Comparing the different types of Markov switching model for Euro to Iran Rial exchange rate

Mahdi Pourrafiee[1], S. Pourmohammad Azizi[2], Marzieh Mohammadi Larijani [3], Ali Pahlevannezhad[4]

[1] Department of Mathematics and Computer Science, Allameh Tabataba'i University, Tehran, Iran.
mahdipourrafiee@gmail.com

[2] Henan Academy of Big Data, Zhengzhou University, Zhengzhou 450001, China.
eazizak@yahoo.com

[3] Department of Mathematics and Computer Science, Allameh Tabataba'i University, Tehran, Iran.
Hamta.mohammadi@gmail.com

[4] Department of Management and Accounting, Allameh Tabataba'i University, Tehran, Iran.
alipahlevannezhad@gmail.com

Abstract:
According to the law of one price, the price of a foreign commodity depends on its price at the origin as well as the exchange rate of that country. According to this rule, if the foreign exchange costs are insignificant, the price of a single commodity will be the same everywhere, and ideally the purchasing power of a currency inside and outside the country will be the same. Due to the effect of the exchange rate on financial assets, the study of regime change in exchange rate fluctuations is importance and Regime Switching model is the most complete and populare regime change. The aim of this research is modeling Euro-Rial exchange rate under the Markov regime switching and Markov random regime switching models. In order to evaluate the achieved results, unit root test, which included the Dickey-Fuller and the Phillips-Peron test, is used to estimates Markov regime switching and Markov random regime switching parameters in order to find the best fluctuations model.

[2] Corresponding author
Received: 2020-08-30 Approved: 2020-11-02

Keywords: Markov regime switching , Markov random regime switching model, Dickey-Fuller test, Phillips-Peron test.

MSC2010 Classifications: 91G50, 91B84.

1 Introduction

In this paper, authors investigate the relationship between Euro and Iran Rial exchange rate and different types of Markov switching models during low and high volatility periods. Recently, study of the interest rate pass-through in crises has drawn attention to itself. Interest rate pass-through may be different among household and non-financial corporate interest rates, therefore onducting monetary policy would be necessary for a central bank to track interest rates for households and corporations. Moreover, the monetary policy actions that cause changes in retail interest rates highlight expectations about the economy and the future monetary policy. Additionally, given that the inflation-targeting regime is a widely implemented policy, the importance of the pass through mechanism increases. Investigating interest rate pass-through in bank rates may reveal useful information about the management strategies followed by banks and the effectiveness of central bank policy.

Before the First World War, the gold standard system was used by the countries to define the exchange rate [1, 2]. In this system, the exchange rate of the countries had relative stability and no fluctuations. In 1944, the World Bank decided to choose fixed exchange rate method and use US dollar as currency's connector [3–5]. In 1971, the relationship between the US dollar and the gold was cut off and the value of dollar depended on global supply and demand [6, 7]. After the adoption of the currency floating exchange system in 1974 [8, 9] inflammation in the currency market became more apparent, and because of its importance has a direct impact on the financial system of the countries [10–13]. The aim of this study is to investigate the fluctuations of the Euro-Rial exchange rate by Markov regime switching and Markov random regime switching models.

For this purpose, in the next section, we present Markov regime switching model and Markov random regime switching models, in section 3, we show numerical examples and analysis.

2 Preliminaries

Some basic definitions and preliminary will be introduced in this section. These definitions including Markov regime switching model and Markov random regime switching model which are used for modeling Euro-Rial exchange rate time series.

2.1 Markov regime switching model

When an economy changes from one regime to another, a fundamental change occurs [14]. In 1989, Hamilton [15]introduced the Markov model in which regimes were hidden. The experimental results of markov switching models of macroeconomic variables related to interest rates are an important evidence for the validity of markov switching models [16] [17] . Kai (1994) fitnessed a markov of GARCH switching model on the return rate of the 3 months Treasury bills and found three years that were strongly fluctuating,the OPEC's oil shock in 1974 and the federal experience between 1979 and 1982 [3]. In markov switching models, the economists wont be able to see the regimes, so we can conclude that the regime can occur at any location [19]. In the discussion of parameter determination, the probability of transition from regime i to j, the regime time t is define as follow [20]:

$$pr(s_t = k\,|s_{t-1} = j) = 1 - \sum_{i=1}^{i=k-1} pr(s_t = i\,|s_{t-1} = j) \qquad (1)$$

It is also important to consider the variables' probabilities. Dimitrios [21] show the regime switching works because the parameters of the general model takes only one regime and not the regime changes into account. The Markov Switching model is define as follows

$$\Delta r_t = a + br_{t-1} + \sigma_i r_{t-1}^{\gamma} \varepsilon_t,\ i \in (1, 2) \qquad (2)$$

The $\sigma_i r_{t-1}^{\gamma} \varepsilon_t$ is random parameters which is represented the fluctuation of the interest rate or in the other word the regime variance , r_t indicates the interest rate, ε_t is an error parameter and γ represents a propagation parameter.

