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Research paper

An online portfolio selection algorithm using beta risk measure and fuzzy clustering

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Abstract:
An online portfolio selection algorithm has been presented in this research. Online portfolio selection algorithms are concerned with capital allocation to several stocks to maximize the portfolio return over the long run by deciding the optimal portfolio in each period. Despite other online portfolio selection algorithms that follow Kelly's theory of capital growth and only focus on increasing return in the long term, this algorithm uses the beta risk parameter to exploit upside risk while hedging downside risk. This algorithm follows the pattern-matching approach, uses fuzzy clustering in the sample selection step, and the log-optimal objective function along with the transaction cost and considering the beta risk measure in the portfolio optimization step. The implementation of the proposed algorithm in this research on a 10-stock dataset from the NYSE market in the period of December 2021 to December 2022 shows the superiority of this algorithm in terms of return and risk and the overall Sharpe ratio compared to the algorithms proposed previously in the literature on online portfolio selection.

Keywords: Pattern-matching approach, Risk-averse model, Fuzzy C-Means, Transaction cost.
Classification: MSC2010 Classifications: 91G10, 68T10, 91B60.

1 Introduction

The use of algorithmic trading techniques has increased in recent years due to the growth of the volume, and the rate of transactions, and the amount of data to make decisions about portfolio selection. Online portfolio selection is one of the algorithmic trading issues, which allocates capital among several stocks and

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updates the stocks weight in each period to maximize the portfolio return over the long run.

There are two major theories in the literature on portfolio selection models. The Markowitz theory [13] is concerned with balancing expected returns (mean) and risk (variance) and is suitable for single-period decisions. In contrast, Kelly's theory [3], which is known as the capital growth theory, is concerned with maximizing the expected log returns and is more suited to multi-period decision-making. As the decision-making in online portfolio selection approaches is performed for several consecutive periods, Kelly's capital growth theory is conventionally used in modeling these algorithms. The nature of algorithms based on Kelly's theory is such that the focus is on increasing portfolio returns in the long term, which may lead to an increase in investment risk.

As yet, three main approaches of follow the winner, follow the loser, and pattern matching have been mentioned in the literature on online portfolio selection [11]. The results of the studies in online portfolio selection suggest that the pattern-matching approaches, which form the portfolio based on similar historical patterns, have empirically been revealed to be better than other approaches [11]. Pattern matching algorithms include the two steps of sample selection and portfolio optimization. In order to select the sample, the recent data is compared with the historical data, and the historical data that resemble the recent data are identified as the selected sample. In the next step, the optimal portfolio is selected based on the selected sample.

Every investor has two essential criteria for portfolio selection. First, obtaining the maximum return and, at the same time, facing the lowest possible risk. Therefore, portfolio selection models seek to maximize returns and minimize risk to obtain the best results according to these criteria. For this purpose, it is necessary to use relevant risk measures in modeling. Considering that the emphasis of the online portfolio selection models is on maximizing investment return in the long term, the investment risk in these models has been given less attention.

The algorithm proposed in this research is an online portfolio selection algorithm following the pattern-matching principle. This algorithm employs fuzzy clustering techniques to find historical patterns similar to the recent pattern and decide the optimal portfolio every day accordingly. The notable point in this research is the use of the beta risk measure in the portfolio selection model in order to exploit the risk measure to obtain the maximum benefit.

The background and research literature on the pattern-matching principle has been presented in section 2. Section 3 is concerned with the parameters, formulation, and issues related to clustering and the use of the risk measure. Section 4 introduces the process of the proposed algorithm. Section 5 is concerned with the application of the proposed algorithm on real data and its evaluation and comparison with the results of the previous algorithms introduced in the literature. Eventually, Section 6 presents conclusions and suggestions for future research.

2 Research Background

The pattern matching principle is one of the basic principles in the literature on online portfolio selection. This principle is concerned with using historical data to make decisions regarding the portfolio in the following period. This approach seeks historical patterns similar to the recent one. Pattern-matching algorithms have been empirically proven to yield the most significant capital growth compared to other online portfolio selection approaches. Usually, pattern-matching algorithms consist of two steps [11].

