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Research paper

Presenting a comparative model of stock investment portfolio optimization based on Markowitz model

Samaneh Mohammadi Jarchelou[1], **Kianoush Fathi Vajargah**[2], **Parvin Azhdari**[3]

[1] Department of Statistics, Islamic Azad University, Tehran North Branch
statistics1985@yahoo.com

[2] Department of Statistics, Islamic Azad University, Tehran North Branch
K_fathi@iau-tnb.ac.ir

[3] Department of Statistics, Islamic Azad University, Tehran North Branch
par_azhdari@yahoo.com

Abstract:
Investment is the selection of assets to hold and earn more profit for greater prosperity in the future. The selection of a portfolio based on the theory of constraint is classical data covering analysis evaluation and ranking Sample function. The investment process is related to how investors act in deciding on the types of tradable securities to invest in and the amount and timing. Various methods have been proposed for the investment process, but the lack of rapid computational methods for determining investment policies in securities analysis makes performance appraisal a long-term challenge. An approach to the investment process consists of two parts. Major is securities analysis and portfolio management. Securities analysis involves estimating the benefits of each investment, while portfolio management involves analyzing the composition of investments and managing and maintaining a set of investments. Classical data envelopment analysis (DEA) models are recognized as accurate for rating and measuring efficient sample performance. Unluckily, this perspective often brings us to get overwhelmed when it's time to start a project. When it comes to limiting theory, the problem of efficient sample selection using a DEA models to test the performance of the PE portfolio is a real discontinuous boundary and concave has not been successful since 2011. In order to solve this problem, we recommend a DEA method divided into business units based on the Markowitz model. A search algorithm is used to introduce to business units and prove their validity. In any business unit, the boundary is continuous and concave. Therefore, DEA models could be applied as PE evaluation. To this end, 25 companies from the companies listed on the Tehran Stock Exchange for the period 1394 to 1399 were selected as the sample size of statistics in data analysis. To analyze the data, after classification and calculations were analysed by MATLAB software, the simulation results show that performance evaluation based on constraint theory based on DEA approach and the Markowitz model presented in this paper is efficient and feasible in evaluating the portfolio of constraint theory.

Keywords: Data envelopment analysis, performance measurement, portfolio optimization, stocks, securities.
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[2]Corresponding author
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1 Introduction

Portfolio performance estimation is a serious issue from both a practical and an academic perspective (see [7, 11, 14]). In addition to the most famous functional measurements of the Trinor index, Sharp index, and Jensen index, which are still used, the portfolio boundary approach is the most important theory in performance estimation [2]. Because the mean-variance (MV) framework proposed by Markowitz [16, 20, 21]. For the boundary approach of a fixed contract basis, numerous researchers have generalized this idea and theory in order to match the real investment situation. One important supposition is that in the Markowitz's basic theory, investors build their portfolios with all the capital accessible in the market [19]. Although, a large number of empirical writing indicates that many investors prefer to limit the number of stock holdings in their portfolio, this crack between theory and actuality encourages many researchers to investigate this issue, which is defined as the problem of portfolio selection with mean variance (CCMV) [15, 17]. When portfolio performance reaches the evaluation stage, the CCMV boundary is required because the portfolio boundary approach is achieved by comparing some distances relative to the efficient boundary.

The issue of selecting the CCMV portfolio is a specific occasion of quadratic theory of constraint optimization (CCQO) problems, which is generally proven by NP-Hard. As Chang and his colleagues developed and generalized the standard model, there is much work to be done to solve the CCMV portfolio problem of selecting and calculating the boundary.

The approach of analysis and selection of securities can be applied after determining the target market of investment and areas of investment. Given that the study area is the Tehran Stock Exchange and capital market, it is suitable for investors who choose the Tehran Stock Exchange for investment. The main question and concern of investors is what share and when and at what price to buy, whether to keep the stock or buy again or sell, as well as what stock and how many shares to sell at what time and at what price, and so on.

Since Markowitz released his model, the model has made many changes and improvements in the way people view investment and portfolios, and has been applied as an effective gadget for portfolio optimization [12]. Markowitz proposed that investors consider risk and return simultaneously and choose capital allocation value between different investment occasions based on the action and reaction between the two. One of the basic subjects in capital markets should be considered investors, including individuals in real or legal, is a matter of choosing the optimal investment portfolio, and pertaining to this, the study of investors to select the best investment portfolio according to the amount of risk and return is done [5], as natural or legal persons, is the discussion of choosing the optimal investment portfolio, and in this regard, the study of investors to select the best investment portfolio according to the certain of risk and return is done. It is generally presumed that investors do not desire risk and evade it and ever seek to invest in items of assets that have the maximum return and minimum risk, that is, investors come back investment as a favorable factor. They look at the variance of returns (risk) as an unpleasant element In portfolio optimization, the important issue is the optimal selection of assets and securities that can be assigned with a given extent of capital. Investment is viewed as a desirable factor and returns to variance of returns (risk) as an unwelcome element. In portfolio optimization, the crucial issue is the optimal selection of assets and securities that can be made with a given capital, although minimizing risk and maximizing return on investment may seem easy, but in workout there are many manners to build an optimal portfolio [2]. The problem of optimizing Markowitz and determining the efficient investment frontier can be solved by mathematical models when the number of assets that can be invested and the market constraints are small [1]. But when the situations and limitations of the real world are taken into account, the problem will be complex and difficult. For many years, advanced mathematics and computers have been helping human beings to solve such complex problems in order to get them out of the situation as much as possible. Environmental uncertainty has contributed to the ambiguity. In this research, we introduce the efficiency and effectiveness of PE portfolio, under the position of constraint theory, search algorithm for data split points, portfolio efficiency in the framework of mean-variance of constraint theory, and finally by simulating a numerical example to aspects New and faster calculations and evaluation of DEA findings of business units in this study are considered to determine the efficiency of portfolio evaluation (portfolio portfolio) theory of constraint and how to implement it. Comparative performance evaluation based on constraint theory based on DEA approach and Markowitz model is presented in this paper and we also hope that this research can be a basis for conducting extensive research for theoretical and research studies.

2 Theoretical foundations and review of research background

2.1 Portfolio

A portfolio means a translated portfolio, a conceptual portfolio that goes beyond a stock portfolio and includes other non-equity investments. Technically, an investment portfolio includes the complete set of real and financial assets of the investment.

