Portfolio Selection by a Non-Radial DEA Model: Evidence from of Tehran Stock Exchange (TSE)

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Abstract:
In this paper, considering risks of a portfolio such as mean return, variance of returns, and moments of higher order as output variables including desirable and undesirable outputs, we introduce a non-radial and slack based score to measure efficiency of portfolios. Using the present measure, ranking of portfolios is provided which is consistent with standard risk-return ratios in finance. We provide illustrations to show the effects of this contribution on the measures of technical efficiency and ranking of portfolios on a sample set of daily prices of banks and credit institutions listed on the first stock market of Tehran Securities Exchange (TSE). The advantage of this paper is to present a model based on stock market returns and risk, which is based on the DEA view of the production possibility set. Of course, in making it, the quadratic property of variance and the origin of coordinates have been used as a moderating point.

Keywords: Data Envelopment Analysis, Portfolio, Efficiency, Mean-Variance.

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Introduction

Initial and influential work of Markowitz [8], on portfolio selection with providing models and approaches for decision-making has been presented in the financial and economic literature to evaluate the performance of portfolios of financial assets. The mean-variance approach in-troduced by Markowitz [8] is based on creating a frontier against which portfolio performance is measured.

In addition to the Markowitz method, Data Envelopment Analysis (DEA) has been developed as a non-parametric tool for performance measurement of decision-making units (DMUs) in the literature of operational and economic research. In this regard, Sengupta [13] proposed a methodology for portfolio selection through quadratic optimization based on DEA. However, this approach was used until Murthi [11] identified DEA as "extremely useful technique for measur-ing efficiency" of mutual funds.

While they used a CCR model on mutual funds, McMullen [9] used a BCC model. Prema-chandra [12] then introduced stochastic DEA and studied stock indexes, and Morey and Morey [10] used DEA for portfolio analysis. Subsequently, several studies have transposed the methodology used in production theory to the study portfolios of financial assets associated with DEA, without necessarily questioning the validity of such transposal. Although these works contributed to the elaboration of a general approach for measuring single-period portfolio effi-ciency in multi-moments framework, some reforms can still be proposed to develop an appro-priate approach to the analysis of financial assets by so much as the definition of the underlying technology or select a model orientation.

At the intersection of risk and lottery theory, some of the literature in economics emphasizes on two major changes, which have not received much attention until now in the literature on multi-criteria decision-making with DEA: multi-moment frameworks ought to replace the simple mean-variance framework and desirability in increases in risk measurements should to be con-sidered. In this article we intend to examine the technology defined by Christine [4] in portfolio analysis and appropriate selection of input and output variables and a non-radial measure based on the methodology proposed by Cooper [5] to efficiency measurement and ranking of portfo-lios.

The sections of the paper are as follows: Section introduces the DEA introductory models. The third section deals with the problem of portfolio evaluation and selecting the appropriate input and output. The proposed methodology is presented in Section , Also in Section we see the effects of the model introduced on a number of Tehran Securities Exchange (TSE) and fi-nally, concludes in Section .

Preliminaries

CCR model

Suppose there are $nDMU$s. Each $DMU_j (j = 1, \ldots, n)$ produces s different outputs $y_{rj} (r = 1, 2, \ldots, s)$ using m different inputs $x_{ij} (i = 1, 2, \ldots, m)$. The CCR ratio model [3], in evaluating the efficiency score of $DMU_o, o \in \{1, 2, \ldots, n\}$ is given by:

$$\begin{aligned}
&\max \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} && (1)\\
&s.t\\
&\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1 && j = 1, \ldots, n\\
&u_r \geq 0 && r = 1, \ldots, s\\
&v_i \geq 0 && i = 1, \ldots, m
\end{aligned}$$

where u_r $(r = 1, \ldots, s)$ and v_i $(i = 1, \ldots, m)$ are weights on outputs and inputs, respectively. Using the transformation on linear fractional programming in model, it can be converted into the following linear problem:

$$
\begin{aligned}
&\theta_o = max \sum_{r=1}^{s} u_r y_{ro} && (2)\\
&s.t. \\
&\sum_{i=1}^{m} v_i x_{io} = 1 \\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 && j = 1, \ldots, n \\
&u_r \geq 0 && r = 1, \ldots, s \\
&v_i \geq 0 && i = 1, \ldots, m
\end{aligned}
$$

where θ_o is the CCR-efficiency of DMU_o which is a number between zero and one. If $\theta_o = 1$, then DMU_o is called CCR-efficient.