Step one, Sample Selection: In this step, the purpose is to find the set C. Set C is the set of price-relative vectors of historical days that are expected to have the same behavior as the following day. Then, each index is assigned a probability P_i, which is usually concerned with the degree of similarity.

Step two, Portfolio Optimization: In this step, an optimal portfolio is selected based on the price vector of similar days obtained in the first step, as follows.

$$b_{(t+1)} = \arg \max_{b \in \Delta_m} U(b; C) \tag{1}$$

Where $U(b; C)$ indicates the utility function of b, under the set C. While there is no similar day to the recent day, the uniform portfolio is chosen as the optimal portfolio.

Pattern-matching algorithms seek to find price-relative vectors similar to the price-relative vector of the following day. To this end, the similarity between the recent time window and historical time windows is calculated, and the behavior of the price-relative vector of the day following similar time windows is expected to be similar to the price-relative vector of the following day, the optimal portfolio of which is to be calculated. After identifying the similar price-relative vectors, the optimal portfolio for the following period is selected accordingly.

The $\mathrm{B^H}$ strategy, which is a combination of two steps, employing the histogram method in the first step and log-optimal in the second, was proposed by Györfi and Schäfer [7]. Györfi et al. [6] introduced $\mathrm{B^K}$ a combination of the two methods of Kernel in the first and log-optimal in the second step. Also, Györfi et al. [13] presented the $\mathrm{B^{NN}}$ strategy, which is a combination of the nearest neighbor in the first step and the log-optimal in the second step. The CORN strategy, a combination of the correlation method in the first and log-optimal in the second step was presented by Li et al. [10]. They proved to have empirically better performance than the three other algorithms mentioned earlier. Moreover, Györfi et al. [5] proposed the $\mathrm{B^S}$ strategy using semi-log-optimal to simplify $\mathrm{B^K}$ calculations. The $\mathrm{B^M}$ strategy a combination of the kernel method in the first and the Markowitz method in the second step- to employ Markowitz's theory and consider the balance between mean (return) and variance (risk) was presented by Ottucsák and Vajda [15]. Györfi and Vajda [6] also introduced the $\mathrm{B^{GV}}$ strategy to include the transaction cost in the calculations. Loonat and Gebbie [12] proposed learning zero-cost portfolio selection with pattern matching. A risk-aversion pattern matching-based portfolio selection, RACORN-K was presented by Wang et al. [17]. Sooklal et al. [16] proposed a modified Corn-K algorithm as DRICORN-K, which is a dynamic risk correlation-driven non-parametric algorithm.

In the sample selection step, pattern-matching algorithms seek for days similar to the following day by detecting time windows similar to the recent one. Clustering algorithms allow for all data to be sifted together, while the presented pattern-matching algorithms consider various distance criteria such as correlation and Euclidean distance to compare historical time windows to the recent one rather than comparing all data simultaneously. In this regard, Abdi and Najafi [4] introduced the spectral pattern-matching algorithm through spectral clustering in the sample selection step and log-optimal in the portfolio optimization step. Khedmati and Azin [9] also developed this model and used the four clustering algorithms of K-medoids, K-means, hierarchical clustering, and spectral clustering while including transaction costs in the model. Abdi et al. [5] presented an online portfolio selection algorithm based on the pattern-matching principle and using fuzzy clustering.

3 Theoretical Basis

In this section, the theoretical principles of the research, including the formulation of the problem and the items used in each step of the algorithm, are explained [11].

3.1 Formulation

Consider an investor who decides to invest the capital in a number of stocks ($m \geq 2$) for n trading periods ($n \geq 1$). The close price of m stocks in each time period t is shown by the vector $p_t = (p_{1,t}, p_{2,t}, , p_{i,t}, , p_{m,t})$. Also, the market price change is represented by an m-dimensional price-relative vector x_t, where each element is calculated by $x_{t.i} = \frac{p_{t.i}}{p_{t-1.i}}$ and shows the return of the i^{th} stock in period t. Thus, an investment in asset i on period t increases by a factor of $x_{t.i}$. In this regard, $x_1^n = \{x_1, , x_n\}$ is a sequence of price-relative vectors for n trading periods, which actually form an $n * m$ matrix, where the n index indicates the trading periods and the m index indicates the stocks. Also, the time-window of price-relative vectors between period s to t is shown as $x_s^e = \{x_s, , x_e\}$, $1 \leq s < e \leq n$.