The word portfolio, in simple terms, refers to a combination of assets formed by an investor for investment. This investor can be an individual or an institution. Technically, a portfolio includes a set of invested real and financial assets of an investor.

2.2 Portfolio optimization investor

Two important components in investing decisions are the rate of risk and the return on capital assets. The choice of the optimal asset set is often made by exchanging risk and return, the maximum risk of the asset set will yield the higher the return investors. Identifying the efficiency of the portfolio of assets allows investors to obtain the maximum expected return on their investment based on the function of their utility and degree of risk aversion and risk-taking. Each investor, based on their risk-taking and risk aversion, selects a point on the efficient frontier and determines the composition of their portfolio with the goal of maximizing returns and minimizing determines the risk.

Portfolio optimization is the selection of the best combination of financial assets in a way that maximizes the return on investment portfolio and minimizes portfolio risk. The crucial idea of recent portfolio theory is that if one invests in assets that are not entirely correlated; The risk of those assets altogether neutralize and a consistent return with lower risk can be attained.

2.3 Classic optimization (Markowitz model)

Markowitz generally showed how portfolio diversification decreases its risk for the investor. Investors can earn an efficient portfolio, which is called mean-variance.

The following data are required to use the Markowitz model:

(i) Expected return for index i denoted by $E(R_i)$.

(ii) Deviation of the expected return criterion for the i-th share, denoted by S_i, as a measure of the risk per share.

(iii) Covariance, as a measure of correlation and kinetic relationship among the rates of return of different stocks, which is indicated by the δ_{ij}.

The cause of why a company's stock is considered as a risky asset is that its total return rate is not fixed (random). Because these rates change over time, the probability distribution function can be formed for them and the criteria required by the Markowitz model such as, standard covariance, mean, deviation and so on gained from it.

The Markowitz model is established on some assumptions: investors are risk opposing and have the expected desirability of incrementality, and the ultimate advisability curve of their property decreases.

Investors select their portfolio according to the expected average return on variance. Thus, their indifference curves are a function of the expected rate of return and variance.

Each investment option is infinitely divisible. Investors have a time horizon (one period) and this is the same for all investors. Investors at a certain level of risk prefer higher returns, and conversely, investors pay attention to two factors in their choice:

[a)]"High expected returns" that is a favourable element. "Uncertainty of return" that is an unfavourable element.

In order to receive the optimal capital portfolio selection in Markowitz procedure, it is assumed minimum variance is for a certain level of return, we have n units of assets in the capital market $\hat{\sigma}_{ij}$, $\hat{r} = (\hat{r}_1, \ldots, \hat{r}_n)$ and $G = \{\hat{\sigma}_{ij}\}_{i,j=1}^{n}$ when $\hat{\sigma}_{ij}$ represents the covariance between the assets i and j. The G covariance matrix is considered to be a positive semi-definite matrix. If $x = (x_1, \ldots, x_n) \in \Omega$ the weight of the invested portfolio for n risky assets where and

$$x = (x_1, \ldots, x_n) \in R^n, \sum_{i=1}^{n} x_i = 1$$

is a practical set of weight portfolios. If K is the number of risky stocks in the portfolio.

The following is CCMV optimization problem

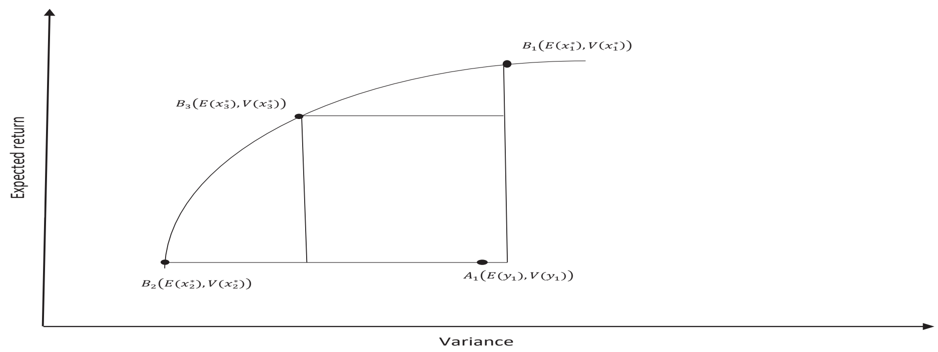
$$\begin{aligned} &\min \sum_{i=1}^{n} \sum_{i=1}^{n} \hat{\sigma}_{ij} x_i x_j && (1a) \\ &\text{S.t:} \\ &\sum_{i=1}^{n} x_i \hat{r}_i, && (1b) \\ &\sum_{i=1}^{n} x_j {=} 1, && (1c) \\ &\sum_{i=1}^{n} |\text{sign}(x_i)| {=} K_j. && (1d) \end{aligned}$$

The final variance of the stock portfolio return optimized by the portfolio function (1a) in which the set $\hat{r}$ of the return portfolio is at level (1b). Equation (1c) ensures that all capital is invested. The expected coefficient is given by Equation (1d) when the number of portfolio assets is limited by a coefficient k.

2.4 Definition of PE

In addition to traditional evaluation indicators, cross-border PE is the most crucial factor in measuring portfolio efficiency. Following the theory of portfolio boundary approach, PE is a portfolio in accordance with the boundary basis. If the boundary is different and continuous, different portfolio returns could be determined using several distances. Consider an example of portfolio K is to be compared for portfolio j, where $(j = 1, 2, \ldots, K)$, suppose that $y_j{=}(y_{1j}, y_{2j}, \ldots, y_{nj})$ represents the weight vector of the portfolio, So the expected return and its variance are $E(Y_J)$ and $V(Y_J)$, respectively. As shown in Figure (1), the variance is shown on the horizontal axis and the expected return is shown on the vertical axis. $A_1(E(Y_1),V(Y_1))$ refers to the evaluated portfolio.