Undesirable outputs models

In accordance with the global environmental conservation awareness, undesirable outputs of productions and social activities, e.g., air pollutants and hazardous wastes, are being increasing-ly recognized as dangerous and undesirable. Thus, development of technologies with less unde-sirable outputs is an important subject of concern in every area of production. DEA usually as-sumes that producing more outputs relative to less input resources is a criterion of efficiency. In the presence of undesirable outputs, however, technologies with better (desirable) outputs and less bad (undesirable) outputs relative to less input resources should be recognized as efficient.

This subsection deals with this environmental efficiency problem by applying a slacks-based measure of efficiency (SBM) introduced by Tone [14]. The SBM is non-radial and non-oriented, and utilizes input and output slacks directly in producing an efficiency measure. Here, the modification of SBM which takes undesirable outputs into account introduced by Cooper [5] is presented.

Suppose that there are $nDMU$s each having three factors: inputs, good outputs and bad (undesirable) outputs, as represented by three vectors $x \in R^m$, $y^g \in R^{s_1}$ and $y^b \in R^{s_2}$, respectively. Then, the matrices X, Y^g and Y^b are defined as follows.

$X = \{x_1, \ldots, x_n\} \in R^{m\times n}$, $Y^g = \{y_1^g, \ldots, y_n^g\} \in R^{s_1\times n}$, and $Y^b = \{y_1^b, \ldots, y_n^b\} \in R^{s_2\times n}$.

It is assumed that $X > 0$, $Y^b > 0$. The production possibility set (P) is defined by

$$P = \left\{(x, y^g, y^b)\middle|\ x \geq X\lambda,\ y^g \leq Y^g\lambda,\ y^b \leq Y^b\lambda,\ \lambda \geq 0\right\}$$

Where $\lambda \in R^n$ is the intensity vector.

Definition 0.8. A $DMU_o(x_o, y_o^g, y_o^b)$ is efficient in the presence of undesirable outputs if there is no vector $(x, y^g, y^b) \in P$ such that $x_o \geq x,\ y_o^g \leq y^g$ and $y_o^b \geq y^b$ with at least one strict inequality.

In accordance with the above definition, the modified SBM under the VRS condition was pre-sented as follows:

$$\tau^* = min \frac{1 - \frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{x_{io}}}{1 + \frac{1}{s_1+s_2}(\sum_{r=1}^{s_1}\frac{s_r^g}{y_{ro}^g} + \sum_{r=1}^{s_2}\frac{s_r^b}{y_{ro}^b})} \tag{3}$$
$$s.t.$$
$$x_o = X\lambda + s^-$$
$$y_o = Y^g\lambda - s^g$$
$$y_o^g = Y^b\lambda + s^b$$
$$1\lambda = 1$$
$$s^- \geq 0, s^g \geq 0,\ s^b \geq 0, \lambda \geq 0$$

The vectors $s^- \in R^m$ and $s^b \in R^{s_2}$ correspond to excesses in inputs and outputs, respectively, while $s^g \in R^{s_1}$ expresses shortages in good outputs. objective function of model (3) is strictly decreasing with respect to $s_i^-\ (\forall i)$, $s_r^g\ (\forall r)$ and $s_r^b(\forall r)$ and the objective value satisfies $0 < \tau^* \leq 1$.

The model (3) is a non-linear model that can be transform to the following equivalent linear program (SMB-Undesirable):

$$\tau^* = \min t - \frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{x_{io}} \tag{4}$$
$$s.t.$$
$$t + \frac{1}{s_1+s_2}\left(\sum_{r=1}^{s_1}\frac{s_r^g}{y_{ro}^g} + \sum_{r=1}^{s_2}\frac{s_r^b}{y_{ro}^b}\right) = 1$$
$$tx_o = X\lambda + s^-$$
$$ty_o = Y^g\lambda - s^g$$
$$ty_o^g = Y^b\lambda + s^b$$
$$1\lambda = 1$$
$$s^- \geq 0, s^g \geq 0,\ s^b \geq 0, \lambda \geq 0, t > 0.$$

Let an optimal solution of the above program be $(\lambda^*, s^{-*}, s^{g*}, s^{b*}, t^*)$, then, DMU_o is efficient in the presence of undesirable outputs if and only if $\tau^* = 1$, i.e., $t^* = 1$, $s^{-*} = 0$,$s^{g*} = 0$ and $s^{b*} = 0$.