At the beginning of the t^{th} period, the proportion of capital invested in m stocks, is specified by a portfolio vector $b_t = (b_{t,1}, , b_{t,m})$ where all the components are positive and also $\Sigma_{i=1}^{m} b_{t.i} = 1, \ \forall t$.

Investment ratios at the beginning of the t^{th} period are computed by observing the past behavior of the market, i.e. the price-relative vectors from the first period to the period before t, and are shown as $b_t = b_t(X_1^{t-1})$. So, $b_1^n = \{b_1, , b_n\}$ is the portfolio strategy for n periods, and also is an output of an online portfolio selection algorithm.

At the end of period t, by choosing the portfolio b_t for this period, a return of S_t which is calculated by $S_t = b_t^{\intercal} x_t = \Sigma_{i=1}^{m} b_{t.i} x_{t.i}$ has been achieved. Since this model reinvests the capital, the portfolio wealth will increase multiplicatively.

From period 1 to n, a portfolio strategy b_1^n increases the initial wealth S_0 by a factor of $\Pi_{t=1}^{n} b_t^{\intercal} x_t$, that is, the final cumulative wealth after a sequence of n periods is calculated as follows.

$$S_n(b_1^n.x_1^n) = S_0 \prod_{t=1}^{n} b_t^{\intercal} x_t = S_0 \prod_{t=1}^{n} \sum_{i=1}^{m} b_{t.i} x_{t.i} \tag{2}$$

Since the model assumes multi-period investment, the exponential growth rate for a strategy b_1^n is defined as follows.

$$W_n(b_1^n) = \frac{1}{n} \log S_n(b_1^n) = \frac{1}{n} \sum_{t=1}^{n} \log b_t^{\intercal} x_t \tag{3}$$

In most of the online portfolio selection algorithms, the goal is maximizing S_n. At the beginning of t^{th} period, based on the previous market window X_1^{t-1}, the investor designs the portfolio b_t for the following trading period. At the end of the period, the return of the selected portfolio is equal to $b_t.x_t$. This procedure is repeated until n^{th} period, and the strategy is finally scored according to the portfolio cumulative wealth S_n.

3.2 Clustering

The aim of clustering is to discover natural groups within data. A cluster refers to a set of data sharing common features. Clustering seeks to find structure within an unclassified dataset by placing data in groups so that the data in a group are significantly more similar to the data in their cluster from specific features compared to data in other clusters. Distance is the similarity criterion in clustering, representing heterogeneity, and helps move through the data space. Whether two data can be placed in the same cluster is indicated by calculating the distance between them and their proximity. Clustering is a way of unsupervised data mining, meaning that no initial labeling has been performed on the information.

Fuzzy C-Means clustering is the developed form of the K-Means method. As a similar logic in K-Means, first, a center for each cluster is set in the C-Means method, and the data are clustered based on their distance from the center. Fuzzy clustering allows each data to belong to several clusters with specific membership levels.

3.3 Risk Measure in Portfolio Optimization

As yet, in some algorithms based on the pattern-matching approach, the risk measure has been considered in the portfolio optimization step. Wang et al. [17] and later Sooklal et al. [16] proposed online risk-averse portfolio selection algorithms based on the CORN algorithm, named RACORN and DRICORN, respectively. The DRICORN algorithm has been exploiting upside risk while hedging downside risk. For this purpose, the beta (β) risk measure, which shows the sensitivity of each share or portfolio to the changes in the overall market, has been used. In this regard, when the market is expected to be bearish, a penalty is considered for the portfolio or stock with a high beta value to reduce the risk of portfolio depreciation, and vice versa when the market is expected to be bullish, a bonus is considered for the portfolio or stock with a high beta value to get maximum returns from positive market changes. Two challenging issues in this algorithm are measuring the sensitivity of stocks or portfolios to market changes and predicting whether the market will be bullish or bearish.

For every stock i, β is calculated as follows.

$$\beta_i = \frac{cov(R_i, R_m)}{var(R_m)} \tag{4}$$

Where R_i and R_m are the daily returns on stock i and the market index respectively.