$B_3(E({x^*}_3),V({x^*}_3))$, $B_2(E({x^*}_2),V({x^*}_2))$ and $B_1(E({x^*}_1),V({x^*}_1))$ are reference points that are calculated using return-based and risk-based measurements, respectively, without the amount of optimal portfolios. Therefore, using different distances, the



(i)

Figure 1: Definition of classical PE for the MV boundary

returns of axis-based, risk-oriented and borderless portfolios can be determined as $A_1(E(Y_1),V(Y_1))$ as follows.

$$PE_E = \frac{E(Y_1)}{E(x_1^*)}, PE_v = \frac{V({x^*}_2)}{V(y_1)}, PE_N = \frac{1-(V(y_1)-V(x_3^*))/V(y_1)}{1+(E(x_3^*-E(y_1))/E(y_1)}.$$

To be more precise, according to the directional function (DDF), $g=(g_v, g_E)$, PE for a portfolio with different y_0 axis is calculated using the following model:

$$\begin{aligned}
&\max \delta \\
&\quad \text{S.t:} \\
&\quad E(y_0)+\delta g_E \leq \sum_{i=1}^{n} x_i \hat{r}_i, \\
&\quad V(y_0)+\delta g_V \geq \sum_{i=1}^{n}\sum_{j=1}^{n} x_i \hat{\sigma}_{ij} x_j, \\
&\quad \sum_{i=1}^{n} x_i = 1, \\
&\quad x_i \geq 0, i=1,\ldots,n.
\end{aligned}$$

Suppose if $g_E=E(Y_0)$,$g_v=0$, where PE defines the recurrent returns as PE$_E=\frac{1}{1+\delta}$. If $g_V=-V(Y_0)$,$g_E=0$, PE is defined as the risk axis and as PE$_V=1-\delta$. If we assume $g_V=-V(Y_0)$,$g_E=E(Y_0)$, then PE without return as PE$_N=\frac{1-\delta}{1+\delta}$ is determined.

One form of the CCMV portfolio optimization problem that seems to be of particular concern is the fact that the effective and efficient CCMV boundary differs significantly from the classical MV boundary. So in general, the efficient CCMV boundary may be discontinuous. Therefore, in the case of CCMV, it can not be possible to determine PE under any orientation at all times, and this depends on the geometric nature of the CCMV boundary. If the boundary is continuous (Figure 2), then it is conceivable to determine both return-driven and risk-driven PE.

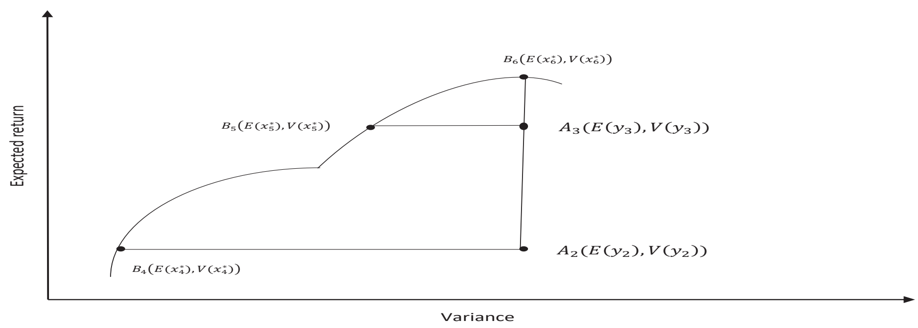


Figure 2: Determination of PE CCMV continuous boundary

As shown in Figure (2), $B_6(E(x_6^*),V(x_6^*))$ is a return axis reference point for $A_2(E(y_2),V(y_2)$) and $A_3(E(y_3),V(y_3))$ if $B_4(E(x_4^*),V(x_4^*))$ and $B_5(E(x_5^*),V(x_5^*))$. The risk-based reference points are for $A_2(E(y_2),V(y_2))$ and $A_3(E(y_3),V(y_3))$. Different can be specified in Figure (3).

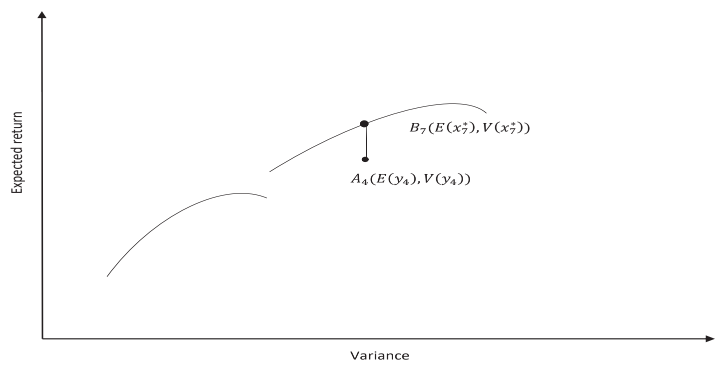


Figure 3: Determination of PE discontinuous CCMV efficiency axis

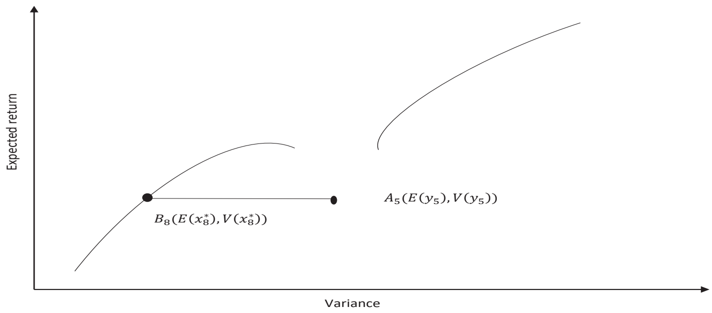


Figure 4: Determination of PE discontinuous CCMV risk-driven boundary

There is a significant difference for discontinuous boundaries, as shown in Figure (3). $A_4(E(y_4), V(y_4))$ refer to the evaluated portfolio and $B_7(E(x_7^*), V(x_7^*))$ to the reference point with variance A_4. Since there is a jump at the CCMV boundary, there is not a point with the same efficiency compared to point A_4. Therefore, only the return axis PE is available for A_4. In contrast, in Figure (4) $A_5(E(y_5), V(y_5))$ on portfolio performance and its estimation using reference point $B_8(E(x_8^*), V(x_8^*))$ refers to. At this stage, the risk is in one direction because there is no point on the efficient boundary for the upper and lower zones of the portfolio.

3 Estimation of PE for CCMV using split DEA model

In this section, an algorithm for determining data division points based on evidence to estimate actual points is examined. As a result, the classic DEA model can be applied in any department. The expected return is assumed as the desired output and the variance is an undesirable output. This output is used as input.

