Finite difference method for basket option pricing under Merton model

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Abstract:
In financial markets, dynamics of underlying assets are often specified via stochastic differential equations of jump diffusion type. In this paper, we suppose that two financial assets evolved by correlated Brownian motion. The value of a contingent claim written on two underlying assets under jump diffusion model is given by two dimensional parabolic partial integro-differential equation (PIDE), which is an extension of the Black-Scholes equation with a new integral term. We show how basket option prices in the jump diffusion models, mainly on the Merton model, can be approximated using finite difference method. To avoid a dense linear system solution, we compute the integral term by using the Trapezoidal method. The numerical results show the efficiency of proposed method.

Keywords: basket option pricing, jump-diffusion models, finite difference method.
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1 Introduction

In practice, the financial instruments such as basket options require a multivariate model with dependence between components. Basket op-

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tions play an important role in risk reduction and financial markets management. Financial investors are interested in the basket option bacause of fewer price in compared to the option price with one asset.

Under actual market conditions, asset prices are exposed to large and sudden changes when reacting to good or bad news. The jump diffusion model is attractive because it induces the discrete economic events such as wars and market crash.

The financial option constructed on two underlying assets leads to the two dimensional parabolic PIDE which is based on jump diffusion process instead of the classical diffusion dynamics with the Black-Scholes model [1]. Merton [2] introduce the option pricing model by a jump diffusion asset dynamics in order to have a more realistic description of the market.

In most cases, the analytical method for the solution of PIDE problems is impossible because it has the integral term. An implicit, finite difference approach for single asset European options was explored by d'Halluin, et al. [4] and on two-factor option pricing was developed by Clift, et al. [3], which avoids a dense linear system solution by combining a fixed point iteration scheme with FFT method.

The aim of this paper is presenting the implicit finite difference method for two-dimensional PIDE arising from the option pricing model with two underlying asset. We have approximated the European basket option price to show the efficiency and accuracy of the proposed method.

2 Basket option model

We assume that the asset price $S_i, i = 1, 2$ follows the risk-natural process

$$\frac{dS_i}{Si} = (r - \lambda\kappa_i)dt + \sigma_i dW_i + (e^{J_i} - 1)dN. \tag{1}$$

where r denotes risk free rate, σ_i and J_i are volatility and jump size of the ith-asset, respectively. Here, W_i is the standard Brownian motion where ρ_{ij} is correlation between W_i, W_j. In the equation (1), dN is a Poisson process with mean arrival rate $\lambda > 0$ and $\kappa_i = E[e^{J_i} - 1]$.

Using the Ito's formula for finite activity jump processes, the contingent claim $v(S, t)$ which is depends on $S = (S_1, S_2)$ can be derived by taking the expectation under the risk natural process. The resulting PIDE is given by

$$\frac{\partial v}{\partial t} = -\frac{1}{2}\sum_{i,j=1}^{2}\sigma_i\sigma_j\rho_{ij}S_iS_j\frac{\partial^2 v}{\partial S_i S_j} - \sum_{i=1}^{2} rS_i\frac{\partial v}{\partial S_i} + rv$$
$$+ \lambda\int_{\tilde{\Omega}}[v(Se^J,t) - v(S,t) - \sum_{i=1}^{2}\frac{\partial v}{\partial S_i}S_i(e^{J_i}-1)]g(J)\mathrm{d}J \quad (S,t)\in\tilde{\Omega}\times[0,T), \tag{2}$$

where the jump magnitudes $J = (J_1, J_2)$ have some known probability density $g(J)$. In the Merton model, the density function for the jump magnitudes follow the bi-variate Normal distribution with mean $\tilde{\mu}$, standard deviation matrix $\tilde{\sigma}$ and correlation factor $\tilde{\rho}$.

Let $y = (y_1, y_2)$, $S_i = e^{y_i}$, $\tau = T - t$ and by the change of variables $v(e^y, t) = u(y, \tau)$ the equation (2) can be rewritten as following parabolic partial integro-differential equation

$$\begin{cases} u_\tau - \mathcal{L}u - \lambda\mathcal{H}u = 0 & (y,\tau)\in\Omega\times(0,T] \\ u(y,0) = \mathcal{I}(y) \end{cases} \tag{3}$$

where $\mathcal{I}$ is payoff condition,

$$\mathcal{L} = \frac{1}{2}\sum_{i,j=1}^{2}\sigma_i\sigma_j\rho_{ij}\frac{\partial^2}{\partial y_i\partial y_j} + \sum_{i=1}^{2}(r - \frac{\sigma_i^2}{2})\frac{\partial}{\partial y_i} - r$$

is linear differential operator term, and

$$\lambda\mathcal{H}u = \int_{\Omega}[u(y+J,\tau) - u(y,\tau) - \sum_{i=1}^{2}(e^{J_i}-1)\frac{\partial u}{\partial y_i}]g(J)\mathrm{d}J$$

is the integral term. The operator $\lambda\mathcal{H}$ represents the effects of finite activity asset price jumps generated by a Poisson process. The typical payoff value for basket put option is given by

$$\mathcal{I}(y) = \max(K - \sum_{i=1}^{d}\alpha_i e^{y_i}, 0) \tag{4}$$

where K is the exercise price of the option and $\alpha_i, i = 1,2$ are given constants correspond to the share of assets on portfolio.

3 Numerical results

For the numerical illustrations through out this section, we have used the diffusion parameters $\sigma_1 = \sigma_2 = 0.3$, $\rho = 1$, $r = 0.1$, jump parameters $\tilde{\mu}_1 = \tilde{\mu}_2 = 0.1$, $\tilde{\sigma}_1 = \tilde{\sigma}_2 = 0.4$, $\tilde{\rho} = 0$ and payoff parameters $\alpha_1 = \alpha_2 = 0.5$, $T = 1$, $K = 1$. We have discretized the operator in space by localization of the computational domain $[-3, 3] \times [-3, 3]$ and the integral term is implemented by Trapezoid role. The arising ODEs system is solved by ode45 command of Matlab. In the experiments, we have used uniform gride points where the number of the grid points in each direction is $N = 40$. The approximated solution for different values of asset price is reported on table 1.

Table 1: European put basket option prices for different value of expected jump arrival rate λ

$v(S_1, S_2, t)$	$\lambda = 0$	$\lambda = 0.1$	$\lambda = 0.2$
$v(0.9, 0.9, 0)$	$1.3680e-01$	$1.4331e-01$	$1.4935e-01$
$v(0.9, 1, 0)$	$1.1276e-01$	$1.1912e-01$	$1.2514e-01$
$v(0.9, 1.1, 0)$	$9.2345e-02$	$9.8403e-02$	$1.0427e-01$
$v(1, 0.9, 0)$	$1.1287e-01$	$1.1905e-01$	$1.2524e-01$
$v(1, 1, 0)$	$9.2252e-02$	$9.8186e-02$	$1.0415e-01$
$v(1, 1.1, 0)$	$7.4980e-02$	$8.0556e-02$	$8.6208e-02$

For the arrival intensity rate $\lambda = 0$, the Merton model is reduced the ordinal Black-Scholes model. The option prices for this case is reported on second column of table 1. It can be seen that the option prices increase when the intensity rate λ increases.

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