2.2 Markov random regime switching model

The Markov random regime switching model is compared with the random model and the markov switching model. The special case of Markov random regime switching model is as follows

$$y_t = x_t + 2\gamma \log r_{t-1} + \log \varepsilon_t^2 \quad x_t = \omega_i + \phi x_{t-1} + \eta_t \quad \omega_i = \log \delta_i^2 \tag{3}$$

In which γ is propagation parameter, $\eta_t \approx iid(0, \sigma_\eta^2)$ is the random parameter which randomizes the process, r_t is the time-dependent interest rate and ε_t^2 is square of error.

3 Numerical examples and analysis

In this section to demonstrate the idea of our model and solve the effectiveness of the models, we conduct the expriments by using our proposed Markov regime switching model and Markov random regime switching model to estimate the best time series which are taken from daily historical data of Euro-Rial exchange rate. Initially, a recent price data beginning from working day of from 2014/11/12 to 2017/12/12 (only business days)

3.1 Time series modeling

A time series variable is non-steady when the mean, variance, and Autocorrelation is constant over time. We used the differentiation method to keep our series steady. The series of return are defined as follows

$$r_t = 100 \times \log \frac{f_t}{f_{t-1}} \tag{4}$$

This formula is used to estimate the fluctuations of Euro-Rial daily exchange. We use jarque bera test in order to understand the distribution of return, the results of jarque bera test as follows

Table 1: Sample statistics of jarque-bera test from daily data (2016- 2019)

Average	Max	Min	Std	Skewness	Kurtosis	Test J-B
0.0115	72.53	1.54	0.725	75.9	5835.94	912+e0.75

The jarque-bera test is used to understand that the time series have normal distribution, and the null hypothesis is that the data has a normal distribution. According to Table 1 the statistic is not normal because its skewness and kurtosis is different with normal distribution.

Markov regime switching model

We consider the parameter γ in three modes of zero, 0.5 and determination estimation and also consider two regimes in the model

$$r_t = a + br_{t-1} + e_t \, , i \in (1, 2) \tag{5}$$

Table 2: Sample statistics of Markov regime switching model from daily data (2016- 2019)

	First Regime		Second Regime	
	r_t	$e_t = \Delta r_t - a - br_{t-1}$	r_t	$e_t = \Delta r_t - a - br_{t-1}$
Mean	2729.20	0.1375	16489.57	1025.414
Median	2728	0.1436	16451	1012.715
Std	179.204	0.07524	2052.128	596.6847
Skewness	-0.6851	-0.4531	-0.45887	-0.005183
Kurtosis	3.142	1.8322	2.65871	1.83124
J-B Test	226.1254	175.2451	165.1248	238.2158

The variance is the only difference between the two models in order to solve this problem, we have the following statement:

$$\sigma^2_{s_t} = \sigma^2_1(1 - s_t) + \sigma^2_1 s_t \tag{6}$$

Markov random regime switching model

In this model, γ is extremely important. In this research, we only consider two modes $\gamma = 0$ and $\gamma = 1$ that shows explosive and non-explosive behavior respectively.We estimated the Markov random regime switching model in three case which is show in table 3.

It is important to consider the transmission probabilities. We consider the probabilities p_{00} and p_{11} for probability of staying in the zero-regime and regime one. The expected duration of the zero-mode regime is $\frac{1}{1-p_{00}} = 2853$ and $\frac{1}{1-p_{00}} = 2853$ is the duration of regime one which shows the days that two regimes are continuous, so probability of staying in the zero-mode regime is 0.9996 and the probability of staying in regime one mode is 0.9997.

