta = 0.95$

Figure 2: *Comparison of risks of models* (4), (5) *and* (6) *for different number of stocks and two different confidence levels for 2016-2020.*



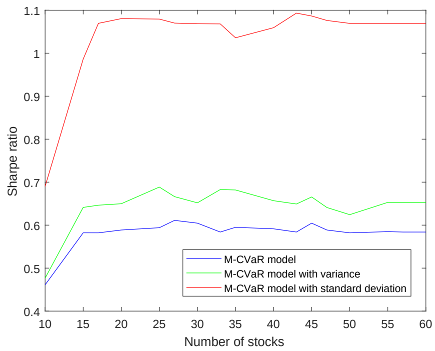
(a) Monthly return, $\beta = 0.90$

(b) Monthly return, $\beta = 0.95$

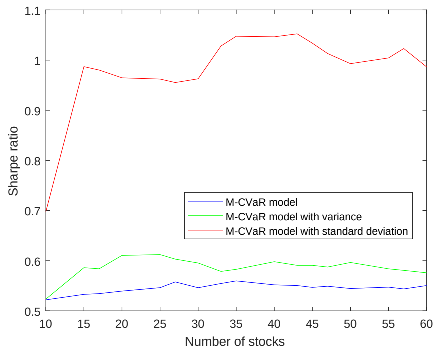
(c) Annual return, $\beta = 0.90$

(d) Annual return, $\beta = 0.95$

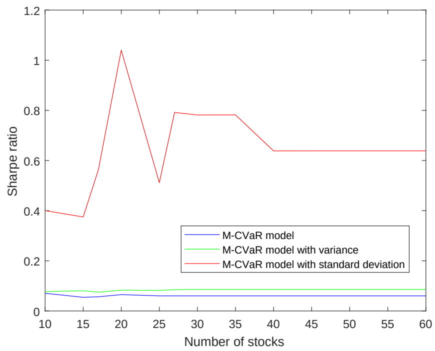
Figure 3: *Comparison of CVaR values of models (4), (5) and (6) for different number of stocks and two different confidence levels for 2016-2020.*



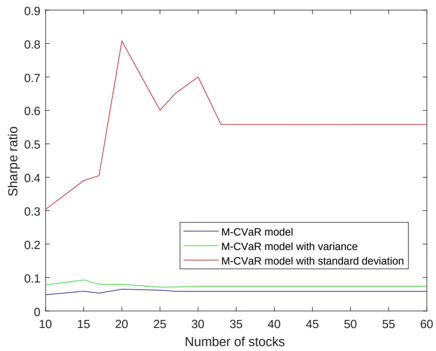
(a) Monthly return, $\beta = 0.90$



(b) Monthly return, $\beta = 0.95$

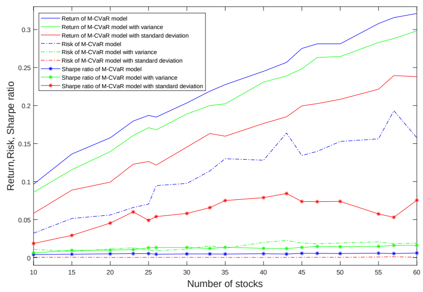


(c) Annual return, $\beta = 0.90$

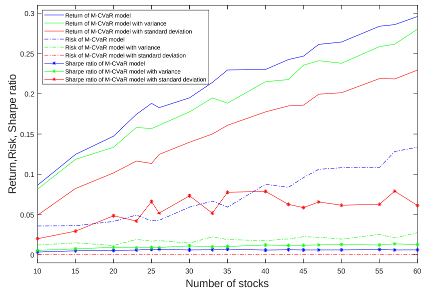


(d) Annual return, $\beta = 0.95$

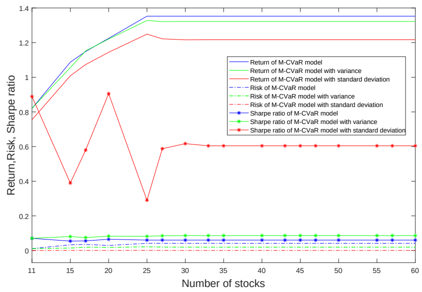
Figure 4: *Comparison of Sharpe ratios of models* (4), (5) *and* (6) *for different number of stocks and two different confidence levels for 2016-2020.*



