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Research paper

Designing an epidemic health insurance

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Abstract:
This paper considers a one-year healthcare insurance contract in which a policyholder protects against an epidemic risk within the policy term and its side effects at most five years later for those who got an infection during the policy period. To derive actuarial computations, it considers a SIDS epidemic model in which infection rates depends on the time. Under such SIDS epidemic model and four health insurance plans fair premium and loss reserves have been evaluated. Finally, through a simulation study practical application of the product has been given.

Keywords: Epidemiological model; Side effects of a disease; Healthcare insurance; Fair premium; Loss reserve.
Classification: 60J28, 91B30, 92D30.

1 Introduction

In light of the recent outbreak of Covid-19, it is important to remember that epidemics raise sanitary as well as financial concerns. Analytically tractable models and cover epidemic risk are needed. The literature on epidemic models is very large and is continuing to expand. Epidemic models are labeled SIR, SIRS, SIS, and SEIRD, where S, E, I, R, and D denote the class of susceptible, exposed, infected, recovered, and died, respectively. More realistic mathematical models for infectious diseases have been dramatically developed lately. More specifically, (1) factors and structures, such as latent periods and time delays, age, infection-age, gender, other physiologic structures, and effects of isolations, quarantine, vaccination, or treatment, have been further included; (2) the dimensions of the models have been greatly increased, which allows for studying epidemic transmission dynamics between populations and species in depth; (3) more thorough and detailed investigations have been conducted on specific infectious diseases, such as AIDS/HIV and vectorborne diseases. Nevertheless, as the epidemic models become closer to reality and more biological and social factors are included, the model

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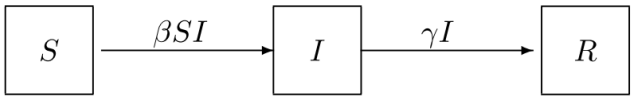

Figure 1: Flow chart of the SIR model

features and behavior become more complex [11].We only introduce some basic mathematical disease models. Discussion about epidemiological models, see [1], [2] and [4].

For example the flow chart of the SIR model is shown in Figure 1. The number of susceptibles who are infected by an infected individual per unit of time, at time t, is proportional to the total number of susceptibles with the proportional coefficient (transmission coefficient) β, so that the total number of newly infectives, at time t, is $\beta S(t)I(t)$. The number removed (recovered) individuals from the infected compartment per unit time is $\gamma I(t)$ at time t, where γ is the recovery rate coefficient, and the recovered individuals gain permanent immunity.

the corresponding model equations are given in the system

$$\frac{dS_t}{dt} = \beta SI$$

$$\frac{dI_t}{dt} = \beta SI - \gamma I$$

$$\frac{dR_t}{dt} = \gamma I$$

For viral diseases, such as influenza, measles, and chickenpox, the recovered individuals, in general, gain immunity to the same virus. Then the SIR model described above is applicable. However, for bacterial diseases, such as encephalitis, and gonorrhea, the recovered individuals gain no immunity and can be reinfected. To study the transmission dynamics of these diseases, researchers proposed an SIS model. The flow chart of an SIS model is shown in Figure 2.

Model equations are given:

$$\frac{dS_t}{dt} = -\beta SI + \gamma I$$

$$\frac{dI_t}{dt} = \beta SI + \gamma I$$

The difference between the SIR and the SIS model is in the SIS model, the infectives are recovered but gain no immunity after recovery. In the SIR model, the infectives obtain permanent immunity to the disease after recovered from infection.

Figure 3 illustrates an SEIRS model with a latent period, where ω is the progression rate coefficient for individuals from compartments E to I, such that $\frac{1}{\omega}$ is the mean latent period. In this models, an exposed compartment, in which all of the

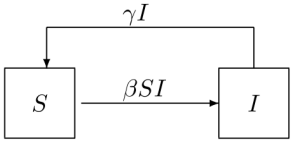

Figure 2: Flow chart of the SIS model

individuals have been infected but have not yet infectious. And model equations are given:

$$\frac{dS_t}{dt} = -\beta SI + \delta R$$

$$\frac{dR_t}{dt} = \gamma I - \delta R$$

$$\frac{dE_t}{dt} = \beta SI - \omega E$$

$$\frac{dI_t}{dt} = \omega E - \gamma I$$

Infectious diseases have always been an important part of human history. Payment of medical expenses are the most significant benefit provided by the infectious disease insurance policy. Insurers often develop new products or tweak existing ones in response to market needs. An organization may be trying to solve a business problem or plan new revenue streams. It is common for new products to be built with the customer in mind. Benet-rich products are easier to sell to customers. A confluence of macro factors has made healthcare one of the most significant concerns facing leaders and citizens in all nations. In todays highly competitive healthcare market, health insurance companies continue to introduce more products. Insurers must be able to respond quickly to their clients demands for products that are tailored to their specific needs to compete effectively. For insurers to meet these challenges, they must be more innovative and agile, for example, by streamlining product development processes to create a broad range of new products, and by responding quickly to market and regulatory changes.