Table 3: Sample statistics of limited, diffusion and non-diffusion Markov random regime switching model from daily data (2016- 2019)

	Parameters	Coefficient	T-Statistics	Significance Level
Limited model	γ	0.5	-	-
	ω_1	3.4162	3.48	0.00
	ω_2	3.5142	6.52	0.00
	ϕ	8E-4	0.37	0.00
Diffusion model	γ	1	-	-
	ω_1	1.3451	2.34	0.01
	ω_2	2.3642	3.67	0.00
	ϕ	-7E-6	-3.57	0.03
Non Diffusion model	γ	0	-	-
	ω_1	3.2256	0.69	0.45
	ω_2	5.3124	-2.35	0.00
	ϕ	-8E-5	6.25	0.00

4 Variation Test

The unit root test is one of the most common tests that is used to detect the stationarity of a time series. The basic assumption of the Dickey-Fuller test is that the error sentences have white-noise and therefore are not correlated. Phillips and Peron introduced another model of this test, and in fact, they generalized the Dickey-Fuller test in which the assumption of lake of white-noise sentences was considered.According to the data in Table 1 the distribution is not normal. Then if the series has a single root, then the market can be efficient. Series tests can be used to check the efficiency and to determine the root of a unit.

From Table 4 we conclude:

1- The values of the Dickey-Fuller test and the Phillips-Peron test are equal at all levels and for both series.

2- In the logarithmic series of the Euro-Rial exchange rate, the Dickey-Fuller and Phillips-Peron tests are greater than critical values at 1percent, 5percent and 10 percent significant levels, then the zero assumption is not rejected and the existence of the unit root is not accepted.

Table 4: Sample statistics of Phillips Perron and Dicky Fuller Test from daily data (2016- 2019)

		Dicky Fuller Test		
	Significance Level	critical value	Statistics	Result
Euro	0.01	-3.48221	-0.84925	Accept
Logarithm	0.05	-2.89654	-0.84925	Unit
Series	0.1	-2.54562	-0.84925	Root
Euro Difference	0.01	-3.48221	-0.84925	Reject
Logarithm	0.05	-2.89654	-0.84925	Unit
Series	0.1	-2.54562	-0.84925	Root
		Phillips Perron Test		
	Significance Level	critical value	Statistics	Result
Euro	0.01	-3.48221	-0.84925	Accept
Logarithm	0.05	-2.89654	-0.84925	Unit
Series	0.1	-2.54562	-0.84925	Root
Euro Difference	0.01	-3.48221	-0.84925	Reject
Logarithm	0.05	-2.89654	-0.84925	Unit
Series	0.1	-2.54562	-0.84925	Root

3- The unit root demolished after differentiation Logarithm Series.

4.1 Comparison of dynamic interest rate fluctuation models

To compare the models, we used three test including Likelihood Ratio Test, mean absolute error, the root mean square error over three different periods. The Likelihood Ratio Test is the first test to compare the regime switching and Markov random regime switching models which is as follows:

$$R^2 = 1 - \frac{\frac{1}{T}\sum_{t=1}^{T}\left(\sigma_t \mid t - 1 r_{t-1}^{\gamma} - |e_t|\right)^2}{\frac{1}{T}\sum_{t=1}^{T} e_t^2} \tag{7}$$

The mean absolute error calculated the distance between the real values and the predicted values. The formula for calculating this statistic is as follows:

$$MSE = \frac{\sum_{i=1}^{N} |x_i - \tilde{x}_i|}{N} \tag{8}$$

Where x_i are real values of time series, $\tilde{x}_i$ is the estimated or predicted values of the time series, represent the sum of the absolute errors and is the total number of data. In this statistic, the lower this amount be, it will be more appropriate:

$$RMSE(\theta) = \sqrt{E((\theta_1 - \theta_2)^2)} = \sqrt{\frac{\sum_{i=1}^{N} (x_{1,i} - x_{2,i})^2}{N}} \tag{9}$$

In whichθ_1 is actual value and θ_2 is estimated value. In the following table, the intended values for each model. The tables of comparison of these two models with three tests of Likelihood Ratio Test, mean absolute error and the root mean square error are given in Table 5.

Table 5: Sample statistics of compare between Markov regime switching and Markov random regime switching models from daily data (2016-2019)

Period	Test	Markov Regime	Markov Random Regime
2016/11/12	MSE	14.6892	17.5123
to	RMSE	3.53	4.507
2017/11/12	r^2	0.9933	0.9918
2017/11/12	MSE	99.23	120.31
to	RMSE	13.24	15.85
2018/11/12	r^2	0.9923	0.99100
2018/11/12	MSE	109.81	111.43
to	RMSE	9.85	12.34
2019/11/12	r^2	0.975	0.968

Based on the results of Table5, the Markov regime switching model, has better than other model.

5 Conclusion

According to the results of the three intended statistics, the most suitable model is the switching regime model, which according to the individual study of each regime as well as the dependence of the parameters in each regime to mentioned regime, better results will be achieved and after this model, the random oscillation model is better one which shows better results than propagation model and random oscillation model.