Thus, it can be interpreted that the β has the same behavior as the regression coefficient in the linear regression model, and it actually indicates the magnitude and direction of changes in the return of the share or portfolio in comparison with market changes.

Also, the β for a portfolio with m stocks, is calculated as follows.

$$\beta_p = \sum_{i=1}^{m} w_i.\beta_i \tag{5}$$

Where w_i represents the weight of stock i in the portfolio.

In this algorithm, β has been incorporated by extending the objective function of the online portfolio selection model based on the market condition as follows.

$$\varepsilon_t(\omega, \rho, \lambda) = \arg \max_{b \in \Delta_m} \prod_{i \in C_t(\omega,\rho)} (b.x_i) \pm \lambda\beta_b \tag{6}$$

Where λ is the beta coefficient as a hyperparameter.

In fact, when a bullish market is expected, the $\lambda\beta_b$ is added to the model and when a bearish market is expected, it will be subtracted from the model in order to obtain the maximum profit from market changes.

In the next step, it is necessary to predict the market condition for the decision period that, according to the data classification, the market will be bullish or bearish in the future.

Seven methods for classifying data and predicting the market condition have been presented, which are divided into three general categories: pure price changes, current vs. lagged moving average analysis on cumulative returns, and moving linear regression analysis on price relative vectors. According to the investigation, the results of the moving linear regression method have been more consistent with reality compared to other methods.

4 The Proposed Algorithm

In this research, an online portfolio selection algorithm has been presented based on the pattern-matching principle. It makes decisions regarding the weight of each stock and performs buys and sales at the beginning of every period. The main output of the algorithm is the total return by the end of the final investment period.

The proposed algorithm is a risk-averse extension of the FCM-Log algorithm [5], which exploits upside risk while hedging downside risk. For this purpose, the beta risk measure has been used in portfolio optimization step.The beta risk measure estimates the portfolio sensitivity to the market. Hence, the proposed objective function in portfolio optimization step rewards high-beta stocks in bullish market, while penalises them in bearish market.

In order to evaluating the performance of the proposed algorithm, the closing price of the 10 most active stocks from the New York Stock Exchange, containing TAL, ET, ITUB, SWN, RIG, VALE, OXY, XOM, F , and BAC in the period from the beginning of December 2021 to the beginning of December 2022 have been extracted. 25% of the data was selected to test the algorithm, and the buy and sell transaction cost was assumed to be zero and 0.02, respectively.

Like all other pattern-matching algorithms, the presented algorithm is composed of the two steps of sample selection and portfolio optimization. The sample selection step deals with searching historical data for time windows that have behaved similarly to the recent time window.

In this regard, the price data matrix of P is entered into the model, and the price-relative matrix of X is developed by the model, which actually shows the return of each stock on each day. Then, considering the train ratio as the input of the model, the training and test data are separated, and based on the size of the time window (TW), the training matrix is split into sub-matrices with the size of TW.

In fact, the sub-matrices are formed in order to cluster and find sub-matrices with the same cluster as the recent sub-matrix. Considering that the input of the clustering algorithms is vectors, the sub-matrices created in the previous step are unwrapped and turned into a vector. In order to cluster the created vectors, a fuzzy clustering algorithm using Euclidean distance has been employed. Also, the number of optimal clusters is determined according to the number of clusters that result in the lowest value of the objective function in the clustering model.

The sub-matrices, placed into the same cluster with the recent sub-matrix, are regarded as similar to the recent sub-matrix, and the day after each similar sub-matrix is saved as the similar day in the matrix called matrix C. Next, the Euclidean distance of similar sub-matrices with the recent sub-matrix is calculated and a weight is assigned to each sub-matrix according to the distance. These weights are saved in a column vector W.

After finding the similar days and the weight assigned to each in the first step, the optimal portfolio of the following day is calculated in the second step according to the price-relative matrix of similar days and the corresponding weight vector.

In this step, according to whether the decision is related to the first day or other days, and also considering whether a similar day or days have been found in the first step, the following different situations are considered:

1. If no similar days have been found in the first step (C is null) and a decision is made regarding the first day, a uniform portfolio is chosen as the optimal portfolio.