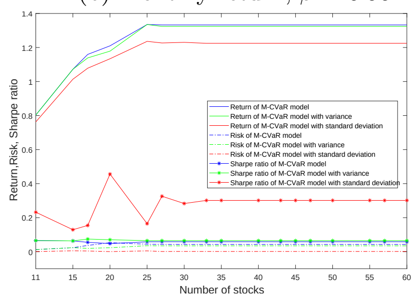
(a) Monthly return, $\beta = 0.90$



(b) Monthly return, $\beta = 0.95$

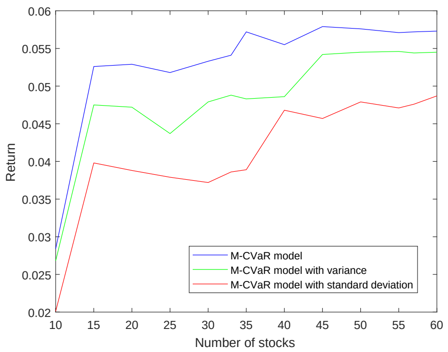


(c) Annual return,$\beta = 0.90$

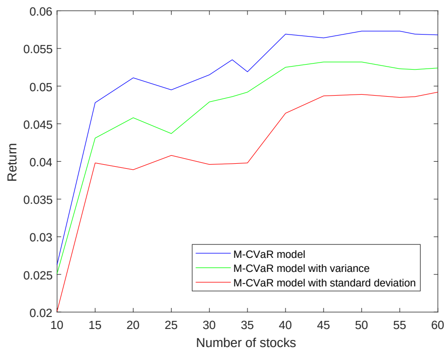


(d) Annual return, $\beta = 0.95$

Figure 5: *Comparison of Sharpe ratios vs returns of models* (4), (5) *and* (6) *for two different confidence levels.*



(a) $\beta = 0.90$

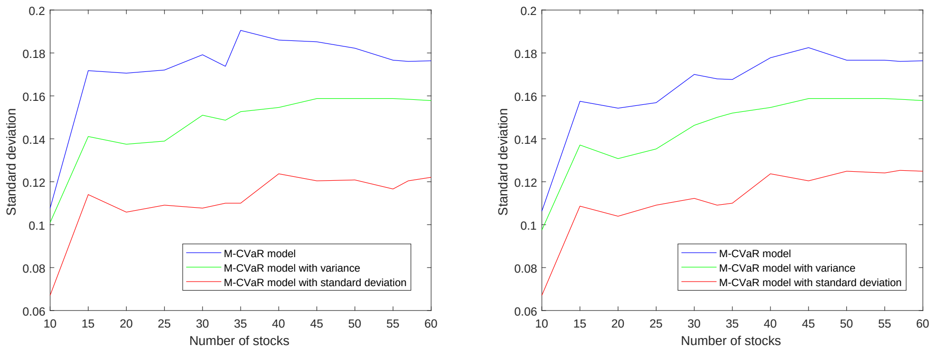


(b) $\beta = 0.95$

Figure 6: *Comparison of out-of-sample returns of models* (4), (5) *and* (6) *for different number of stocks and two different confidence levels.*