[5] developed the actuarial applications in epidemic models. They designed insurance products for two well-known epidemics: the Great Plague in England and the SARS epidemic in Hong Kong. [10] applied actuarial methods to propose a life insurance plan protecting against epidemic disease. They extended SIR epidemic model in which the removal and infection rates may depend on the number of registered removals. [13] studied SIDRS epidemic model and used actuarial techniques and principles to determine the financial obligations of the insurance parties. [3] studied determining the optimal insurance premium rate for healthcare in deterministic and stochastic SEIR models. Their results showed how the vaccination program affects insurance costs by comparing the savings in benefits with the expenses for vaccination. [6] used an actuarial approach for modeling pandemic risk. He used the SIR model and calculate fair pure premium in Belgium, Germany, Italy and Spain.

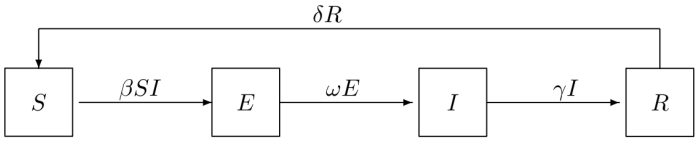

Figure 3: Flow chart of the SEIRS model

[12] considered SEIR model and formulated the level net premiums of infinite term infectious disease. This study proposes epidemic insurance plans that include hospitalization coverage, death benefits, and the side effects of disease coverage. Moreover, it provides a mathematical model to calculate the insurers and policyholders liabilities in the epidemic insurance policy. This paper considers annuity and lump sum for hospitalization coverage and also it considers the side effects of the disease in the insurance plan. We discuss the present value of the premium as the financial obligation of the policyholder. The structure of this paper is as follows: in Section 2, the model of the spread of the epidemic disease is introduced. Section 3 introduces actuarial models for epidemic diseases. The details of the designed product, the theoretical basis of the product, and how to value it are discussed in this section. In Section 4, a numerical example is presented. Finally, Section 5 will present the conclusions and suggestions.

2 Epidemic model

Dynamic models for infectious diseases are mostly based on compartment structures that were initially proposed by [8] and [9] and developed later by many other biomathematicians. To formulate a dynamic model for the transmission of an epidemic disease, the population in a given region is often divided into several different groups or compartments.

In this paper, we use a SIDS model. This model classifies the population into susceptible individuals S, infected I, and died D. Note that the recovered individuals have only temporary immunity after they recovered from infection and return to the group of susceptible. In this paper, we use the proposed epidemic model by [6]. He proposed a new deterministic model in which the contagion rate is inversely proportional to time instead of to the susceptible population.

$$I_t = Ne^{-(\alpha+\mu)^t}(\beta t)^{\gamma} \tag{1}$$

$\beta(day^{-1})$ represents the transmission rate to the infected compartment I; the μ is the transition rate from compartment I to the deceased compartment D. α is the recovery rate from the disease. We assume that the recovered peoples have only temporary immunity after they recovered from infection. If the number of people who lose their immunity is αI, they enter the susceptible compartment again. $\frac{\gamma}{t}$ is the contagion rate per capita.

Let D_t be the total number of deaths up to time t and it is solution of the

following ordinary differential equation:

$$\frac{dD_t}{dt} = \mu I$$

the size of susceptible individuals denoted by S_t and it is solution of the following ordinary differential equation:

$$\frac{dS_t}{dt} = \alpha I_t - \frac{\gamma}{t} I_t$$

We assume that the total population size, N, is constant and $S_t + I_t + D_t = N$. There is no entry into or departure from the population, except possibly through death from the disease.