2. If no similar days have been found in the first step (C is null) and a decision is made on a day other than the first day, no change is made to the previous day's portfolio. It means that no transaction is set to be made, and the optimal portfolio for the following day would be the adjusted portfolio at the end of the previous day.

3. If similar day(s) have been found in the first step (C is not null), the optimal portfolio for the following day is calculated by solving the following optimization model.

$$\begin{aligned} \max Z &= E\{\log (b \odot (1 - TC)).x \mid x_j, j \in C(x_1^t)\} \pm \lambda \beta_p \\ &= \sum_{j \in C(x_1^t)} w_j \log (b \odot (1 - TC)).c_j \pm \lambda \sum_{i=1}^{m} w_i \beta_i \end{aligned} \tag{7}$$

$$s.t. \sum_{i=1}^{m} b_i = 1$$

$$l \leq b_i \leq u, \ \forall i$$

In the above objective function, which is a log-optimal, the matrix C and the vector W are obtained from the first step, representing the price-relative matrix and the weight vector of similar days, respectively. TC is the vector of transaction cost rate, which is obtained as follows.

$$\begin{aligned} TC &= \mid \gamma \mid \odot \tau \\ \gamma &= B_{adj}(t-1) - b \\ \tau &= \begin{Bmatrix} TC\ Buy,\ \gamma < 0 \\ TC\ Sell,\ \gamma > 0 \\ 0,\ \gamma = 0 \end{Bmatrix} \end{aligned} \tag{8}$$

Also, β_p is incorporated into the model as a portfolio beta, along with the λ coefficient as a hyper-parameter. The $\lambda\beta_b$ term is added to the above model when the market is expected to be bullish and subtracted when the market is expected to be bearish to gain the maximum profit from market changes. In order to predict the market condition, a uniform, and exponential weighted moving linear regression is calculated on the data of the studied market index. Because of the higher importance of recent data, in the exponential weighted method, higher weight is given to the recent data. If the slope of the calculated regression is positive, the term $\lambda\beta_b$ will be added to the model, and if the slope is negative the term $\lambda\beta_b$ will be subtracted from the model.

By optimizing the above model, the value of b^* is obtained, which is the optimal portfolio for the following day.

Adjusted portfolio (B_{adj}). As the price of each stock in the portfolio has been changed during the day, the weight of each one in the portfolio will change without making any transactions. This new weight at the end of the day is called the adjusted portfolio. The adjusted portfolio is obtained as follows.

$$B_{adj}(t) = \frac{B^*(t) \odot x(t)}{B^*(t)x(t)} \tag{9}$$

It is worth mentioning that the size of the time window used in the algorithm is not predetermined, and the algorithm modifies the optimal time window size in each run according to the results of the optimization model in the previous periods.

As the next day passes and the price data of the market is revealed on that day, the return resulting from the decision for that day is determined, the current period is transferred to the training matrix, and this process is repeated until the end of the investment period. At the end, the evaluation criteria of the model are calculated.

In this research and according to the described algorithm, two algorithms of FCM-DRLog(UW) and FCM-DRLog(EW) have been presented. In these algorithms, uniform and exponential-weighted moving linear regression have been used, respectively, in order to predict whether the market will be bullish or bearish.

5 Results

In this section, the results of the proposed algorithm for real data and its results compared to other algorithms proposed in the literature so far, have been presented. For this purpose, the data and assumptions in Abdi et al. [5] have been used to implement the proposed algorithm. Also, in this study, the rate of treasury bonds is assumed to be 2.5% in order to calculate the annual Sharpe ratio.

In addition to the mentioned parameters that are similar to the past algorithms, some specific parameters of the proposed algorithm are assumed as follows:

- In calculating the portfolios beta, the data related to the NYSE Composite

	FCM-DRLog(EW)	FCM-DRLog(UW)
Total Cumulative Return (S_n)	1.28	1.27
Annual Percentage Yield (APY)	1.70	1.60
Annual Standard Deviation of Returns	0.45	0.53
Annual Sharpe Ratio (SR)	3.72	2.95

Table 1: The results of the proposed algorithm

(NYA) index has been used as the market index.

- The time period to calculate the linear regression on the NYSE Composite (NYA) index data has been considered 60 days.
- In weighted linear regression, exponential weighting is used so that the weight of recent data is greater than that of historical data.
- The value of λ, as the beta coefficient in the model, is assumed to be 0.001.