Here, we need to concentrate on the volume of national and foreign currency in circulation in the economy: by lowering the supply of national money and increasing the supply of foreign currency in circulation in the economy, we will lower the national and foreign exchange rates. In general, changes in the regime are often based on political and social problems, and the shocks that come about in this area are very important influential. Therefore, it is necessary to succeed in these areas if we be willing that the exchange rate to be far from excessive jumps and abundant shocks; but, it is not possible to fully expect these areas to be fully covered, because some series of conditions origins from outside the country. Future studies in this field can investigate the forecast for the exchange rate of other random series, such as the Fourier series, or to apply the existing model for other valid currencies

Bibliography

[1] S. Ye, J. Du, Can markov switching based hybrid models improve the performance of forecasting exchange rates?, Procedia computer science 139 (2018) 321–328.

[2] Y. Ning, L. Zhang, Modeling dynamics of short-term international capital flows in china: A markov regime switching approach, The North American Journal of Economics and Finance 44 (2018) 193–203.

[3] S. Papadamou, T. Markopoulos, Interest rate pass through in a markov-switching vector autoregression model: Evidence from greek retail bank interest rates, The Journal of Economic Asymmetries 17 (2018) 48–60.

[4] J.-T. Wu, Markov regimes switching with monetary fundamental-based exchange rate model, Asia Pacific management review 20 (2) (2015) 79–89.

[5] S. M. E. P. Azizi, A. Neisy, Mathematic modelling and optimization of bank asset and liability by using fractional goal programing approach, International Journal of Modeling and Optimization 7 (2) (2017) 85.

[6] G. C. Aye, T. Chang, R. Gupta, Is gold an inflation-hedge? evidence from an interrupted markov-switching cointegration model, Resources Policy 48 (2016) 77–84.

[7] A. H. Nafei, S. M. E. P. Azizi, R. A. Ghasempour, An approach in solving data envelopment analysis with stochastic data, in: International workshop on Mathematics and Decision Science, Springer, 2016, pp. 154–162.

[8] B. M. Lucey, F. A. O'Connor, Do bubbles occur in the gold price? an investigation of gold lease rates and markov switching models, Borsa Istanbul Review 13 (3) (2013) 53–63.

[9] S. M. E. P. M. Azizi, A. Neisy, A new approach in geometric brownian motion model, in: International workshop on Mathematics and Decision Science, Springer, 2016, pp. 336–342.

[10] H.-Y. Lee, S.-L. Chen, Why use markov-switching models in exchange rate prediction, Economic Modelling 23 (4) (2006) 662–668.

[11] R. Mohamadinejad, J. Biazar, A. Neisy, Spread option pricing using two jump-diffusion interest rates, UNIVERSITY POLITEHNICA OF BUCHAREST SCIENTIFIC BULLETIN-SERIES A-APPLIED MATHEMATICS AND PHYSICS 82 (1) (2020) 171–182.

[12] M. Karimnejad Esfahani, A. Neisy, S. De Marchi, An rbf approach for oil futures pricing under the jump-diffusion model, Journal of Mathematical Modeling (2020) 1–12.

[13] A. H. Nafei, W. Yuan, H. Nasseri, Group Multi-Attribute Decision Making Based on Interval Neutrosophic Sets, Vol. 28, Infinite Study, 2019.

[14] A. Z. Baharumshah, S.-V. Soon, M. E. Wohar, Markov-switching analysis of exchange rate pass-through: Perspective from asian countries, International Review of Economics & Finance 51 (2017) 245–257.

[15] H. Fahmy, Regime switching in commodity prices, Ph.D. thesis, Concordia University (2011).

[16] J. D. Hamilton, Specification testing in markov-switching time-series models, Journal of econometrics 70 (1) (1996) 127–157.

[17] P. Shahrestani, M. Rafei, The impact of oil price shocks on tehran stock exchange returns: Application of the markov switching vector autoregressive models, Resources Policy 65 (2020) 101579.

[18] J. Gonzalo, J.-Y. Pitarakis, et al., Estimation and inference in threshold type regime switching models, Handbook of Research Methods and Applications in Empirical Macroeconomics, edited by Nigar Hashimzade and Michael A. Thornton (2013) 189–205.

[19] W. Dai, A. Serletis, On the markov switching welfare cost of inflation, Journal of Economic Dynamics and Control 108 (2019) 103748.

[20] H. Tong, Threshold models in time series analysis30 years on, Statistics and its Interface 4 (2) (2011) 107–118.

[21] D. V. Paliouras, Comparing regime-switching models in time series: Logistic mixtures vs. markov switching, Ph.D. thesis (2007).