Remark 2.1. Note that the variables $S, I, D, \beta, \gamma, \mu$ and α used in the model are assumed to only depend on time t. In particular, they are assumed not to depend on the age and gender of the individuals. A key quantity in the epidemic model is the basic reproductive number, R_0. A disease dies out if $R_0 < 1$ and spreads if $R_0 > 1$. R_0 is the average number of secondary infections due to a single infectious individual during the mean course of infection in a completely susceptible population. In our model, the reproduction number is a function of time and it equals to

$$R_0(t) = \frac{\gamma}{t(\alpha + \mu)}$$

3 Actuarial models for infectious disease insurance

In this paper, the designed health insurance product provides the policyholder with a lump sum (or annuity benefits) if the insured individual catches an epidemic illness which is specified by the policy conditions. This product is modeled by a multiple-state model with state space S, I, and D. States which have already been defined in the previous section. In the stochastic model of the SIDS discrete Markov chain model, it is assumed that the time step is very small so that one change per time unit is possible. Transfer possibilities only depend on the current situation. So the Markov chain process is like a birth-death process.

3.1 Product specifications

In our epidemic insurance plan, the insurer has obligations in three situations: hospitalization benefits, death benefits, and side effects of the disease. At the beginning of the contract, a healthy policyholder (susceptible), state S, pays the fixed premium, π, to the insurance company. If the policyholder becomes infected she/he will be to state I. People who are in state I will go to state S if they recover, and if not, they will go to the state of D. Note that the recovered individuals have only temporary immunity after they recovered from infection. One of the side effects of the epidemic disease is damage to organs. People who become severely ill with Covid-19 may experience organ damage that affects the heart, kidneys, skin, and brain. Inflammation and immune system problems, may also occur. It

is not known how long these effects may last. These effects can also lead to new conditions, such as diabetes or heart or nervous system disease. In this insurance product, the side effects of the disease will be covered for up to ve years after the end of the insurance period.

Model Assumption 1. Assume:

A_1) *The insurer pays the cost of hospitalization at a constant rate of* c_1 *in the form of an annuity or* c_3 *in a lump sum.*

A_2) *The insurer pays the lump sum of the amount at a constant rate* c_2 *to the deceased policyholders.*

A_3) *The insurer pays the lump sum of the amount with a constant rate* c_4 *to the policyholders who have the side effects of the disease.*

A_4) *The side effects of the disease must occure within 5 years after the infectious disease and be caused by the insured infectious disease.*

A_5) *In the side effects of the disease policyholder can only make one claim. The benefit is only paid once and coverage is terminated upon payout of benefits.*

A_6) *The insurance premium is payable during the period when the insured remains susceptible.*

3.2 Theoretical representation of the product

For modeling, we use international actuarial notations and concepts similar to life insurance, except that instead of conditioning the probability of payments on death, we condition this probability on illness.

Fair insurance premium

In this subsection, we aim to determine a fair insurance premium rate for the infectious insurance policy in SIDS model. Insurance premium is the price demanded by the insurance company for transferring the risk of loss from the insured to the insurer. Numerous principles can be applied at the policy level. We refer the reader to study [7] for more details about premium principles. In this paper, we use the standard equivalence principle:

E(Present value of benefits)=E(Present value of benefits premiums)

We use the similar notation of Boado-Penas et al. [3] to calculate the actuarial present value of premium payments from the insured and the benefits payments from the insurer.

Definition 3.1. For an infectious disease insurance plan under the SIDS model and the force of interest $\delta > 0$, the actuarial present value (APV) is given by,

(i) APV of continuous premium payments of 1 unit per year for a T-year period from individuals in class S is:

$$\bar{a}^{s}_{T\rceil} = \int_0^T e^{-\delta t} S_t dt. \tag{2}$$

(ii) APV of continuous benefit payments of 1 unit per year for a T-year period

to individuals in class I is:

$$\bar{a}^i_{T\rceil} = \int_0^T e^{-\delta t} I_t dt. \tag{3}$$

(iii) APV of lump sum benefit payment to individuals in class I is:

$$\bar{A}^i_{T\rceil} = \int_0^T e^{-\delta t} \frac{\gamma}{t} I_t dt. \tag{4}$$

(iv) APV of lump-sum benefit payment to individuals in class D is:

$$\bar{A}^d_{T\rceil} = \int_0^T e^{-\delta t} D_t dt. \tag{5}$$

We mention here some results of Hainaut [9] which are crucial throughout this paper.

Lemma 3.2. *For $\delta \geq 0$, we have*

$$\bar{a}^i_{T\rceil} = \frac{N\beta^\gamma}{\theta^{\gamma+1}} \Gamma(\gamma+1, T\theta),$$

where $\theta = \delta + \alpha + \mu$ and $\Gamma(\gamma+1, x) = \int_0^x u^\gamma e^{-u} du$ is the lower incomplete gamma function.