As shown in Table 1 , the total cumulative return of the portfolio (S_n) for the FCM-DRLog(EW) algorithm is equal to 1.28, and for the FCM-DRLog(UW) algorithm is equal to 1.27. Also, the annual percentage yield in the FCM-DRLog(EW) and FCM-DRLog(UW) algorithms is equal to 170% and 160% respectively.

Figure 1 and Figure 2 show the daily return and the daily cumulative return trend of the FCM-DRLog(UW) and FCM-DRLog(EW) algorithms respectively.

As mentioned in Table 1 , the annual standard deviation of returns of the portfolio is calculated as 0.45 in the FCM-DRLog(EW) algorithm and 0.53 in the FCM-DRLog(UW) algorithm. Additionally, the annual Sharpe ratio for the FCM-DRLog(EW) algorithm is 3.72, and for the FCM-DRLog(UW) algorithm is 2.95.

Considering that the proposed algorithm follows the pattern-matching principle, the results of the current algorithm are compared with other algorithms presented in the literature and benchmark algorithms to show the improvement in the results compared to previous ones. The comparison is presented in Table 2 and also shown graphically in Figure 3.

According to the results of previous algorithms in the literature in the total cumulative return of the portfolio (S_n), which are shown in Table 2 , the proposed algorithm has been outperformed, and the algorithms return is close to the benchmark algorithm, BCRP.

According to the APY criterion, which represents the annual percentage yield, if the algorithm has a continuous performance, the APY of the proposed algorithms will be higher than the previous algorithms.

Also, the annual standard deviation of returns of the portfolio for proposed algorithm, as a measure of risk, is lower than the benchmark algorithm, BCRP.

One of the most important criteria for evaluating models is the annual Sharpe ratio, which indicates the excess return of the risk-free return with regard to the

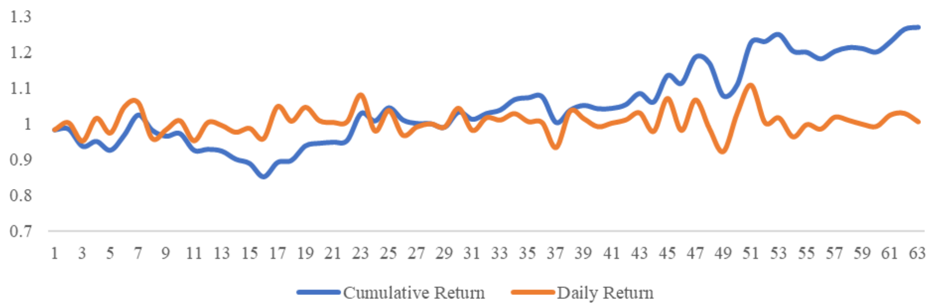

Figure 1: The daily return and the daily cumulative return trend of the FCM-DRLog(UW) algorithm

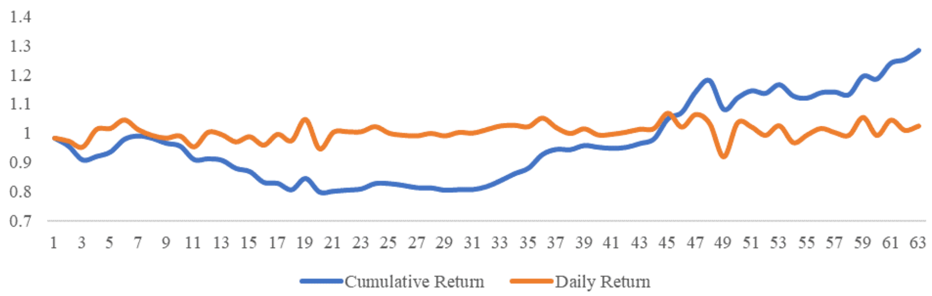

Figure 2: The daily return and the daily cumulative return trend of the FCM-DRLog(EW) algorithm