Input-Output selection in portfolio problem

In portfolio analysis, decision-making units (DMUs) can either be individual securities or port-folios of securities such as investment funds or indexes, depending on the purpose of the study. One of the features of these DMUs is the Pearson linear correlation between the distribution of prices or returns. This correlation, as long as the definition of the "financial technology" is based on characteristics of the distribution of returns on financial assets, can be contrary to the implicit assumption of independence between DMUs in production theory. The linear correlation between DMUs' distributions leads to the disintegration assumption the convex combina-tions of technology set, which in turn leads to the creation non-convex technology sets. As Christine [4] have pointed out, the degree of linear dependence between financial assets affects the level of risk of any convex combination of financial assets and can be reduced by diversifi-cation. The minimum level of risk, when measured by the variance of returns distribution, is a convex quadratic function of the mean returns. The non-convexity resulting of the set frontier is consequently a problem only when it measures performance using a direction vector that fol-lows an expansion path. These dependence relationships between financial assets lead to an additional feature of portfolio analysis with DEA, although they consist of efficient portfolios, the frontier may well be comprised of portfolios made of inefficient assets.

Several scholars explored on proper input and output such as Basso [2], Galagedera [7], Bas-so [1] and Eling [6] in which risk measures are considered as input variables and return measures as output variables. On the one hand, production decision-making is based on reducing inputs and increasing outputs, and decision-making in finance is generally based on risk reduction and increased returns. On the other hand, the effective portfolio frontier is similar to that of a pro-duction frontier, then for a long time there is a comparison between efficiency analysis in pro-duction and performance analysis in finance. This analogy and desirability for return and the disadvantage of commonly accepted risk has led many authors to consider the risk-return rela-tionship of financial assets as equivalent to an input-output relationship. Christine [4] proposed a framework to treat risks of various orders as outputs. Thus, any analysis based on a multi-moment framework implies working on the output correspondence of the production possibility set.

Now suppose an output vector is $y = (f_1, f_2, \ldots, f_n)$ with f_i the random variable "risk of order n". We also assume that portfolio returns are random values of realized variables. In terms of the simple mean-variance framework, we can assume that f_1 and f_2 are the first two non-standardized moments that determine the portfolio's distributions of returns. For an initial investment (or a vector of inputs $x \in \mathbb{R}_+^n$ with $n \in \mathbb{N}$ the number of input variables), the output vector can then be defined under $y = (f_1, f_2) = (\mu.\sigma^2)$ and the financial technology set T can be defined as Output correspondence $P(x)$ is defined as the equation (2) [15]

$$T\left(x.\mu.\sigma^2\right) = \left\{\left(x.\mu.\sigma^2\right) : x \text{ returns with the first two moments } \left(\mu.\sigma^2\right)\right\}$$
$$p(x) = \left\{\left(\mu.\sigma^2\right) : \left(x.\mu.\sigma^2\right) \in T\right\} \text{ for all } x \in \mathbb{R}^+$$

It is also important to notice that the distribution of returns is generally expressed not in mone-tary units but as rates of return on investment. In this case, the output sets would be the same for any level of input x, which would translate into the following equality [15]

$$P(x) = P = \left\{(\mu, \sigma^2) : (\mu, \sigma^2) \in T\right\} \text{ for all } x \in \mathbb{R}^+.$$

Let R_j be the return on DMU_j with $j \in \{1, \dots, n\}$ at time t and consider R_j as a random variable defined on theprobability space (Ω^j, F^j, P^j), with Ω^j the sample space of the variable R_j, $F^j = \left(F_1^j, F_2^j, \dots, F_E^j\right) = \{F_e^j : e \in E\}$ the set of events that can influence the outcomes of the variable R_j, with $E \in \mathbb{N}^*$ the number of possible events and $P^j = \left(P_1^j, P_2^j, \dots, P_E^j\right) = \{P_e^j : e \in E\}$ for all j the assignment of probabilities to every event contained in F^j. Suppose that for any DMU_t, μ_t is themean return of a distribution of returns, q_t is the share of DMU_t in a portfolio, σ_t is the standard deviation of the distribution of periodic returns and ρ_{tj} is the coefficient of linear correlation with the distribution of a DMU_j. The set of admissible activity vectors that represents all possible combinations of shares q_j of initial investment in portfolio j can be defined as follow [15]

$$\mathfrak{I} = \left\{ q \in R^J : \sum_{j=1}^{J} q_j \leq 1, q_j \geq 0 \ \forall j \right\} \tag{5}$$