□

Lemma 3.3. *The cumulated number of deceases caused by the epidemic disease at time $t \geq 0$ is given by*

$$D_t = N\mu\beta^\gamma (\alpha+\mu)^{-\gamma-1} \Gamma(\gamma+1, t(\alpha+\mu)).$$

If $\theta = \delta + \alpha + \mu$, we have

$$\bar{A}^d_{T\rceil} = \frac{N\mu\beta^\gamma}{\theta^{\gamma+1}} \Gamma(\gamma+1, T\theta).$$

□

By the fact that $S_t + I_t + D_t = N$, the size of the susceptible population at time $t \geq 0$ is given by

$$S_t = N - Ne^{-(\alpha+\mu)t}(\beta t)^\gamma - N\mu\beta^\gamma(\alpha+\mu)^{-\gamma-1}\Gamma(\gamma+1, t(\alpha+\mu)). \tag{6}$$

Lemma 3.4. *For $\delta \geq 0$, we have*

$$\bar{a}^s_{T\rceil} = \frac{N}{\delta}(1-e^{-\delta T}) - \frac{N\beta^\gamma}{\theta^{\gamma+1}}\Gamma(\gamma+1, T\theta)(1-\frac{\mu}{\delta}) + \frac{N\mu\beta^\gamma}{\delta(\alpha+\mu)^{\gamma+1}} e^{-\delta T}\Gamma(\gamma+1, T(\alpha+\mu)).$$

□

Corollary 3.5. *For side effects of the disease under assumptions A_4 and A_5 from Model Assumption (1) the actuarial present value of 1 unit lump-sum benefit payment is*

$$\bar{A}^{sa}_{5\rceil} = \frac{N\gamma\beta^\gamma}{\theta^\gamma}(\Gamma(\gamma, 6\theta) - \Gamma(\gamma, \theta)).$$

Proof. From assumption A_5, we have that

$$\begin{aligned} \bar{A}^{sa}_{5\rceil} &= \int_1^6 e^{-\delta t}\frac{\gamma}{t} I_t dt \\ &= \int_1^6 \frac{e^{-\delta t}}{t} N e^{-(\alpha+\mu)t}(\beta t)^{\gamma} dt \\ &= \frac{N\gamma\beta^{\gamma}}{\theta^{\gamma}}(\Gamma(\gamma, 6\theta) - \Gamma(\gamma, \theta)). \end{aligned}$$

We consider $\theta = \alpha + \mu + \delta$ and perform a change of variable $\theta t = u$ and calculate the integral. □

We consider four insurance plans for covering the infectious disease:

- **Plan 1.** Annuity for hospitalization and lump sum for death benefit.
- **Plan 2.** Lump sum for hospitalization and lump sum for a death benefit.
- **Plan 3.** Annuity for hospitalization, a lump sum for side eects of the disease, and a lump sum for a death benefit.
- **Plan 4.** Lump sum for each hospitalization, side effects of the disease, and death benefit. In the following theorem, we develop expressions for a net level premium, π, based on the equivalence principle for four proposed plans. We will use a one-year term for the policy.

Theorem 3.6. *For the SIDS model, the net level premium for one-year term insurance is to be collected continuously from individuals in the susceptible class*

(i) to pay level benefits continuously while in the infectious class under Plan 1 is given by

$$\pi_1 = \frac{(c_1 + \mu c_2)\Gamma(\gamma+1, \theta)}{\frac{\theta^{\gamma+1}}{\delta\beta^{\gamma}}(1 - e^{-\delta}) - \Gamma(\gamma+1, \theta)(1 - \frac{\mu}{\delta}) + \frac{\mu\theta^{\gamma+1}}{\delta(\alpha+\mu)^{\gamma+1}} e^{-\delta}\Gamma(\gamma+1, (\alpha+\mu))}. \tag{7}$$

(ii) to pay lump sum for hospitalization and lump sum for a death benefit under Plan 2 is given by

$$\pi_2 = \frac{c_3\gamma\Gamma(\gamma, \theta) + c_2\theta^{-1}\mu\Gamma(\gamma+1, \theta)}{\frac{\theta^{\gamma}}{\delta\beta^{\gamma}}(1 - e^{-\delta}) - \frac{1}{\theta}\Gamma(\gamma+1, \theta)(1 - \frac{\mu}{\delta}) + \frac{\mu\theta^{\gamma}}{\delta(\alpha+\mu)^{\gamma+1}} e^{-\delta}\Gamma(\gamma+1, (\alpha+\mu))}. \tag{8}$$