	Presented algorithm		Benchmark		Pattern-Matching algorithms				
	FCM-DRLog(EW)	FCM-DRLog(UW)	BCRP	Uniform-BAH	FCM-Log	BK	BNN	Corn-U	Corn-K
Total Cumulative Return	1.28	1.27	1.33	1.07	1.25	1.01	1.01	0.94	0.99
Annual Percentage Yield	1.70	1.60	2.11	0.32	1.45	0.04	0.04	-0.20	-0.05
Annual Standard Deviation of Returns	0.45	0.53	0.56	0.34	0.56	0.38	0.42	0.49	0.42
Annual Sharpe Ratio	3.72	2.95	3.71	0.86	2.54	0.04	0.03	-0.46	-0.17

Table 2: The results of the proposed algorithm and the comparison with previous ones in the literature

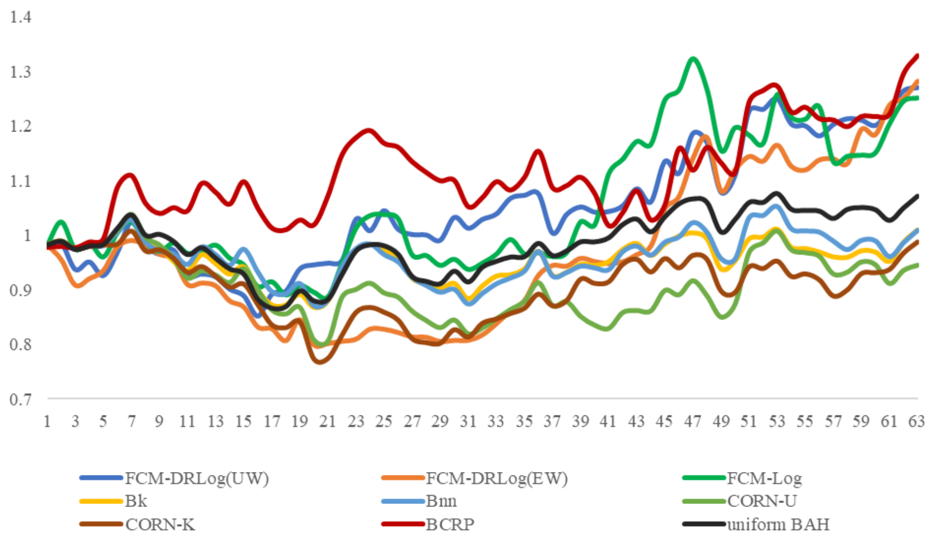

Figure 3: The daily cumulative return trend of the proposed algorithms existing ones in the literature

tolerated risk. The higher the value of the annual Sharpe ratio of the algorithm, the higher the return obtained compared to the tolerated risk. The proposed algorithm has the highest Sharpe ratio compared to other algorithms in the literature, and it is almost equal to the Sharpe ratio of the benchmark algorithm, BCRP.

6 Conclusions and Grounds for Future Research

In this research, an online portfolio selection algorithm has been presented based on the pattern-matching principle. This algorithm calculates the optimal portfolio for each day by using two steps: sample selection and portfolio optimization. In the first step, historical data is separated into time windows, and a fuzzy clustering algorithm is implemented in order to find similar days. In this regard, the time windows of the same cluster with the recent time window are considered similar, and the following days will be considered as days similar to the day set to be decided upon. In the second step, in order to optimize the portfolio, the log-optimal objective function has been used, taking into account the transaction cost and considering the beta risk measure. According to the bullish or bearish prediction of the market on the day of the decision, the beta risk measure is used in such a way that in the bullish market, the goal is to maximize the beta of the portfolio, and in the bearish market the goal is to minimize the portfolio beta so that the maximum return can be obtained in this way. In fact, in this research, the goal is to use the beta risk measure optimally in order to obtain maximum return.

In order to implement the algorithm, the New York Stock Exchange market data was used for 10 most active market stocks in the period from the beginning of December 2021 to the beginning of December 2022. According to the comparison of the proposed algorithm with other algorithms in the literature, the proposed algorithm has a better performance in terms of return, risk, and risk-adjusted return compared to the previous algorithms.

In order to provide suggestions for future research, considering other risk measures and minimizing them in the model can be used as a basis for future research. Also, considering stock liquidity in the model will lead to more realistic results. Using other data mining methods or considering other parameters in order to find similar data in the first step can also lead to higher efficiency.

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