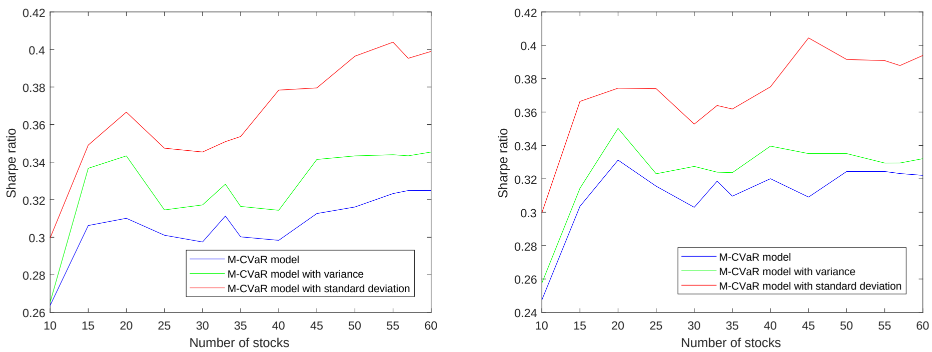
Table 3: Comparison of out-of-sample returns, standard deviations and Sharpe ratios for models (4), (5) and (6) for S&P index data with different confidence levels (β) and different number of scenarios (m) when $K = 30$.

β	m	Model (4)			Model (5)			Model (6)		
		Return	Standard deviation	Sharpe ratio	Return	Standard deviation	Sharpe ratio	Return	Standard deviation	Sharpe ratio
$\beta = 0.9$	m=100	0.0446	0.1191	0.3745	0.0429	0.1011	0.4244	0.0415	0.0996	0.4296
	m=500	0.0445	0.1096	0.4061	0.0445	0.1091	0.4079	0.0444	0.1082	0.4104
	m=1000	0.0445	0.1148	0.3877	0.0443	0.1037	0.4272	0.0387	0.0902	0.4279
	m=2000	0.0446	0.1085	0.4110	0.0441	0.1002	0.4401	0.0438	0.0992	0.4415
	m=3000	0.0444	0.1108	0.4006	0.0425	0.1031	0.4122	0.0294	0.0678	0.4336
	m=4000	0.0445	0.1112	0.4002	0.0421	0.1006	0.4184	0.0341	0.0787	0.4333
	m=5000	0.0443	0.1149	0.3856	0.0398	0.1002	0.3972	0.0295	0.0712	0.4143
$\beta = 0.95$	m=100	0.0451	0.1111	0.3970	0.0429	0.1075	0.3991	0.0299	0.0741	0.4036
	m=500	0.0453	0.1126	0.4023	0.0445	0.1082	0.4113	0.0398	0.0912	0.4364
	m=1000	0.0456	0.1136	0.4013	0.0414	0.1003	0.4129	0.0295	0.0712	0.4143
	m=2000	0.0456	0.1136	0.4014	0.0402	0.1002	0.4102	0.0299	0.0701	0.4265
	m=3000	0.0456	0.1139	0.4004	0.0439	0.1049	0.4185	0.0399	0.0856	0.4661
	m=4000	0.0451	0.1117	0.4038	0.0399	0.0971	0.4109	0.0285	0.0632	4509
	m=5000	0.0445	0.1126	0.4032	0.0391	0.0959	0.4077	0.0285	0.0659	0.4325



(a) $\beta = 0.90$ (b) $\beta = 0.95$

Figure 7: *Comparison of out-of-sample standard deviations of models (4), (5) and (6) for different number of stocks and two different confidence levels.*



(a) $\beta = 0.90$ (b) $\beta = 0.95$

Figure 8: *Comparison of out-of-sample Sharpe ratios of models (4), (5) and (6) for different number of stocks and two different confidence levels.*

5 Conclusions

In conclusion, this paper presents a novel approach to portfolio optimization by incorporating standard deviation in the Mean-CVaR model with cardinality constraints and short selling, instead of using variance. The proposed model is shown to outperform the MV-CVaR model in terms of standard deviation, under certain conditions. Furthermore, computational experiments on the SP index data set from 2016-2020 with varying numbers of stocks and confidence levels demonstrate that the new model has higher Sharpe ratios than both the Mean-CVaR and MV-CVaR models. Future research could extend the proposed model to include other constraints, such as transaction costs and options.