Estimating the standard deviation (M-SD) of the portfolio boundary may result the selection of diverse DEA models. In risk-free assets such as BCC bank deposits, the option will be appropriate. The CCR model is used instead of the NIRS DEA model. The BCC model is suitable for MV issues in MV issues because the MV boundaries are always concave. In the BCC case, for the CCMV problem, the portfolio boundary function is set to $r = F(\hat{\sigma})$. Point A refers to the coordinates $(\hat{\sigma}_i, \hat{r}_i), \hat{r}_i = F(\hat{\sigma}_i)\,(i = 1, 2, \ldots)$ which indicates that will be established under the following conditions:

(i) $F(\hat{\sigma})$ The CCMV boundary refers to the interval $[\hat{\sigma}_i, \hat{\sigma}_{i+1}]$ $(i = 1, 2, 3, \ldots)$

which is smooth.

(ii) The points are in two ways:

[Type (I):]A discontinuous point that has the following conditions:

$$\lim_{\hat{\sigma}\to\hat{\sigma}_i} F(\hat{\sigma}_i) \neq \lim_{\hat{\sigma}\to\hat{\sigma}_i} F(\hat{\sigma}_i).$$

Notice that $F'(\hat{\sigma}_i^+) > F'(\hat{\sigma}_i^-)$. Continuous point with inequality between derivatives as $\lim_{\hat{\sigma}\to\hat{\sigma}_i} F(\hat{\sigma}_i) = \lim_{\hat{\sigma}\to\hat{\sigma}_i} F(\hat{\sigma}_i)$.

The search algorithm has two parts, the first part of which has 3 steps to reach the farthest points of the layer from the main sample points. The search procedure is then specified to put the data split points. The search algorithm determines the data split points based on discrete sample points, so DEA models can be applied in any segment.

The search algorithm determines the data split points established on the discrete sample points. As a result, DEA models can be used in any part. Use data split points to divide the CCMV boundary until a suitable DEA model is used.

3.1 DEA models for PE estimation

Split points can be fully estimated by the data split points embedded in the search algorithm. Therefore, the boundary of the CCMV portfolio can be broken down into several continuous and concave boundaries. DEA models can be applied to estimate PE in each section. It is also worth noting that there may be more data splitting points than splitting points, but this will not lead to erroneous estimation, however, it will increase computational work.

Assume that there are a total of m portfolios. Between the data division points of section i and $i+1$ $(i = 1, 2, 3, \ldots)$ there is a number of m's in the examined and estimated portfolio. If $\sum_i m'_i = m$. Suppose $y_j^I = (y_{1j}^I, y_{2j}^I, \ldots, y_{nj}^I)$ represents the weight vector of the portfolio and for the portfolio $(j = 1, 2, 3, \ldots, m')$ is and have:

$$E(y_j^I) = \sum_{i=1}^{n} y_{ij}^I \hat{r}_i, \qquad V(y_j^I) = \sum_{i=1}^{n} \sum_{k=1}^{n} y_{ij}^I \hat{\sigma}_{ik} y_{ij}^I,$$

which is described as expected returns and variance, respectively. In each part of the BCC models, if chosen correctly, it can estimate the efficiency well, where DMU0 is a sample point under estimation.

[A)]Risk-based BCC model:

$$
\begin{aligned}
&\min\ \theta\\
&\quad \text{S.t:}\\
&\quad\quad \sum_{j=1}^{m'} \lambda_j^I V(y_j^I) \leq \theta V(y_0^I),\\
&\quad\quad \sum_{j=1}^{m'} \lambda_j^I = 1,\\
&\quad\quad \lambda_j^I \geq 0, j=1,2,\ldots,m'.
\end{aligned}
$$

BCC efficiency return model:

$$
\begin{aligned}
&\max\ \phi\\
&\quad \text{S.t:}\\
&\quad\quad \sum_{j=1}^{m'} \lambda_j^I V(y_j^I) \leq V(y_0^I),\\
&\quad\quad \sum_{j=1}^{m'} \lambda_j^I V(y_j^I) \geq \phi E(y_0^I),\\
&\quad\quad \lambda_j^I \geq 0, j=1,2,\ldots,m'.
\end{aligned}
$$

The BCC model used for the CCMV portfolio problem is a classic DEA model, and modifications will be applied to the data.

3.2 Evaluating the performance of decision units based on data envelopment analysis

One of the efficient tools in evaluating and performing decision units (DMUs) is data envelopment analysis (DEA) which is a non-parametric method. The purpose of these models is to measure and compare the relative efficiency of a set of decision units with similar (homogeneous) inputs and outputs (such as bank branches, schools, etc.) in comparison with each other. In data envelopment analysis Any organization that wants to be evaluated is considered as a decision unit. In evaluation with data envelopment analysis, the importance of each unit relative to other units is obtained, which is called efficiency.

Productivity is an important issue in data envelopment analysis. In economic analysis, one of the indicators that has always been considered in the study of total factor productivity growth is the Malmquist productivity index, which is named after Professor Malmquist.

The Malmquist index was introduced by Stan Malmquist [18], a Swedish economist. It was then used for the first time in production theory by Keys and Christine Sen & Divert [4]. The Malmquist Productivity Index calculates the relative performance of a decision unit over time periods using the technology of that period. Farr et al. [4] combined the Farrell [10] efficiency measure with the Keyz productivity measure to generate the Malmquist Productivity Index based on data envelopment analysis and then break it down into two components. The first measures the efficiency change index (EC) and the second measures the technology change index (TC). Chen and Ali [6] presented a new perspective on the Malmquist productivity index based on data envelopment analysis and went into more detail about the second component (technology change). The Malmquist Productivity Index based on data envelopment analysis has been recognized as a useful tool for measuring decision units over the past few decades. For example, Faro et al. [9] used the Malmquist Productivity Index to analyze productivity growth in industrialized countries. There have also been many studies in the insurance industry of different countries, for example, examining the growth of productivity and technical efficiency in the insurance industry of Italy on life-life insurance companies [8], evaluating the productivity of non-life insurance companies in Japan [13], data envelopment analysis has also been used to evaluate the performance of the Iranian insurance industry, for example, calculating the technical efficiency of insurance companies [22], and evaluating their performance.