(iii) to pay level benefits continuously while in the infectious class, a lump sum for side effects of the disease, and a lump sum for death benefit under Plan 3 is given by

$$\pi_3 = \frac{(c_1 + c_2\mu)\Gamma(\gamma+1, \theta) + c_4\gamma\theta e^{-\delta}(\Gamma(\gamma, 6\theta) - \Gamma(\gamma, \theta))}{\frac{\theta^{\gamma+1}}{\delta\beta^{\gamma}}(1 - e^{-\delta}) - \Gamma(\gamma+1, \theta)(1 - \frac{\mu}{\delta}) + \frac{\mu\theta^{\gamma+1}}{\delta(\alpha+\mu)^{\gamma+1}} e^{-\delta}\Gamma(\gamma+1, (\alpha+\mu))}. \tag{9}$$

(iv) to pay a lump sum for hospitalization, a lump sum for side effects of the disease and a lump sum for death benefit under Plan 4 is given by

$$\pi_4 = \frac{(c_3\gamma\theta + c_2\mu)\Gamma(\gamma+1, \theta) + c_4\gamma\theta e^{-\delta}(\Gamma(\gamma, 6\theta) - \Gamma(\gamma, \theta))}{\frac{\theta^{\gamma+1}}{\delta\beta^{\gamma}}(1 - e^{-\delta}) - \Gamma(\gamma+1, \theta)(1 - \frac{\mu}{\delta}) + \frac{\mu\theta^{\gamma+1}}{\delta(\alpha+\mu)^{\gamma+1}} e^{-\delta}\Gamma(\gamma+1, (\alpha+\mu))}. \tag{10}$$

Proof. (i) The net level premium for a policy of one-year term for Plan 1 is given by:

$$\pi_1 = \frac{c_1 \bar{a}^i_{1\rceil} + c_2 \bar{A}^d_{1\rceil}}{\bar{a}^s_{1\rceil}}.$$

The proof is straightforward from Lemma (3.2), Lemma (3.3), and Lemma (3.4). Combining these results leads to Equation (7).

(ii) The net level premium for a policy of one-year term for Plan 2 is given by:

$$\pi_2 = \frac{c_3 \bar{A}^i_{1\rceil} + c_2 \bar{A}^d_{1\rceil}}{\bar{a}^s_{1\rceil}}. \tag{11}$$

$\bar{A}^d_{1\rceil}$ and $\bar{a}^s_{1\rceil}$ are obtained from Lemma (3.3) and Lemma (3.4) with $T = 1$, respectively. It remains to calculate $\bar{A}^i_{1\rceil}$.

$$\begin{aligned} \bar{A}^i_{1\rceil} &= \int_0^1 e^{-\delta t} \frac{\gamma}{t} I_t dt \\ &= \int_0^1 e^{-\delta t} \frac{\gamma}{t} N e^{-(\alpha+\mu)t} (\beta t)^\gamma dt \\ &= \frac{N\gamma\beta^\gamma}{\theta^\gamma} \Gamma(\gamma, \theta). \end{aligned}$$

We consider $\theta = \alpha + \mu + \delta$ and perform a change of variable $\theta t = u$ and calculate the integral. By substituting these results to Equation (11) we obtained the desired result Equation (8).

(iii) The net level premium for Plan 3 is

$$\pi_3 = \frac{c_1 \bar{a}^i_{1\rceil} + c_2 \bar{A}^d_{1\rceil} + c_4 e^{-\delta} \bar{A}^{sa}_{5\rceil}}{\bar{a}^s_{1\rceil}}.$$

The proof is straightforward from Lemma (3.2), Lemma (3.3), Lemma (3.4) and Corollary (3.5).

(iv) The net level premium for Plan 4 is

$$\pi_4 = \frac{c_3 \bar{A}^i_{1\rceil} + c_2 \bar{A}^d_{1\rceil} + c_4 e^{-\delta} \bar{A}^{sa}_{5\rceil}}{\bar{a}^s_{1\rceil}}.$$

The proof is straightforward from Lemma (3.3), Lemma (3.4) and Corollary (3.5). $\bar{A}^i_{1\rceil}$ is given in part (ii) in this theorem.