Bibliography

[1] Artzner, P., Delbaen, F., Eber, J.M., Heath, D, *Coherent measures of risk*, Math. Finance., 9 (1999), pp. 203–228.

[2] Chang, K., Young, M., Liu, C., and Chung, H, *Behavioral stock portfolio optimization through short-selling*, Int. j. model. optim., 8(2018), pp. 125–130.

[3] Dai, Z., Wen, F, *A generalized approach to sparse and stable portfolio optimization problem*, J. Ind. Manag. Optim., 14 (2018), 1651.

[4] Elahi, Y., Abd Aziz, M.I, *Mean-variance-cvar model of multiportfolio optimization via linear weighted sum method*, Math. Probl. Eng., 2014 (2014).

[5] Goodwin, T.H, *The information ratio*, Financ. Anal. J., 54 (1998), pp. 34–43.

[6] Grant, M., Boyd, S., Ye, Y, *Cvx: Matlab software for disciplined convex programming*, version 2.0 beta (2013).

[7] Hamdi, A., Khodamoradi, T., Salahi, M, *A penalty decomposition algorithm for the extended mean-variance-cvar portfolio optimization problem*, Discrete Math. Algorithms Appl., (2023).

[8] Khodamoradi, T., Salahi, M, *Extended mean-conditional value-at-risk portfolio optimization with PADM and conditional scenario reduction technique*, Comput. Stat., (2022), pp. 1–18

[9] KHODAMORADI, T., SALAHI, M., NAJAFI, A.R, *A note on CCMV portfolio optimization model with short selling and risk-neutral interest rate*, Stat. Optim. Inf. Comput., 8 (2020), pp. 740–748.

[10] KHODAMORADI, T., SALAHI, M., NAJAFI, A.R, *Cardinality-constrained portfolio optimization with short selling and risk-neutral interest rate*, Decis. Econ. Finance., (2021), pp. 1–18,

[11] KHODAMORADI, T., SALAHI, M., NAJAFI, A.R, *Multi-intervals robust mean-conditional value-at-risk portfolio optimization with conditional scenario reduction technique*, Int. J. Appl. Decis., (2022), pp. 1–18

[12] KIM, J.H., LEE, Y., KIM, W.C., FABOZZI, F.J, *Mean–variance optimization for asset allocation*, J. Portf. Manag., 47 (2021), pp. 24–40.

[13] KOBAYASHI, K., TAKANO, Y., NAKATA, K, *Bilevel cutting-plane algorithm for cardinality-constrained mean-cvar portfolio optimization*, J Glob Optim., 81 (2021), pp. 493–528.

[14] KONNO, H., WAKI, H., YUUKI, A, *Portfolio optimization under lower partial risk measures*, Asia-Pac. Financial Mark. , 9 (2002), pp. 127–140.

[15] LO, A.W, *The statistics of sharpe ratios* Financ. Anal. J., 58 (2002), pp. 36–52.

[16] PINEDA, S., CONEJO, A, *Scenario reduction for risk-averse electricity trading*, IET Gener. Transm. Distrib., 4 (2010), pp. 694–705.

[17] ROCKAFELLAR, R.T., URYASEV, S, *Optimization of conditional value-at-risk*, J. Risk., 2 (2000), pp. 21–42

[18] ROMAN, D., DARBY-DOWMAN, K., MITRA, G., *Mean-risk models using two risk measures: a multi-objective approach*, Quant Finance., 7 (2007), pp. 443–458.

[19] SHI, Y., ZHAO, X., YAN, X, *Optimal asset allocation for a mean-variance-cvar insurer under regulatory constraints*, Am. j. ind. bus. manag., 9 (2019), 1568.

How to Cite: Maziar Salahi[1], Tahereh Khodamoradi[2], Abdelouahed Hamdi[3], *Mean-standard deviation-conditional value-at-risk portfolio optimization*, Journal of Mathematics and Modeling in Finance (JMMF), Vol. 3, No. 1, Pages:83–98, (2023).