Suppose N decision unit produces s output x_{ij}^t, y_{rj}^t like $x_{ij}^{t+1}, y_{rj}^{t+1}$ respectively the amount of inputs and outputs DMU_j (decision unit j) in time periods t and $t+1$ where t $i = 1, \ldots, m$ and $r = 1, \ldots, s$ and $j = 1, \ldots, n$. The following linear programming model known as the BCC model introduced by Bunker, Charans, and Cooper [3] is used to calculate the efficiency size DMU_p ($p \in \{1, \ldots, n\}$) assuming a return to the variable production scale.

$$
\begin{aligned}
TE_p = {} & \min\ \theta \\
& \text{S.t:} \\
& \quad \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \theta x_{ip}, && \forall i = 1, \ldots, m, \\
& \quad \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro}, && \forall r = 1, \ldots, s, \\
& \quad \sum_{j=1}^{n} \lambda_j = 1, \\
& \quad \lambda_j, s_i^-, s_r^+ \geq 0, \qquad \forall i = 1, \ldots, m, j = 1, \ldots, n, r = 1, \ldots, s.
\end{aligned}
\tag{1}
$$

If $TE_p = 1$ and $s_i^* = 0$ and $s_r^* = 0$ and $i = 1, \ldots, m$ and $r = 1, \ldots, s$ are the optimal solutions of model (1), then DMU_P is efficient, otherwise it will be inefficient. Now, to calculate the Malmquist productivity index based on data envelopment analysis, assuming returns on a variable scale, we must solve the BCC models at t and $t+1$, models (2) and (3), and linear programming models (4) and (5).

$$
\begin{aligned}
D_p^t(x_p^t, y_p^t) = &\min\ \theta \\
&\text{S.t:} \\
&\quad \sum_{j=1}^{n} \lambda_j x_{ij}^t \leq \theta x_{ip}^t, \qquad \forall i = 1, \ldots, m, \\
&\quad \sum_{j=1}^{n} \lambda_j y_{rj}^t \geq y_{rp}^t, \qquad \forall r = 1, \ldots, s, \\
&\quad \sum_{j=1}^{n} \lambda_j = 1, \\
&\quad \lambda_j \geq 0, \qquad \forall j = 1, \ldots, n.
\end{aligned}
\tag{2}
$$

$$
\begin{aligned}
D_p^{t+1}(x_p^{t+1}, y_p^{t+1}) = &\min\ \theta \\
&\text{S.t:} \\
&\quad \sum_{j=1}^{n} \lambda_j x_{ij}^{t+1} \leq \theta x_{ip}^{t+1}, \qquad \forall i = 1, \ldots, m, \\
&\quad \sum_{j=1}^{n} \lambda_j y_{rj}^{t+1} \geq y_{rp}^{t+1}, \qquad \forall r = 1, \ldots, s, \\
&\quad \sum_{j=1}^{n} \lambda_j = 1, \\
&\quad \lambda_j \geq 0, \qquad \forall j = 1, \ldots, n.
\end{aligned}
\tag{3}
$$

$$
\begin{aligned}
D_p^t(x_p^{t+1}, y_p^{t+1}) = &\min\ \theta \\
&\text{S.t:} \\
&\quad \sum_{j=1}^{n} \lambda_j x_{ij}^t \leq \theta x_{ip}^{t+1}, \qquad \forall i = 1, \ldots, m, \\
&\quad \sum_{j=1}^{n} \lambda_j y_{rj}^t \geq y_{rp}^{t+1}, \qquad \forall r = 1, \ldots, s, \\
&\quad \sum_{j=1}^{n} \lambda_j = 1, \\
&\quad \lambda_j. \geq 0 \qquad \forall j = 1, \ldots, n.
\end{aligned}
\tag{4}
$$

$$
\begin{aligned}
D_p^{t+1}(x_p^t, y_p^t) = &\min\ \theta \\
&\text{S.t:} \\
&\quad \sum_{j=1}^{n} \lambda_j x_{ij}^{t+1} \leq \theta x_{ip}^t, \qquad \forall i = 1, \ldots, m, \\
&\quad \sum_{j=1}^{n} \lambda_j y_{rj}^{t+1} \geq y_{rp}^t, \qquad \forall r = 1, \ldots, s, \\
&\quad \sum_{j=1}^{n} \lambda_j = 1, \\
&\quad \lambda_j \geq 0. \qquad \forall j = 1, \ldots, n.
\end{aligned}
\tag{5}
$$

So that model (2) and model (3) are the efficiency size DMU_p, $(p \in \{1, \ldots, n\})$ respectively in time periods t and $t+1$. Model (4) is the efficiency measure in time period $t+1$ using the production technology of time period t, and model (5) is the measure of efficiency in period t with period production technology of $t+1$. Based on the above efficiencies, Farohmokaran [4] proposed the Malmquist Productivity Index DMU_p as follows

$$MPI_p = \left[\frac{D_p^t(x_p^{t+1}, y_p^{t+1})}{D_p^{t+1}(x_p^{t+1}, y_p^{t+1})} \times \frac{D_p^t(x_p^t, {y_p}^t)}{{D_p}^{t+1}({x_p}^t, {y^t}_p)} \right]^{\frac{1}{2}} . \quad (6)$$

The above equation measures productivity changes DMU_p over time t to $t+1$. According to Farohmakaran [4], if $MPI_p > 1$ it indicates productivity growth and $MPI_p = 1$ indicates the amount of productivity unchanged and if $MPI_p < 1$ it shows a decrease in productivity.

Consider 25 companies as one decision unit in 1995-99. To determine their productivity in these years, we must first run the linear models (2), (3), (4) and (5), and then calculate the amount of productivity with the formula mentioned above.

4 Simulation of data analysis

In order to prove the validity of the proposed solution plans using the data obtained from the site www.codal.ir has been prepared and prepared, we have selected 25 shares for this purpose and their daily return data in a time interval We use 1395-1399 to estimate their mean and variance. We will also review the results by estimating the data under the cover model and comparing the two models. Table 1 shows the statistical characteristics of the stock portfolio.

Related issues, especially CCMV issues were calculated using special software and DEA by GAMS software. We first generated the investment weights in a discrete uniform distribution and used them to create sample points with a specific expected variance and return. Then the PE of each sample point is derived by comparing its distance from the optimal point on the boundary. DEA values are calculated using DEA models in each section and the results are divided by the results obtained from the applicability and priority of the DEA approach and the correlation coefficient of PE values and DEA values as well as the correlation coefficients of their ranks under different sample sizes. We compared the DEA analysis based on the Markowitz model. In addition, the estimation of PEs is used through the split DEA approach.