□

Reserve valuation

Prediction of loss reserve is an important problem from insurers' and regulators' viewpoints. Reserves are a critical tool for insurers to measure their liabilities toward policyholders. In classical life insurance, reserves build-up from the beginning of the policy term, as the insurer accumulates premiums, to ultimately run out at the end of the policy term, when all benefits have been paid out to the policyholders. In the proposed health insurance plans, loss reserves may be calculated

Occurrence time	Development time				
	0	1	…	J-1	J
1	$X_{1,0}$	$X_{1,1}$	…	$X_{1,J-1}$	$X_{1,J}$
2	$X_{2,0}$	$X_{2,1}$	…	$X_{2,J-1}$	
. :					
K-1	$X_{K-1,0}$	$X_{K-1,1}$			
K	$X_{K,0}$				

Figure 4: Run-off triangle.

using various standard and accepted actuarial methods, such as the chain ladder technique. Mack chain ladder model for loss reserving has been deeply studied in the literature; see, e.g., [14]. The typical data structure for loss reserve problem is given in Figure 1 where the rows corresponds to the occurrence date $k = 1, 2, , K$ and the column corresponds to the loss development date $j = 0, 1, , J$, where J is the maximum possible development. We consider a sequence of random variables $X_{k,j}$, where $X_{k,j}$ denotes the incremental payments made for the k^th accident time up to the j^th development time.

Cumulative payments $C_(k, j)$ for accident time k and after j development time is given by

$$C_{k,j} = \sum_{j=0}^{J} X_{k,j}$$

The claim's development process $C_{k,j}$ in mack chain ladder satisfies the following assumptions.

Model Assumption 2. Assume:

B_1) *Cumulative claims $C_{k,j}$ of different accident time k are independent.*

B_2) *There exist development factors $f_0, \cdots, f_{j-1} \geq 0$ such that*

$$E(C_{k,j} \mid C_{k,0}, ..., C_{k,j-1}) = f_{j-1} C_{k,j-1}$$

where $\hat{f}_j = \frac{\sum_{k=0}^{K-j-1} C_{k,j+1})}{\sum_{k=0}^{K-j-1} C_{k,j}}$.

B_3) *There exist variance parameters $\sigma_0^2, \cdots, \sigma_{j-1}^2$ 0 such that*

$$Var(C_{k,j} \mid C_{k,j-1}) = {}^{2}_{j-1} C_{k,j-1}$$

where $\sigma_j^2 = \frac{1}{K-j-1} \sum_{k=0}^{K-j-1} C_{k,j} (\frac{C_{k,j+1}}{C_{k,j}} - \hat{f}_j)^2$.

Let's D_k being the information available when the reserves are estimated, the filtration generated by the aggregated claim costs i.e.

$$D_k = \{C_{k,j}; k + j \leq K, j \leq K\}$$

Under Model Assumption (2), we have

$$\hat{C}_{k,j}^{CL} = \hat{E}(C_{k,j} \mid D_k) = C_{k,K-k} \prod_{l=K-k}^{k-1} \hat{f}_l$$

The loss reserve for accident time k is given by $\hat{R}_k = \hat{C}_{k,j}^{CL} - C_{k,K-k}$, and the total loss reserve is $\hat{R}^{total} = \sum_{k=1}^{K} \hat{R}_k$.

4 A practical application

This section provides the practical application of the findings. Assume that the constant recovery rate is $\alpha = 0.13$, the mortality rate is $\mu = 0.05$, the infection rate is $\beta = 0.75$, $\gamma = 0.75$ and the force of interest is $\delta = 0.002$. Moreover, assume that the insurance company provides $c_1 = 1000\$$ per day for hospitalization costs to the individuals in compartment I. The additional compensation for a dead individual is $c_2 = 1000\$$. The insurance company provides $c_3 = 10000\$$ lump sum for hospitalization costs to the individuals in compartment I and the costs for side effects of disease are $c_4 = 1000\$$.

Given that we do not have the data related to the epidemic disease during 5 years, so to calculate the loss reserve, we will generate the number of claims related to the hospitalization costs, the side effects caused by the disease, and death benefits using the simulation method. We generate synthetic data by using the Algorithm 2.

Algorithm 2 Generate run off triangle

1: **Require:** Inpute λ_k, p_j, c_m
2: **Ensure:** Output full run-off triangle
3: Set $k \leftarrow 1$;
4: While $k \leq 5$ do
5: Use the Poisson distribution (with intensity λ_k) to generate the number of claims for the accident year k, and call it N_k;
6: for $j \leftarrow 0$ to 4
7: Using the Multinomial distribution with parameters $(N_k, p_0, \cdots, p_j)$ to generate vector $(N_{k,0}, \cdots, N_{k,j})'$;
8: for $l \leftarrow 1$ to $N_{k,j}$
9: Set $X_{k,j} = \sum_{l=1}^{N_{k,j}} c_m^{(l)}$;
10: Set $k \leftarrow k + 1$.