Depending on the theoretical framework selected, the representation of the set of possible portfolios is then expressed as the set of all the related measures such that $q \in \mathfrak{I}$. The portfolio possibility set defined in (5) on the output correspondence can then be redefined from the sample set of observed DMUs and a set of admissible activity vectors $\mathfrak{I}$ as subsets of output vectors $\mathfrak{I}$ as in equation (6) below, if free disposability was assumed on outputs.

$$P = \left\{ \left(\mu, \sigma^2\right) : \ q^T \mu \geqq \mu, \quad q^T \Omega q \leqq \sigma^2, \quad q \in \mathfrak{I} \ \right\} \tag{6}$$

with:

- μ the $(n \times 1)$ vector of mean returns of the n observed DMUs.
- Ω the $(n \times n)$ matrix of covariances of the n observed DMUs and σ^2 the $(n \times 1)$ vector ofvariance of returns of the n DMUs.
- q^T the transpose of q.

The representation of the set of possible portfolios in the mean-variance framework can also be defined as the set of all mean-variance combinations of portfolios such that $q \in \mathfrak{I}$ as in equation (7), with GR_{MV} a non-convex set.

$$GR_{MV} = \{(E_P, V_P) : q \in \mathfrak{I}\} \tag{7}$$

With $E_P = \sum_{j=1}^{J} q_j \mu_j$ the mean return and $V_P = \sum_{j=1}^{J} \sum_{j=1}^{J} q_j q_k \rho_{jk} \sigma_j \sigma_k$ the variance of returns.

Presented approach

In this section, we suggest an efficiency measure based on SBM-Undesirablemodel (see section). The main privilege of the presented model is that it is non-oriented then there doesn?t need to focus on one of the mean return reduction or variance of returns accretion.

Now, let each time series of the portfolios returns was characterized by a mean return μ^t and a variance ofweekly returns σ^{2^t} per time window t. Each joint distribution was as well characterized by a covariance of weekly returns. In the following SBM model, these variables are considered as output variables, with the variance of weekly returns being undesirable output.

According to the portfolio possibility set which is defined as P in equation (6) and model (4), we suggest the following nonlinear programming to estimate the efficiency of the o^{th} portfolios per time t.

$$\begin{aligned}
\varphi^* \quad &= \max s_1 + s_2 \\
&s.t. \quad \tau \sum_{j=1}^{n} q_j \mu_j - s_1 = \mu_o \\
&\qquad \tau^2 \sum_{j=1}^{n} \sum_{k=1}^{n} q_j q_k \sigma_{jk} + s_2 = \sigma_o^2 \\
&\qquad \sum_{j=1}^{J} q_j = 1 \\
&\qquad s_1, s_2, q_j \geq 0 \qquad j = 1, \ldots, n \\
&\qquad 0 \leq \tau \leq 1
\end{aligned} \tag{8}$$

where s_1 and s_2 are slacks in mean and risk returns, respectively. Whenever these slacks get zero values in the optimal solution, the under-evaluation portfolio lies on the efficient frontier, hence it is an efficient portfolio. As a result, φ^* measure the portfolio?s efficiency and it is a value nonnegative and $\varphi^* = 0$ identifies efficient portfolios.

Case Study

To illustrate the impact of the various changes proposed in the previous section we extracted from www.tsetmc.com daily prices of 11 Tehran Securities Exchange of banks and credit institutions listed on the first stock market (main leader board) over one years from (21 March 2015-20 March 2016). After extracting the required information each time series of the portfolios? returns was then characterized by a mean return μ^t and a variance of daily returns σ^{2^t} per time window t. Each joint distribution was as well characterized by a covariance of daily returns. In the following models, these variables are considered as output variables, with the variance of daily returns being either a desirable or undesirable output (in models oriented towards the expansion path or variance reduction, respectively). For any time window t, we defined the technology set on the output correspondence as in equation (9), with $\left(\mu_j^t, \sigma_j^{2^t}\right)$ the mean-variance combinations of each of the 11 portfolios.

$$P^t = \left\{ \left(\mu_j^t, \sigma_j^{2^t}\right) : \tau \left(\mathbf{q}^{\mathbf{T}^t} \mu^t\right) \geqq \mu^t, \tau^2(\mathbf{q}^{\mathbf{T}^t} \mathbf{\Omega}^t \mathbf{q}^t) \leqq \sigma^{2^t}, 0 \leq \tau \leq 1 \text{ }, \mathbf{q}^t \in \mathfrak{I} \right\} \tag{9}$$

Table 1: Return & Covariance of banks and credit institutions of TSE.