4.1 Results obtained

Table 2 shows the amount of productivity in the years 95-96. As can be seen, the productivity score for all companies is less than 1, which indicates a decrease in productivity in these years. In the last column of Table 1, companies are ranked based on productivity scores. Companies 24, 7 and 16 are ranked 1, 2 and 3, respectively. Companies 10 and 14 jointly ranked 14th. Companies 6,1,9,11,12 and 18 also received a joint ranking of 15. Table 3 shows the productivity of companies in

Table 1: Statistical characteristics of the portfolio

	1	2	3	4	5
Annual average	00083/0	00105/0	000524/0	0/0055208	0/0081672
Covariance	1	2	3	4	5
1	4/699148	4/494126	5/000364	6/194968	17/17975
2	-0/90328	1/199191	5/815177	4/825477	3/914382
3	1/44181	1/303029	1/830588	-7/93914	15/45611
4	-1/52338	6/124905	1/694362	5/528612	3/578619
5	-1/34701	0/530996	1/1603	1/538539	5/769759
6	2/02881	1/417203	1/198725	-0/03873	-9/94383
7	4/730941	5/723324	6/696573	8/377112	10/04389
8	0/530008	0/80912	1/675104	-0/35566	16/79323
9	5/556621	0/306724	0/14487	6/395029	15/13838
10	0/823005	0/800784	0/934236	4/538211	2/197316
11	0/417736	1/429774	1/959675	-47/8236	16/59298
12	0/641386	1/164244	4/284856	8/059914	5/38418
13	2/192181	2/608478	8/624315	3/401866	14/66905
14	0/744191	3/16741	2/460988	-2/91326	21/74265
15	2/31154	1/984777	3/50445	-2/49429	34/36774
16	3/525735	1/941432	3/423097	12/35034	114/3003
17	3/570526	3/269675	6/723696	21/72965	20/60252
18	1/312939	0/220722	0/389728	0/847453	1/006653
19	0/109208	4/01398	3/647944	19/77661	35/06708
20	0/658183	6/873195	13/83219	17/14644	-80/9952
21	0/931363	7/110057	1/123853	4/379989	53/96027
22	4/010083	10/33248	5/210395	129/8676	6/302027
23	0/808658	1/619865	0/72155	0/874889	2/906439
24	7/692401	9/841177	6/44211	23/45495	20/54376
25	0/865913	14/50813	3/033952	-14/6617	11/54056

Table 2: Determining the productivity of the year 95-96 and ranking the companies

DMU	$D_p^t(x_p^t, y_p^t)$	$D_p^{t+1}(x_p^{t+1}, y_p^{t+1})$	$D_p^t(x_p^{t+1}, y_p^{t+1})$	$D_p^{t+1}(x_p^t, y_p^t)$	MPI	rank
1	1	1	0.99	3.42	0.538	15
2	0.962	1	0.99	3.42	0.548	12
3	0.910	1	0.99	3.42	0.564	9
4	0.855	1	0.99	3.42	0.582	7
5	0.883	1	0.99	3.42	0.573	8
6	1	1	0.99	3.42	0.538	15
7	0.536	0.97	0.99	3.42	0.724	2
8	0.963	1	0.99	3.42	0.548	12
9	1	1	0.99	3.42	0.538	15
10	0.994	1	0.99	3.42	0.539	14
11	1	1	0.99	3.42	0.538	15
12	1	1	0.99	3.42	0.538	15
13	1	0.98	0.99	3.42	0.533	15
14	0.986	0.99	0.99	3.42	0.539	14
15	0.821	1	0.99	3.42	0.594	6
16	0.651	0.99	0.99	3.42	0.663	3
17	0.675	0.98	0.99	3.42	0.648	4
18	1	1	0.99	3.42	0.538	15
19	1	0.97	0.99	3.42	0.530	16
20	0.982	0.95	0.99	3.42	0.529	17
21	0.982	1	0.99	3.42	0.543	13
22	0.648	0.91	0.99	3.42	0.637	5
23	0.954	1	0.99	3.42	0.551	11
24	0.275	0.91	0.99	3.42	0.979	1
25	0.946	1	0.99	3.42	0.553	10

the years 96-97. Unfortunately, the productivity score of all companies is less than one and they are ranked based on the scores in the last column of companies. Some companies have the same rating. Table 4 shows the productivity score of companies in the year 97-98. Companies with a score higher than one have progressed and companies with a score lower than one have had a setback. Companies 22, 15 and 8 ranked first to third, respectively. In the last column, the companies are ranked in order. Companies 6, 18, 19, 20 and 23 jointly ranked 6th. Table 5 shows the productivity of companies in the year 98-99. Companies 25, 16 and 24 ranked first to third, respectively, and advanced with a high productivity score of one. Companies 4, 6, 8, 18, 20, 22 and 23 with a productivity score of 1,017 were equally ranked 9th.

According to the results, the productivity of companies in the years 95-96 and 96-97 was very low because the companies got a low score of one. But in 1997-98 and 1998-99, many companies made an improvement by earning a high performance

Table 3: Determining the productivity of the year 96-97 and ranking the companies

DMU	$D_p^t(x_p^t, y_p^t)$	$D_p^{t+1}(x_p^{t+1}, y_p^{t+1})$	$D_p^t(x_p^{t+1}, y_p^{t+1})$	$D_p^{t+1}(x_p^t, y_p^t)$	MPI	rank
1	1	0.978	1.422	1.804	0.878	16
2	1	1	1.422	1.804	0.888	8
3	1	0.991	1.422	1.804	0.884	10
4	1	0.987	1.422	1.804	0.882	11
5	1	1	1.422	1.804	0.888	8
6	0.996	1	1.422	1.804	0.889	7
7	0.972	1	1.422	1.804	0.901	5
8	0.998	0.984	1.422	1.804	0.882	12
9	1	1	1.422	1.804	0.888	8
10	1	0.995	1.422	1.804	0.886	9
11	0.997	0.982	1.422	1.804	0.881	13
12	1	1	1.422	1.804	0.888	8
13	0.978	1	1.422	1.804	0.898	6
14	0.987	0.987	1.422	1.804	0.888	8
15	1	0.975	1.422	1.804	0.877	17
16	0.995	0.974	1.422	1.804	0.878	15
17	0.976	0.961	1.422	1.804	0.881	14
18	1	1	1.422	1.804	0.888	8
19	0.968	1	1.422	1.804	0.902	3
20	0.947	1	1.422	1.804	0.912	1
21	1	1	1.422	1.804	0.888	8
22	0.906	0.95	1.422	1.804	0.909	2
23	1	1	1.422	1.804	0.888	8
24	0.910	0.937	1.422	1.804	0.901	4
25	1	0.991	1.422	1.804	0.884	10

score. For example, this change in the 25th company is quite obvious. In the first two periods, it had a low productivity score of one, but in the period of 97-98, it improved, and in the last period, it reached the highest progress and gained the rank of 1.