We assume that individuals claimed the hospitalization costs to the insurance company in accident year k, N_k^h, in Poisson distribution with intensity $\lambda_k^h = (51, 136, 149, 232, 341)$. For the side effects caused by epidemic disease, we assume that individuals claimed to the insurance company in accident year k, N_k^{sa}, in Poisson distribution with intensity $\lambda_k^{sa} = (45, 120, 132, 205, 320)$, and the individuals claimed death to the insurance company in accident year k, N_k^d, in Poisson distribution with intensity $\lambda_k^d = (47, 126, 139, 215, 317)$. For each data generation, the number of claims reported for each accident year k is distributed in the cells of the run-off triangle based on a multinomial distribution with probability $p_j = (0.2508, 0.4304, 0.1931, 0.0700, 0.0555)$. The benefits are placed in the number of $N_{k,j}$ in each cell (k, j) of the run-off triangle. We simulated a run-off triangle for hospitalization and death benefits (data for Plan 1 and Plan 2), and a run-off triangle for hospitalization, side effects, and death benefits (data for Plan 3 and Plan 4). Table 2 shows the cumulative run-off triangle for the coverage of Plan 1

and Plan 2.

Table 1: Cumulative claims, $C_{k,j}$, run-off triangle for Plan 1 and Plan 2 in \$.

j/k	0	1	2	3	4
1	114,000	357,000	475,000	523,000	549,000
2	406,000	1,101,000	1,469,000	1,568,000	
3	254,000	927,000	1,226,000		
4	594,000	1,739,000			
5	939,000				

The chain ladder model has been applied to the data in Table 2 to predict the amount of loss reserve. To use the chain ladder method, we must first calculate the development factors. The development factor estimates $\hat{f}_j$ for Plan 1 and Plan 2 are calculated by assumption B_2 of Model Assumption (2) and presented in Table 3. Applying these development factor estimators to the data set of Table 2 leads

Table 2: Cumulative claims, $C_{k,j}$, run-off triangle for Plan 3 and Plan 4 in \$.

j/k	0	1	2	3	4
1	125,000	391,000	514,000	568,000	593,000
2	436,000	1,211,000	1,604,000	1,711,000	
3	295,000	1,019,000	1,346,000		
4	645,000	1,861,000			
5	1,028,000				

Table 3: Development factor estimator for Plan 1, Plan 2, Plan 3 and Plan 4.

$\hat{f}_j^{Plans1\&2}$	3.0146	1.3291	1.0772	1.04373	1
$\hat{f}_j^{Plans3\&4}$	2.9860	1.3216	1.0760	1.0440	1

to the chain ladder loss reserves. For this aim, we employ the ChainLadder package of the R software and predict the lower part of Table 2 which is shown in red color in Table 4. The difference between the numbers in the last column of Table 4 and the numbers on the diagonal of Table 2 in each accident year gives the yearly loss reserves. The yearly loss reserves under Plan 1 and Plan 2 are presented in Table 2. The total loss reserve is obtained from the sum of the yearly loss reserves. In the same way, calculations for Plan 3 and Plan 4 are presented in Table 5

The results of net level premium under Theorem (3.6) are shown in the second column of Table (7). The loss reserve for these plans is presented in the third column of Table (7). Plan 3 and Plan 4 provide more coverage than Plan 1 and Plan 2, so the insurance company faces more risk in Plan 3 and Plan 4, and for this reason, it must keep more loss reserves.

Table 4: Prediction of loss reserve under Plan 1 and Plan 2 coverages in $.

j/k	0	1	2	3	4
1					
2					1,636,563
3				1,320,599	1,378,344
4			2,311,375	2,489,722	2,598,588
5		2,830,728	3,762,435	4,052,747	4,229,958

Table 5: Prediction of loss reserve under Plan 3 and Plan 4 coverages in $.

j/k	0	1	2	3	4
1					
2					1,786,308
3				1,448,316	1,512,063
4			2,459,559	2,646,523	2,763,007
5		3,069,618	4,056,908	4,365,294	4,557,429

Table 6: Yearly loss reserves for Plan 1, Plan 2, Plan 3 and Plan 4 in $.