DMUs		DMU1	DMU2	DMU3	DMU4	DMU5	DMU6	DMU7	DMU8	DMU9	DMU10	DMU11
BANKS		Tejarat Bank	Saderat Bank	Pasargad Bank	Karafarin Bank	Sina Bank	Middle East Bank	Eghtesad Novin Bank	Parsian Bank	Post Bank	Mellat Bank	Ansar Ban
Covariance	Tejarat Bank	11603.277										
	Saderat Bank	5533.4204	9278.8856									
	Pasargad Bank	6408.105	2850.3417	86301.29949								
	Karafarin Bank	−1222.83	1720.5815	−23412.93204	38793.76							
	Sina Bank	10911.555	6113.6171	8387.199665	−3391.869	20835.921						
	Middle East Bank	9275.1844	9242.3753	−96827.98911	22804.181	27936.115	209545.92					
	Eghtesad Novin Bank	11579.934	9179.1509	50578.20428	−14792.94	11615.139	−35568.02	62803.97				
	Parsian Bank	8616.2492	−1021.416	63779.8104	−44136.64	25593.213	−24712.28	27732.74	118729.83			
	Post Bank	19946.242	8308.6203	41336.26162	−21716.32	24805.296	14736.2	46657.121	59666.932	71377.732		
	Mellat Bank	13274.541	8093.7066	51572.75617	−10879.07	15896.982	−33236.37	41182.925	31928.314	38271.665	42832.623	
	Ansar Bank	8668.3813	6761.6335	62030.29638	−17698.62	10318.102	−62258.3	41105.667	34566.671	28858.52	40069.25	55457.554
Returns		0.07398	0.11395	−0.05802	0.28292	0.12824	0.34390	0.0758	−0.19655	−0.08074	0.026317	0.027569

http://www.tsetmc.com

The results of model (8) implementation on the data in Table 1 are shown in Table 2. DMUs 1 to 4 have efficient performance because the optimal value of their objective function is zero. In fact, it has not been possible to increase returns or reduce risk during this period.

Table 2: The results of the proposed model and its risk reduction rate.

	φ^*	τ	S_1	S_2	q_1	q_2	q_3	q_4	q_5	q_6	q_7	q_8	q_9	q_10	q_11	Rank
$DMU1$	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	1
$DMU2$	0	1	0	0	0	0	0	0.18	0	0	0.82	0	0	0	0	1
$DMU3$	0	1	0	0	0	0	0	0	0	0	0.15	0	0.85	0	0	1
$DMU4$	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	1
$DMU5$	0.15	1	0.15	0	0	0	0	1	0	0	0	0	0	0	0	2
$DMU6$	0.15	1	0.15	0	0	0	0	1	0	0	0	0	0	0	0	2
$DMU7$	0.21	1	0.21	0	0	0	0	1	0	0	0	0	0	0	0	3
$DMU8$	0.21	1	0.21	0	0	0	0	1	0	0	0	0	0	0	0	3
$DMU9$	0.36	1	0.36	0	0	0	0	1	0	0	0	0	0	0	0	5
$DMU10$	0.26	1	0.26	0	0	0	0	1	0	0	0	0	0	0	0	4
$DMU11$	0.26	1	0.26	0	0	0	0	1	0	0	0	0	0	0	0	4

Due to the negative returns, the origin of the coordinates is practically in the production possibility set, and in all estimates the value of the variable t is one.

The noteworthy point in the above evaluation is that only DMU4 is self-referential. Because Model (8) is nonlinear, we do not expect reference units to be efficient. For example, in the evaluation of DMU1, DMU7 is a reference, while this unit is not efficient.

As you can see, DMU5 is inefficient and could increase returns by 0.15 ($\frac{0.12824+0.15}{0.12824} = 2.17\%$) without taking any more risk.

In fact, the above optimal answer is a local optimal answer obtained by GAMS software. Model linearization (8) can be interesting for those interested in the continuation of this research.

conclusions

The mean return and variance return are two importantoutputs of any portfolio. Although these factors are considered as outputs, minimizing variance return as the risk of portfolio is desired. Then this factor is an undesirable output. In this paper, unlike the classical DEA approaches in which undesirable output considered as input, we introduced a non-radial measure to evaluate the efficiency of portfolios. The presented model is a non-linear programming model. The pre-sented model can be applied in real applications to identify efficient and inefficient portfolios.

The advantage of this paper is to present a model based on stock market returns and risk, which is based on the DEA view of the production possibility set. Of course, in making it, the quadrat-ic property of variance and the origin of coordinates have been used as a moderating point.

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