In this section, diagrams related to each sector are drawn for companies. After reviewing the results of the analysis, we found that Markowitz model showed that the most important factor in choosing the optimal stock portfolio is the two factors of return and risk. The results of this study show. Portfolio optimization sought to select the stock portfolio that has the highest returns and the lowest risk. The main disadvantage of the Markowitz model is that it does not take into account all the factors that are effective in measuring risk and return in the real world. In the real world, to measure risk and stock returns, in addition to the internal factors of companies, external factors are also effective in selecting an optimal portfolio. In general, the factors affecting the development of securities can be divided into

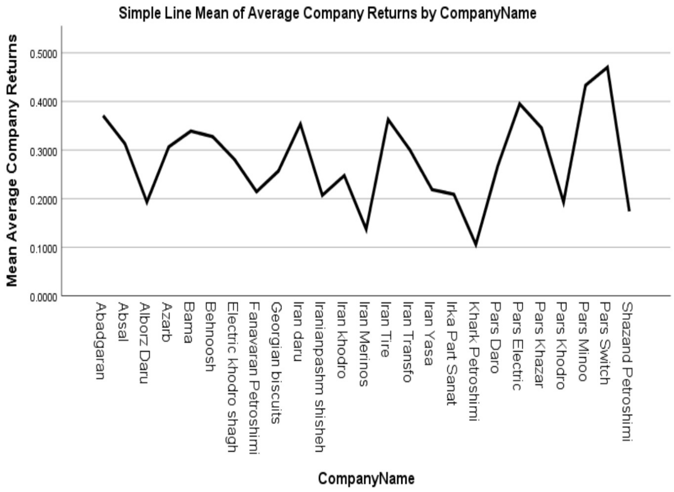


Figure 5: Annual returns for companies

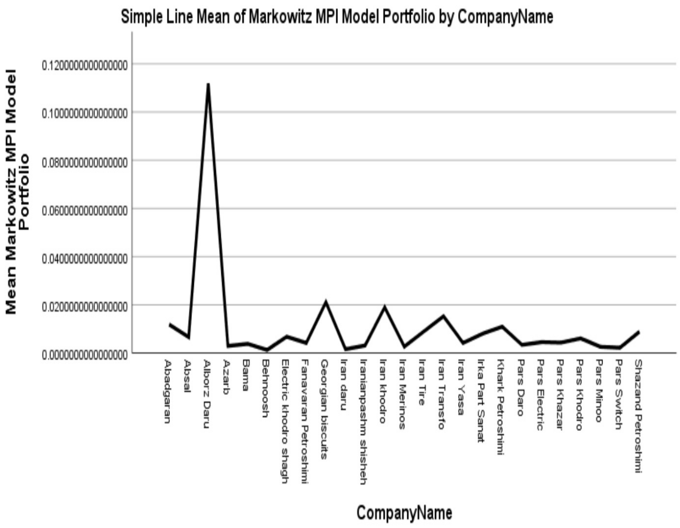


Figure 6: Markowitz model portfolio of 25 selected companies

Table 4: Determining productivity in 97-98 and ranking companies

DMU	$D_p^t(x_p^t, y_p^t)$	$D_p^{t+1}(x_p^{t+1}, y_p^{t+1})$	$D_p^t(x_p^{t+1}, y_p^{t+1})$	$D_p^{t+1}(x_p^t, y_p^t)$	MPI	rank
1	0.98	0.94	10.7	4.95	1.439	11
2	1	0.96	10.7	4.95	1.441	10
3	0.99	0.94	10.7	4.95	1.433	12
4	0.99	1	10.7	4.95	1.478	5
5	1	0.99	10.7	4.95	1.463	7
6	1	1	10.7	4.95	1.470	6
7	1	0.94	10.7	4.95	1.425	13
8	0.98	1	10.7	4.95	1.485	3
9	1	0.94	10.7	4.95	1.425	14
10	0.99	0.96	10.7	4.95	1.448	9
11	0.98	1	10.7	4.95	1.485	4
12	1	0.92	10.7	4.95	1.410	15
13	1	0.97	10.7	4.95	1.448	9
14	0.99	0.98	10.7	4.95	1.463	7
15	0.97	1	10.7	4.95	1.493	2
16	0.97	0.88	10.7	4.95	1.400	16
17	0.96	0.80	10.7	4.95	1.342	19
18	1	1	10.7	4.95	1.470	6
19	1	1	10.7	4.95	1.470	6
20	1	1	10.7	4.95	1.470	6
21	1	0.98	10.7	4.95	1.455	8
22	0.95	1	10.7	4.95	1.508	1
23	1	1	10.7	4.95	1.470	6
24	0.94	0.79	10.7	4.95	1.348	18
25	0.99	0.87	10.7	4.95	1.378	17

three categories: internal factors of companies, non-economic external factors and external factors of macroeconomics. It can be solved using mathematical formulas and a quadratic equation, but in practice and in the real world, given the large number of choices available in the capital markets, the mathematical approach used to solve this model requires calculations and Extensive planning. Given that stock market behavior does not follow a linear pattern, for this reason, common linear methods can not be used to describe this behavior and be useful. Given the conditions of investor uncertainty in determining the factors affecting the investment process, including the exact amount of return and stock risk, and the study of the model by nonlinear planning and its solution to provide optimal portfolio selection and since the purpose of An investment is to have a minimum risk in return for an acceptable amount of return, so an optimization model is used to minimize the undesirable risk and based on a certain amount of return. Data and Markowitz model are under data envelopment analysis, the superiority of their usefulness (re-