$\hat{R}_k^{Plans1\&2}$	0	68,563	152,344	859,588	3,290,958
$\hat{R}_k^{Plans3\&4}$	0	75,308	166063	902,007	3,529,429

Table 7: Fair premium and loss reserve for health insurance under four plans in $

Type of plan	Premium	Loss reserve
Plan 1	$\pi_1 = 20.42$	4,371,452
Plan 2	$\pi_2 = 36.34$	4,371,452
Plan 3	$\pi_3 = 81.40$	4,672,806
Plan 4	$\pi_4 = 64.61$	4,672,806

5 Conclusion and suggestions

This paper combines epidemiological models with actuarial literature and develops a theoretical model to design new health insurance products with some specific benefits for epidemic disease. This paper proposes four health insurance plans. For each plan, a policyholder pays a premium and receives somehow constant hospitalization cost coverage (per day or lump sum), constant coverage for one side effects caused by epidemic disease, and lump sum for the death of epidemic disease. The only difference between these four plans is their coverage conditions. The first plan provides coverage for hospitalization costs in annuity and lump sum for death benefit. In the second plan, the insurance company pays the hospitalization costs

in a lump sum and the death benefit is paid in lump sum such as Plan 1. We believe that the coverage of side effects of the disease may be more attractive for policyholders who want to get more coverage for her/his fear of the epidemic disease. The third plan pays the hospitalization costs in annuity, a lump sum for side effects of the disease, and a lump sum for a death benefit. The fourth plan is the same as the third plan but it pays the hospitalization costs in a lump sum. This paper employed a simple actuarial method to determine a fair insurance premium for four health insurance proposed plans. To this aim, we concentrate on the simplest SIDS epidemic model. We calculate the premium based on the equivalence principle. We investigate how one can calculate loss reserve for the epidemic health insurance plans. To predict the loss reserve, the chain ladder method was used. As we know, one of the problems caused by infectious diseases is the occurrence of side effects after contracting the disease. These side effects of the disease may cause critical illnesses for the insured. In this paper, attention has been paid to these side effects, and it was suggested that the side effects caused by the infectious disease mentioned in the insurance policy that occurs up to five years after the expiration of the insurance policy will be covered. Because we do not have data related to the epidemic disease for five years, we designed an algorithm to generate the data so that we can predict the loss reserve. In this paper, the costs related to the side effects caused by the disease are considered fixed. It is suggested that in future research these costs should be considered variable according to the inflation rate. Different epidemic compartment models will have different premiums. It is recommended that future research consider the calculation of premiums for other epidemic models. It can be useful to predict the reserve of epidemic diseases based on real data. In this paper, the chain ladder method was used to predict the loss reserve for the epidemic disease. The use of other reserving methods, especially micro-level models that include more information details in the reserving model, are suggested as future works.

Bibliography

[1] L. J. S. ALLEN *Stochastic population and epidemic models: persistence and extinction*, Springer, 2015.

[2] T. BRITTON AND E. PARDOUX, *Stochastic epidemic models with inference*, Springer, 2019.

[3] A. A. CHERNOV, A. A. SHEMENDYUK AND M. Y. KELBERT, *Fair insurance premium rate in connected SEIR model under epidemic outbreak*, Mathematical Modelling of Natural Phenomena, 16 (2021), doi:10.1051/mmnp/2021028.

[4] D. J. DALEY AND J. GANI, *Epidemic modeling: an introduction*, Cambridge University Press, 2001.

[5] R. FENG AND J. GARRIDO, *Actuarial applications of epidemiological models*, North American Actuarial Journal, 15 (2011), pp. 112-136.

[6] D. HAINAUT, *An actuarial approach for modeling pandemic risk*, Risks, 9 (2021), pp. 1-28.

[7] R. KASS, M. GOOVAERTS, J. DHAENE AND M. DENUIT, *Modern actuarial risk theory*, Second Edition, Springer, 2008.

[8] W. O. KERMACK AND A. G.MCKENDRICK, *A contribution to the mathematical theory of epidemics*, Proc Roy Soc Lond, 115 (1927), pp. 700-721.

[9] W. O. KERMACK AND A. G. MCKENDRICK, *Contributions to the mathematical theory of epidemics*, Proc Roy Soc Lond, 138 (1932), pp. 55-83.

[10] C. Lefévre, P. Picard and M. Simon, *Epidemic risk and insurance coverage*, Journal of Applied Probability, 54 (2017), pp. 286-303.

[11] Zh. Ma, J. Li, *Dynamical modeling and analysis of epidemics*, Worls Scientific, 2009.

[12] H. S. Nam, *Mathematical and actuarial analysis on a deterministic SEIR model*, Global Journal of Pure and Applied Mathematics, 18 (2022), pp. 453-464.

[13] C. I. Nkeki and G. O. S. Ekhaguere, *Some actuarial mathematical models for insuring the susceptibles of a communicable disease*, International Journal of Financial Engineering, 7 (2020), pp.1-45.

[14] M. V. Wüthrich and M. Merz, *Stochastic claims reserving methods in insurance*, John Wiley & Sons, 2008.

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