Table 5: Determining the productivity of the year 98-99 and ranking the companies

DMU	$D_p^t(x_p^t, y_p^t)$	$D_p^{t+1}(x_p^{t+1}, y_p^{t+1})$	$D_p^t(x_p^{t+1}, y_p^{t+1})$	$D_p^{t+1}(x_p^t, y_p^t)$	MPI	rank
1	0.94	0.84	0.9	0.87	0.961	13
2	0.96	0.97	0.9	0.87	1.022	8
3	0.94	0.86	0.9	0.87	0.972	11
4	1	1	0.9	0.87	1.017	9
5	0.99	0.98	0.9	0.87	1.012	10
6	1	1	0.9	0.87	1.017	9
7	0.94	1	0.9	0.87	1.049	5
8	1	1	0.9	0.87	1.017	9
9	0.94	0.86	0.9	0.87	0.973	11
10	0.96	1	0.9	0.87	1.038	6
11	1	0.90	0.9	0.87	0.965	12
12	0.92	0.98	0.9	0.87	1.050	4
13	0.97	0.86	0.9	0.87	0.958	14
14	0.98	0.80	0.9	0.87	0.919	16
15	1	0.73	0.9	0.87	0.869	17
16	0.88	1	0.9	0.87	1.084	2
17	0.80	0.81	0.9	0.87	1.023	7
18	1	1	0.9	0.87	1.017	9
19	1	0.66	0.9	0.87	0.826	18
20	1	1	0.9	0.87	1.017	9
21	0.98	0.86	0.9	0.87	0.953	15
22	1	1	0.9	0.87	1.017	9
23	1	1	0.9	0.87	1.017	9
24	0.79	0.85	0.9	0.87	1.055	3
25	0.84	0.97	0.9	0.87	1.093	1

turn and risk) were also investigated and the results of analysis and return of 25 companies in 5 consecutive years were reviewed and compared. For companies with acceptable and reliable returns have the same performance but in others are different. Also, the ranking of companies is different in different years. And efficiency of data envelopment analysis model 0.32184 and rate The variance of 0.8212 and the standard deviation or risk is 0.90620 and by comparing the portfolio return and its standard deviation or risk for both methods, it can be concluded that the return of Markowitz model is higher than the return of data envelopment analysis model as opposed to standard deviation (risk). The data envelopment analysis model is lower and its level of reliability is higher than the Markowitz model, which results in the efficiency of the Markowitz model compared to other models.

Table 6: Markowitz model portfolio as well as corporate returns

MPI	Markowitz MPI Model Portfolio	Average Company Returns	Company Name
0.954	0.111892324626792	0/193	Alborz Daru
0.974	0.00681388644889462	0/2804	Electric khodro shagh
0.963	0.00901035811540836	0/363	Iran Tire
0.989	0.0152780010738819	0/2998	Iran Transfo
0.984	0.0190285474402835	0/2476	Iran khodro
0.978	0.0016126704004456	0/3534	Iran daru
1.024	0.00268757539904957	0/137	Iran Merinos
0.983	0.00418669406972062	0/2186	Iran Yasa
0.956	0.00807345740913488	0/209	Irka Part Sanat
0.977	0.0118630023523022	0/3708	Abadgaran
0.966	0.00663786847894245	0/313	Absal
0.971	0.00297554671506265	0/3068	Azarb
0.959	0.00383789219164641	0/339	Bama
0.952	0.00128782015038280	0/3276	Behnoosh
0.913	0.0211382941756879	0/257	Georgian biscuits
1.006	0.00455041823360505	0/395	Pars Electric
0.973	0.00435084570707152	0/3452	Pars Khazar
0.978	0.00608628406323449	0/1928	Pars Khodro
0.932	0.00341320276747106	0/2672	Pars Daro
0.982	0.00221268923312611	0/4702	Pars Switch
0.959	0.00261228731602103	0/433	Pars Minoo
1.017	0.0109384633697158	0/1058	Khark Petroshimi
0.981	0.00888704266995961	0/1738	Shazand Petroshimi
1.070	0.00418784650685694	0/2144	Fanavaran Petroshimi
0.977	0.00315051992490723	0/207	Iranianpashm shisheh

5 Conclusion

As mentioned earlier, the purpose of this study was to select a stock portfolio from the shares of listed companies using data envelopment analysis optimization models and Markowitz model under data envelopment analysis. In studies, an algorithm has the advantage of having a larger ratio, which indicates the selection of the optimal stock portfolio. The result of this research is the efficiency of the Markowitz model compared to other models. By examining the different methods mentioned in this article, new and faster aspects of calculations and evaluation of DEA findings were investigated to determine the efficiency of the portfolio evaluation (stock basket) of the theory of constraints and how to implement it, and to evaluate the comparative performance based on the theory of constraints based on The DEA approach and the Markowitz model in this article showed that each investor based on his risk tolerance and risk aversion, chose a point on the efficient frontier and

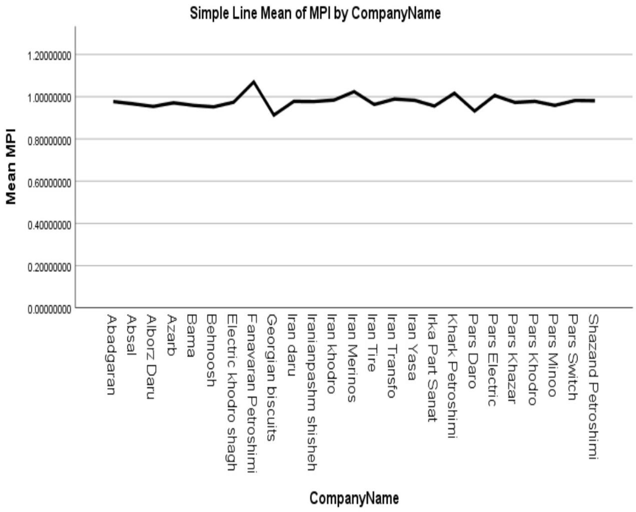


Figure 7: MPI for 25 selected companies

his portfolio composition aimed at maximizing returns and minimizing risk. Determinantly, a basket was selected in which the assets were not completely correlated and investment was made on these assets; The risk of those assets offset each other, resulting in a consistent, lower-risk return that was reviewed and evaluated each year relative to the previous year

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